CRACK ONSET AND PROPAGATION ALONG FIBRE-MATRIX ELASTIC INTERFACES UNDER BIAXIAL LOADING USING FINITE FRACTURE MECHANICS

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Keywords: weak interface, LEBIM, FFM, coupled stress and energy criterion, biaxial loads

Abstract
A fracture criterion able to predict the failure loads producing crack onset and propagation along weak interfaces between solids is presented. The procedure is based on the Linear Elastic-(Perfectly) Brittle Interface Model (LEBIM) combined with a Finite Fracture Mechanics (FFM) approach. The procedure imposes the simultaneous fulfillment of two criteria to produce crack onset and/or growth: for the incremental energy release rate due to the debond originated between the solids and for the stresses at the potential crack path before the crack onset. Each criterion, by itself, represents a necessary but not a sufficient condition to produce the debond. The procedure is implemented in a 2D Boundary Element Method (BEM) code. The interface between the solids is modelled by a continuum spring distribution. This approach offers an improvement in relation to other methods already existing, which is an adequate characterization of the interface stiffness. The present approach also allows to model a debond in mixed fracture mode, thus onset and propagation of a debond at a fibre–matrix interface under biaxial transversal loads can be studied.

1. Introduction

Debonds among the fibres and matrix in a unidirectional composite laminate under loads transverse to the fibres is a frequent failure mechanism and as such it has been deeply and extensively studied by many authors. This is the reason why an adequate modelling of the interface becomes an important issue, when the failure in a composite is studied. An extensive review of works that studied this problem can be found in [1, 2] and references therein. Following the work of several authors, see [2, 3, 4, 5], in the present work the interface is modelled as continuum linear elastic-brittle spring distribution. Normal and shear stresses in the undamaged springs are proportional to relative normal and tangential displacements, respectively. The model includes a brittle failure criterion, as in [2, 4, 5]. This approach is usually referred to as the Linear Elastic-(Perfectly) Brittle Interface Model (LEBIM). The aim of the present work is to develop and implement in a computational code a new LEBIM formulation able to predict debond onset and propagation at an interface of an isolated fibre–matrix system under remote transverse loads.

The novelty introduced in the LEBIM is based on recent works [6, 7], where a coupled stress and energy criterion of the Finite Fracture Mechanics (FFM) is applied to the present problem but considering a perfect fibre-matrix interface, and work [8], where the coupled criterion of FFM is applied in the LEBIM in a semianalytical study.

2. Finite Fracture Mechanics applied to Linear Elastic-Brittle Interface

LEBIM, originally proposed in [2, 4] is a model able to predict interface debonds between two solids. Nevertheless, LEBIM may not be able to characterize adequately a problem with a very stiff interface due to the fact that the fracture toughness, interface stiffness and the maximum critical stress are directly related in the original LEBIM criterion. As mentioned above, Mantič and García [6, 7] and Cornetti et al. [8] applied the FFM concepts to perfect and linear elastic interfaces, respectively, and following these works the original LEBIM proposal can be improved. The present novel approach is based on the coupled criterion of stress and incremental energy release rate, each of them representing a necessary but not sufficient condition to produce crack onset and propagation.

In the present model the interface is characterized by a spring distribution whose normal and shear stiffnesses are defined as k_n and k_t, respectively. So the normal and shear stresses σ and τ, at a point x on an undamaged part of the interface are proportional to the relative normal and tangential displacements (δ_n and δ_t):

$$\sigma = k_n \delta_n, \quad \text{and} \quad \tau = k_t \delta_t. \tag{1}$$

Therefore, the energy stored in a spring (per unit area) and which can be released is given as

$$G = G_I + G_{II}, \quad \text{where} \quad G_I = \frac{\langle \sigma \rangle_+^2}{2k_n} \quad \text{and} \quad G_{II} = \frac{\tau^2}{2k_t}. \tag{2}$$

The stress and energy based fracture-mode-mixity angles are defined, respectively, as

$$\tan \psi_\sigma = \frac{\tau}{\sigma}, \qquad \text{and} \qquad \tan^2 \psi_G = \frac{G_{II}}{G_I} = \frac{k_n}{k_t} \tan^2 \psi_\sigma \quad (\text{for } \sigma \geq 0). \tag{3}$$

