APPLICATION OF FINITE FRACTURE MECHANICS TO COMPOSITES: CRACK ONSET IN A STRETCHED OPEN HOLE POLYMER PLATE WITH NONLINEAR BEHAVIOUR

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Abstract
Fracture of open hole polymer plates in tension is addressed by the coupled stress-energy criterion of Finite Fracture Mechanics, using Finite Element simulations with different material models. Although the final objective is to understand the behaviour of stretched open hole polymer matrix plates, in the present work, linear elastic and nonlinear elastic material models of PMMA are studied. Predictions of the size effect on the critical load obtained are compared with experimental results.

1. Introduction

The actual failure criteria for composites are still not able to fully characterize and predict the onset of the damage in form of cracks or the crack propagation in these materials. Finite Fracture Mechanics (FFM) [1] introduces a new approach to characterize crack onset and may provide a new insight into the analysis of the failure mechanisms in composites.

In the present work, the coupled stress and energy criterion of FFM is applied to characterize damage in a simple configuration: a polymer rectangular plate with a central hole subjected to uniaxial tension at the outer boundary. This study aims to contribute to a deeper understanding of crack onset phenomenon at a stress concentration in a polymer matrix. This could help to understand matrix dominated damage in composites, e.g. in the case of transverse loading, in the presence of holes, where it is expected that the matrix is going to fail first. In previous works, [2, 3, 4, 5, 6, 7], the coupled criterion of FFM was applied to plates with holes in tension, assuming linear elastic material model. In the present work, a nonlinear elastic model will be introduced, and the application of the coupled criterion of FFM will be addressed by means of Finite Element Analysis. Finally, in order to elucidate the size effect on failure load, a comparison of the semi-analytic and numerical predictions with the experimental results obtained for several hole diameters is done.

2. Material, plate geometry and experimental results

2.1. Material and plate geometry

The plate material is Polymethyl Metacrylate (PMMA), an amorphous thermoplastic polymer. To characterize properly the material, standard tension and fracture tests were performed. Finally, tests of open hole plates under tension were carried out up to their failure. The dimensions of the open hole plates, see Figure 1 a), were: W = 40 mm, L = 300 mm, and hole diameters, 2R = 0.5, 1, 2, 3, 4.25, 7 and 10 mm. The plate thickness, t, was between 7.4 and 8.8 mm, due to plate thickness variation [5]. Part b) of the same figure, will be referred later on this work, where the coupled criterion of FFM is addressed.

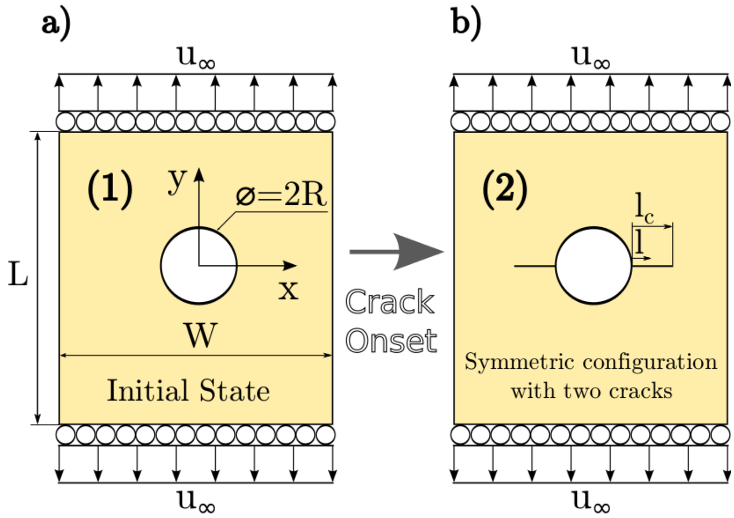
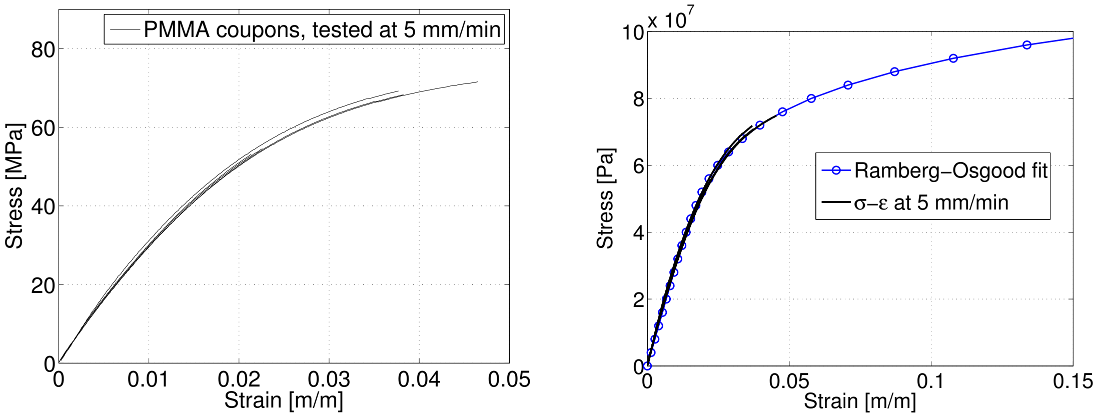


Figure 1: a) Plate geometry and loading system. b) Plate after symmetrical crack onset.

2.2. Material properties and material models

In order to define elastic properties of the PMMA tested, the standard ASTM D638-03 was used as guideline. Thus, five coupons of type III were machined and tested under tension at 5 mm/min velocity. The temperature and relative humidity was controlled to be 22 °C and 50 %, respectively. The thickness of the each specimen was around 7.4 mm. Figure 2 a) shows the stress-strain curves obtained from the tension tests. It can be seen that the curve is not completely linear, namely for high strains a clear deviation from the linear elastic behaviour is observed. The Young's modulus found, in the initial linear elastic part of stress-strain curves, had an average value of 2.82 GPa (it was calculated from the initial slope of stress-strain curve for each specimen). For the numerical studies this Young's modulus value was slightly changed to 3 GPa to achieve a better fit of the Ramberg-Osgood curve. The average failure stress was 63.4 MPa and Poisson ratio was 0.34.

Two material models were employed in the numerical studies: (1) Linear elastic and (2) Nonlinear elastic. The material properties of the linear elastic model have been given above. The second approach was addressed due to the visible nonlinear behaviour of PMMA tension test results, as depicted in Fig. 2 a). The tension tests data were fitted by the power-law following



(a) $\sigma - \epsilon$ curves at v = 5 mm/min. (b) Ramberg-Osgoog fit to $\sigma - \epsilon$ curves

Figure 2: Stress-strain curves at v = 5 mm/min and the Ramberg Osgood power-law fit to these curves.

the Ramberg-Osgood relationship,

$$\epsilon = \frac{\sigma_y}{E}\left(\frac{\sigma}{\sigma_y} + \chi\left(\frac{\sigma}{\sigma_y}\right)^n\right) \tag{1}$$

see Fig. 2 b), slightly adjusting PMMA properties from [8]. The data used in the Ramberg-Osgood curve fit are E = 3 GPa, $\nu = 0.34$, $\sigma_y = 53$ MPa, $\chi = 0.164$ and n = 6.5.

The fracture properties were obtained from compact-tension specimens, according to ASTM D5045-07. The overall resultant values for fracture energy are somehow low [5]. The highest value was 270.88 J/m^2, giving $K_{Ic} = 0.75$ MPa $\sqrt{m}$. This is in agreement with the minimum values for PMMA in the bibliography and it will be taken as basis for the analytic/numerical calculations.

2.3. *Fracture tests for open hole plates*

A batch of open hole specimens was tested in tension. Some of the broken specimens are shown in Figure 3. Five coupons for each diameter were fabricated.

(a) $\varnothing 3 - Nr.4$ (b) $\varnothing 7 - Nr.5$ (c) $\varnothing 10 - Nr.3$

Figure 3: Fracture aspect of plates with holes with 3, 7 and 10 mm diameter.

Also straight coupons without hole were tested in order to evaluate the fracture stress of those coupons. The test velocity was 5 mm/min in order to be comparable to the tension tests. The test machine had a 100 kN load cell. The rupture was abrupt and the fractured surface had no indications of evident plastification.

3. Coupled stress and energy criterion

Referring to Figure 1, a coupled criterion, which combines energy and stress conditions, has been used [1] in order to predict the remote stress needed to originate the onset of two transverse symmetrical cracks, at the hole border.