In order to produce a crack onset and propagation the following incremental energy criterion must be fulfilled:

$$\int_0^{\Delta a} G(a)\,\mathrm{d}a \geq \int_0^{\Delta a} G_c(\psi(a))\,\mathrm{d}a, \tag{4}$$

where $G(a)$ is the Energy Release Rate (ERR) associated to the crack tip at the position $x = a$, essentially it equals the energy (per unit area) stored at the spring located at the crack tip, cf. [3, 9], and is defined by (2) using the tractions $\sigma(a)$ and $\tau(a)$ at the crack tip. $G_c(\psi(a))$ gives the fracture toughness (fracture energy) associated to the crack tip at the position $x = a$. Function $G_c(\psi)$ is usually defined by a phenomenological law, as, e.g., the following one [10]:

$$G_c(\psi_G) = G_{Ic}(1 + \tan^2(1 - \lambda)\psi_G), \tag{5}$$

with G_{Ic} denoting the fracture toughness in pure mode I, λ is the fracture mode sensitivity parameter ($0.2 \leq \lambda \leq 0.3$ is the typical range for interfaces with moderately strong fracture mode dependence).

Besides, for crack onset and propagation a stress criterion must be fulfilled too, the following one being used in the present work:

$$\min_{0 \le x \le \Delta a} \frac{t(x)}{t_c(\psi(x))} \ge 1, \tag{6}$$

where the traction vector modulus at a point x and its critical value are,

$$t(x) = \sqrt{\sigma(x)^2 + \tau(x)^2} \quad \text{and} \quad t_c(\psi(x)) = \sqrt{\sigma_c(\psi(x))^2 + \tau_c(\psi(x))^2}. \tag{7}$$

In the present work, following [2, 5], the energy fracture-mode-mixity angle is used defining

$$\sigma_c(\psi_G) = \bar{\sigma}_c \sqrt{1 + \tan^2[(1-\lambda)\psi_G]} \cos\psi_G \quad \text{and} \quad \tau_c(\psi_G) = \sqrt{\frac{k_t}{k_n}} \bar{\sigma}_c \sqrt{1 + \tan^2[(1-\lambda)\psi_G]} \sin\psi_G, \tag{8}$$

with $\bar{\sigma}_c$ being the critical stress for pure mode I. The present problem is governed by the following dimensionless parameter defined in [8]:

$$\mu = \frac{2k_n G_{Ic}}{\bar{\sigma}_c^2}. \tag{9}$$

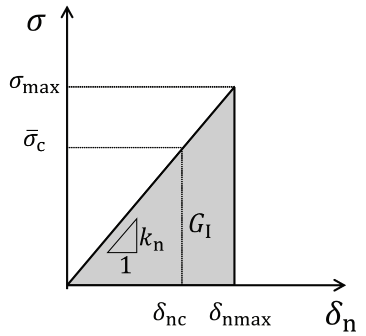

Figure 1. FFM+LEBIM law for pure mode II.

According to Fig. 1, $\mu = \sigma_{\max}^2/\bar{\sigma}_c^2$, with $\sigma_{\max}$ and $\bar{\sigma}_c$ being, respectively, the maximum and critical stresses associated to the energy and stress criteria. When $\mu = 1$, the solution of the present model should revert to the solution of the original LEBIM. For increasing μ, the interface becomes stiffer, so $\mu \to \infty$ leads to the perfect interface. As follows from the above, fracture toughness, strength and stiffness of the interface are independent in the present FFM+LEBIM model, in opposite to the original LEBIM, where these quantities are related by an equation.

3. Cylindrical inclusion under under biaxial transverse loads

A plane strain problem of an infinite fibre embedded in a matrix is considered, an undamaged interface is considered initially and the fibre-matrix system is subjected to a biaxial remote loading. Fibre radius a and $2H$ side square matrix with $H/a = 200/3$ are used, see Fig. 2.

Both the inclusion and matrix are considered to be isotropic linear elastic materials, whose characteristics are presented in Table 1. The value of k_n given in Table 1 corresponds to $\mu = 1$. For larger values of μ, k_n increases proportionally. The applied remote loads, σ_x^∞ and σ_y^∞ with $\sigma_x^\infty \ge \sigma_y^\infty$, are shown in Figure 2.

	E_f(GPa)	ν_f	E_m(GPa)	ν_m	G_{Ic}(Jm^{-2})	$\bar{\sigma}_c$(MPa)	k_n(MPa/μm)	k_t/k_n
Glass-Epoxy	70.8	0.22	2.79	0.33	2	90	2025	0.25

Table 1. Material and interface properties (k_n for $\mu = 1$)

The following general load-biaxiality parameter is used to represent the biaxility relation between the remote loads:

$$\chi = \frac{\sigma_x^\infty + \sigma_y^\infty}{2\max\{|\sigma_x^\infty|, |\sigma_y^\infty|\}}, \quad -1 \leq \chi \leq 1, \tag{10}$$

To solve this problem a Boundary Element Method (BEM) code is used. The code is a modification of the code developed in [2, 4, 5]. A uniform mesh of linear boundary elements is used to discretize the fibre-matrix interface, each element corresponding to a polar angle 0.1°.

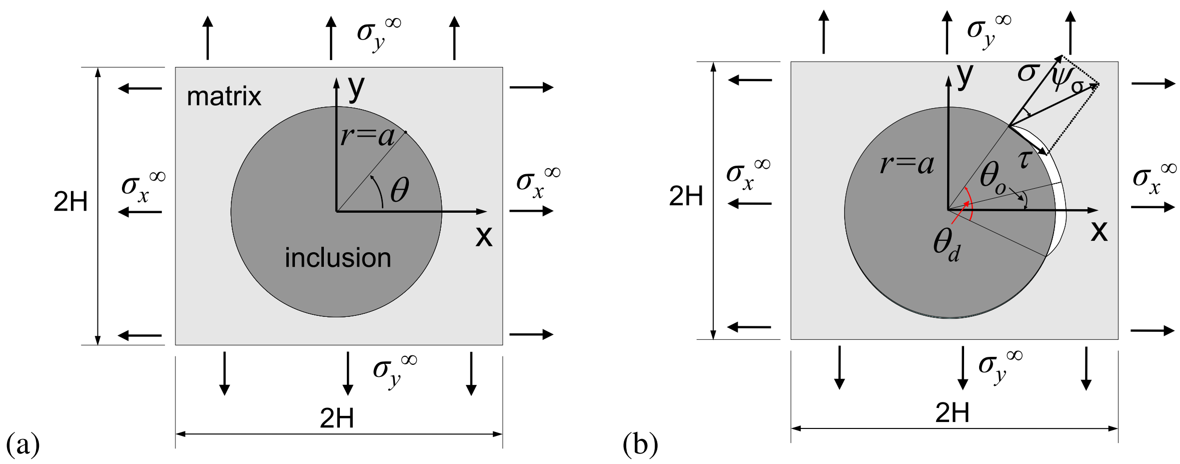

Figure 2. Inclusion problem configuration under remote biaxial transverse tension (a) without and (b) with a partial debond.

The position where debond onset initiates is defined by the polar angle θ_o. The debond length is denoted by the polar angle θ_d, see Fig. 2(b). It should be noted that debond growth is not always symmetric, thus θ_o angle may not be in the centre of the arc defined by θ_d in some stages of the debond growth.

In each step, of the present crack advancing procedure, the intersection point of the curves representing the stress and energy criteria defines the critical remote stress $\sigma_c^\infty > 0$ and the length (polar angle) of crack tip advance $\Delta\theta$. Then, a next step of the procedure begins. The first two steps of this crack advancing procedure are depicted in Fig. 3(a), for μ =4 and χ =0.5 (uniaxial case), where it is seen how the curves of stress and energy criteria advance with each $\Delta\theta_i$, subscript i representing the step number.

The predicted evolution of the onset and propagation of a crack along the fibre-matrix interface for χ =0.5 is shown in Fig. 3(b), for different interface stiffnesses, μ =2 and 4. In this graph the results are compared with the original LEBIM with μ =1. In the first steps for μ greater

(a) (b)