3.1. Stress criterion

The stress criterion assumes the onset of cracks when the vertical component of the stresses near the hole, σ_y, along the symmetry axis of the plate (x), for $0 \leq l \leq l_c$, is higher or equal than the critical stress σ_c (material tensile strength):

$$\sigma_y(l) \geq \sigma_c \quad , \quad 0 \leq l \leq l_c \tag{2}$$

3.2. Energy criterion

On the other hand, the energetic counterpart is imposed by the condition for the incremental Energy Release Rate, given by

$$G_{incr} = -\frac{\Delta\Pi}{\Delta A} \geq G_c, \qquad \text{being} \qquad \Delta\Pi = \Pi_2 - \Pi_1 \tag{3}$$

where $\Delta\Pi$ is the potential energy decrease, due to crack advance at each side of the hole of the plate (see Figure 1), ΔA is the variation of crack area. G_c is the critical Energy Release Rate.

3.3. Coupled criterion for linear elastic material model

The stress criterion, for the linear elastic model, uses Kirsch's expression, for stresses along the symmetry axis (perpendicular to the load) of the infinite plate, with a circular hole:

$$\sigma_c \leq \frac{\sigma_\infty^c}{2}\left(2 + \frac{R^2}{(R+l)^2} + 3\frac{R^4}{(R+l)^4}\right) \quad , \quad 0 \leq l \leq l_c \tag{4}$$

The energy criterion, for this case, is given by the following condition for the onset of crack propagation for half of the plate (one crack):

$$G_{incr}\Delta A = \int_0^{l_c} G(l)t\mathrm{d}l \geq G_c t l_c \tag{5}$$

Stress intensity factor in Mode I, K_I, for two cracks having length l emanating from a hole, is given by Newman's expression:

$$K_I = \sigma_\infty^c \sqrt{\pi l}\left(1 + 0.5\frac{R}{R+l}\right)\left[1 + 1.243\left(\frac{R}{R+l}\right)^3\right] \tag{6}$$

and is used in Irwin's relation between ERR (G), Young Modulus (E) and K_I, for plane stress and strain. The expression in (6) can be substituted into inequality (5).

Due to the dependence of G on the square of K_I, and consequently on the square of σ^c_∞, one can write $G = (\sigma^c_\infty)^2 \hat{G}$. By considering a coupled criterion, defined by the previous conditions, we obtain the system of two nonlinear equations for two unknowns σ^c_∞ and l_c. Eliminating σ^c_∞ from the system we arrive to the following nonlinear equation for the unknown l_c:

$$\left(\frac{2\sigma_c}{\left(2 + \frac{R^2}{(R+l_c)^2} + 3\frac{R^4}{(R+l_c)^4}\right)} \right)^2 \int_0^{l_c} \hat{G}(l)\mathrm{d}l - G_c l_c = 0 \tag{7}$$

This equation was solved numerically by the computer algebra software *Mathematica*. Once the value of l_c is computed, the corresponding value of σ^c_∞ is easily obtained from (4) or (5) considered as equalities.

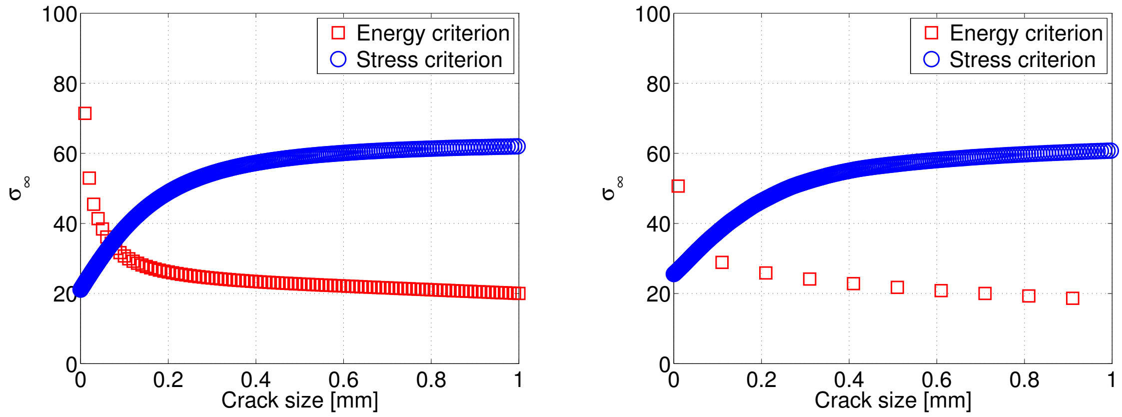
3.4. Coupled criterion for nonlinear elastic material model

In the nonlinear (NL) elastic Finite Element (FE) model (one quarter of the plate with symmetric conditions), data were computed for each u_∞ (several displacement increments) until a final displacement of 3.9 mm (approximately half of the measured total displacement in the experimental tests). For each u_∞ the corresponding σ_∞ is computed.

Referring to the stress criterion, the nodal stresses, σ_y, were obtained along the potential crack line, for each crack size and for each displacement increment u_∞. In order to find the critical σ_∞ for each crack size, a polynomial interpolation of the nodal values of σ_y was made for each crack, and the condition (2) was used to find the corresponding critical σ_∞, for each crack size.

Referring to the energy criterion, also by using the FE model, it was possible to calculate Π_1 and Π_2 for the respective cases of uncracked (1) and cracked (2) plate (see Figure 1). This was accomplished by calculating the resultant of nodal forces at the top plate border, F_∞, for the respective displacement, u_∞, and for the several displacement increments. $\Delta\Pi$ was then computed for all the displacement increments and each crack. It was multiplied by a factor of two, in order to include the energetic effect of the half of lower crack face and plate. After a polynomial interpolation was made for each crack, with the values of $\Delta\Pi$ and using $\Delta\Pi = -G_c \Delta A$, the corresponding critical σ_∞, for each crack size was found.

After that, the FE model was used to validate the linear elastic predictions by the coupled criterion made with *Mathematica*. The results for each of the calculated critical stresses, for each crack size and for the linear and nonlinear elastic cases are plotted in Figure 4 a) and b), respectively. The coupled criterion prediction for the critical load is obtained from the intersection of the two curves, giving the pair (σ^c_∞ , l_c). For the moment, due to computational time and converging issues, for the nonlinear elastic case we have results for the first six diameters and ten crack lengths ranging from 0.01 mm until 1 mm, increased by 0.1 mm. For the linear case, the study was made for one hundred cracks, starting at 0.01 mm and ending at 1 mm, with steps of 0.01 mm.

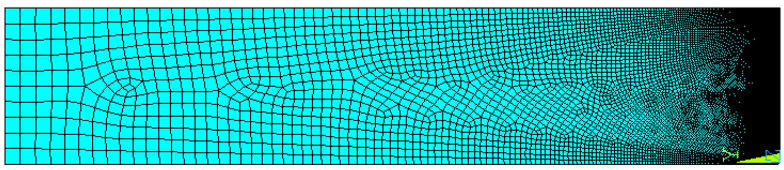


(a) Linear Elastic FE (b) Nonlinear Elastic FE

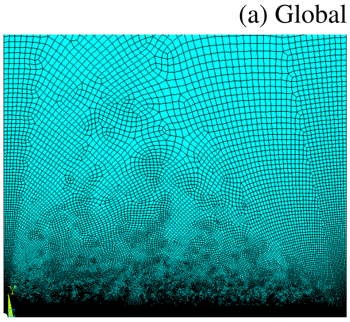
Figure 4: Energy and stress criterion curves of FFM for linear and nonlinear elastic models for ⌀0.5 mm.

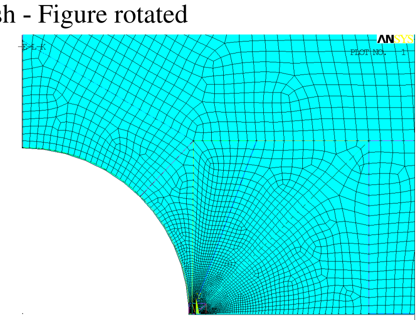
4. Numerical Model

The numerical studies were carried out by using *Ansys* software, with APDL scripts. The insertion of the nonlinear elastic true stress-strain points was made by using the Multilinear Elastic (MELAS) material model table, where up to one hundred data points are possible to be inserted. Due to symmetry, only one quarter of the geometry was modelled. Linear and quadratic plane elements were used and convergence studies done. The quadratic elements, PLANE183, were preferred for subsequent studies due to better convergence results. The mesh was completely programmed in *Matlab* software, the line element sizes and geometrical progressions for all the lines were automatically set for a desired mesh density. As an example, for the case of 0.5 mm hole diameter and crack length 0.01 mm, a sufficiently refined mesh has 72979 elements and 220950 nodes (see Figure 5).



(a) Global mesh - Figure rotated





(b) Lower part plate detail (c) 0.5 mm hole mesh detail

Figure 5: FE mesh and mesh details.

5. Comparison of predictions and experiments. Conclusions

In Figure 6 the failure stresses for open hole plate obtained from the experimental tests, numerical model results and analytical results, for plane stress (p. stress) and plane strain (p. strain), are compared. One can observe some hole size effect: the failure stress decreases with increasing hole diameter, although in the holes with diameter of 7 mm some alteration of this tendency is seen. The experimental results for this diameter have small dispersion, similarly as the specimens with 0.5 and 1 mm hole diameter, whereas the results for the rest have more dispersion.

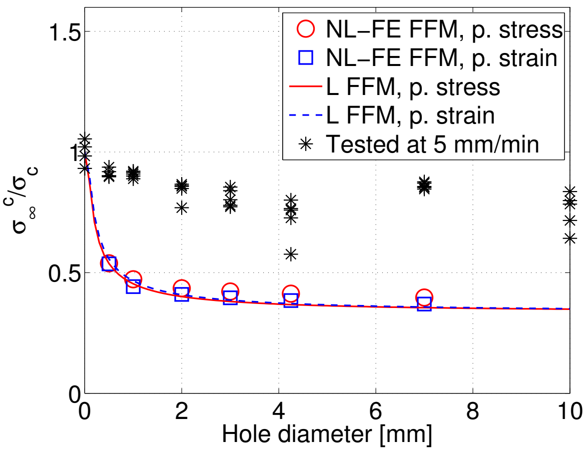
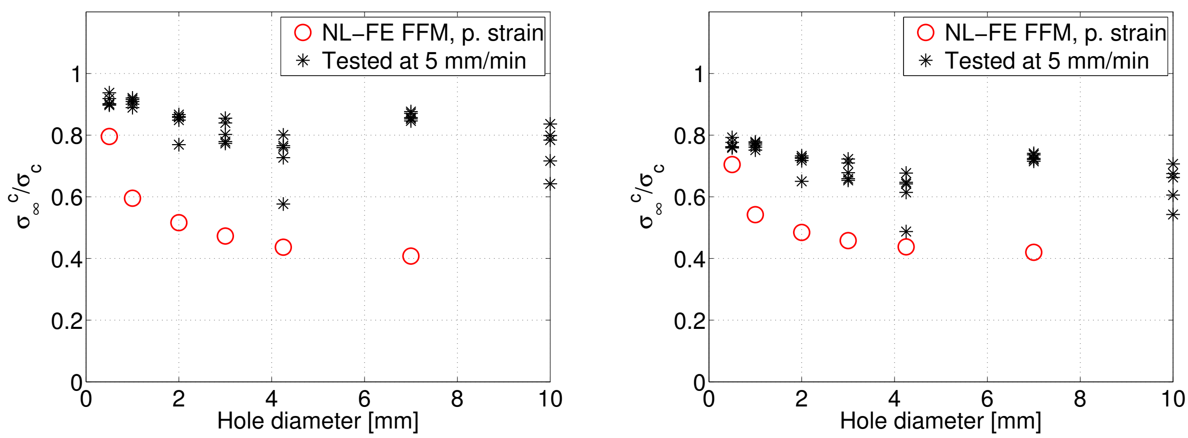


Figure 6: Comparison of the FFM predictions using linear (L) and nonlinear (NL-FE) elastic material models in plane stress and plane strain with experiments.

Due to the poor correlation of numerical and experimental results observed, an attempt to modify strength and fracture toughness data, in order to better capture the hole size effect, was done. Other authors [3, 9, 10] also made similar simulations, justifying some toughening and hardening effects, due to blunt stress concentrators and nonlinear effects of PMMA. Those results are presented in Figure 7. A better correlation of results is obtained.



(a) NL-FE: Gc=1150 J/m^2, σ_c =63.4 MPa

(b) NL-FE: Gc=1150 J/m^2, σ_c = 75 MPa

Figure 7: Comparison of FFM predictions using nonlinear (NL-FE) elastic material model, with modified strength and fracture toughness data, and experimental results.

In view of a quite large difference between the predictions and experimental results, some modifications of the nonlinear elastic model will still be carried out, and also other material rheologies will be explored, trying to achieve an agreement between predictions and experiments. In particular some linear viscoelastic, elastoplastic and viscoplastic material models will be considered, in order to better understand the behaviour of a polymer material in the neighbourhood of stress concentration points and about its influence on the failure load, and specially on the corresponding size effect.

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