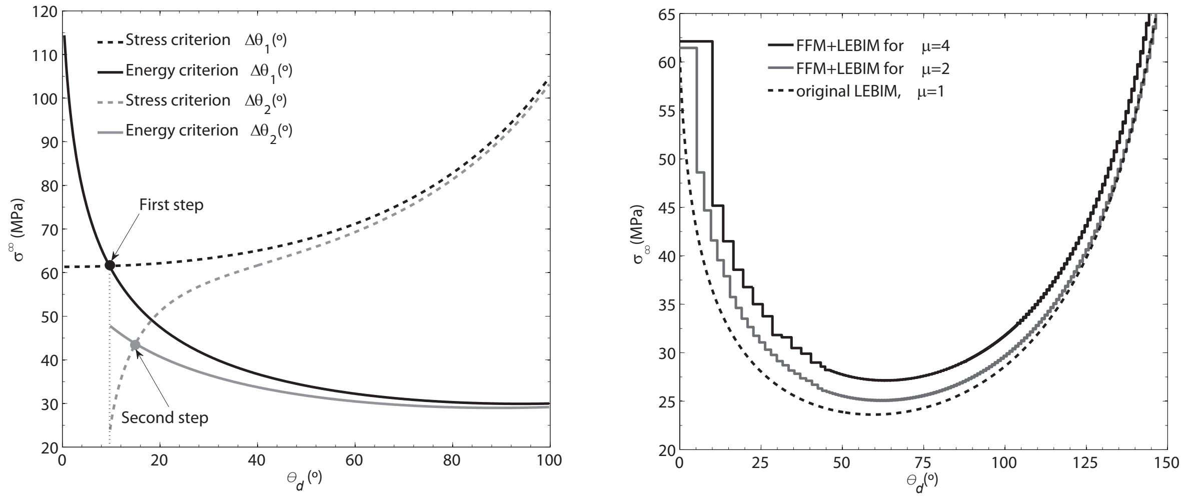

Figure 3. (a) First two steps of the present crack advancing procedure for a cylindrical inclusion embedded in a matrix under a transverse tension for μ =4 and χ =0.5 (uniaxial case).(b) Applied remote stress with respect to semidebond angle for different μ values with χ =0.5 (uniaxial case).

than one, the jumps due to the intersection of the stress and energy criteria curves are clearly observed, but when the solution arrives to the minimal remote stress, the criteria curves do not intersect, and the minimum remote stress that verifies both criteria is typically given by the minimum of the energy criterion curve.

Fig. 3(b) shows that the results obtained by the FFM+LEBIM do not vary significantly with respect to the results obtained by the original LEBIM, even for larger values of μ. In particular, the maximum critical load and the final debond angle achieved under load control present only small variations.

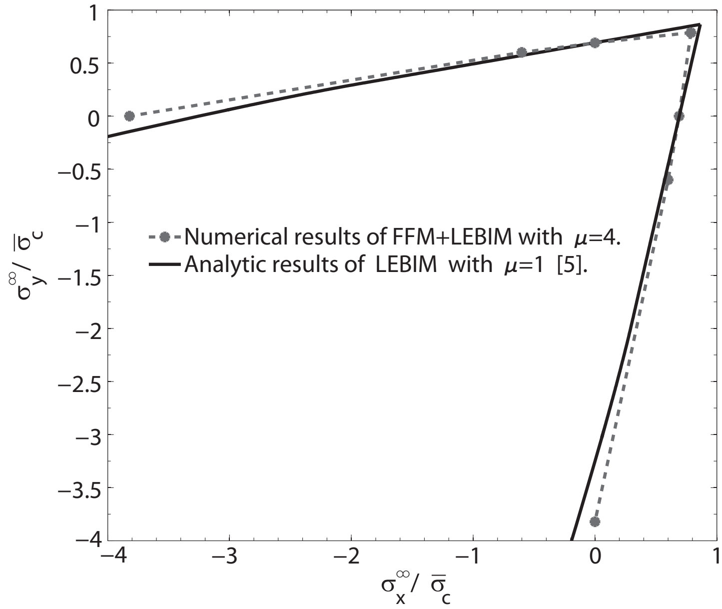

Figure 4. Failure curves for a glass fibre embedded in an epoxy large matrix under biaxial transverse loads.

	χ =0.75	χ =0.5	χ =0	χ =-0.25	χ =-0.5
μ =1	0.00°	0.00°	0.00°	0.00°	13.20°
μ =2	0.00°	0.00°	0.00°	6.00°	17.85°
μ =4	0.00°	0.00°	9.15°	16.05°	20.55°

Table 2. Debond onset angle θ_o for different μ and χ

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Fig. 4 shows the failure curve obtained for the normalized remote biaxial loads which cause a debond at an initially undamaged inclusion-matrix interface with $\mu = 4$. The curve is compared with the analytical curve obtained by LEBIM with $\mu = 1$ in [5] using Gao's elastic solution [11]. Surprisingly only small differences are observed in the results when the interface stiffness is increased, while the tendencies in both curves are the same. The failure curves show that a secondary compressive load makes easier the debond onset originated by a primary tensional load.

In Table 2, the position where the debond onset initiates (angle θ_o) for different μ and χ values is presented. It can be observed that initially the onset position is symmetrical ($\theta_o = 0°$). Nevertheless, when the interface stiffness increases (μ value increases) and a secondary compressive far field load becomes larger, the change of position becomes more evident too. This change in position was also observed in previous works by Correa et al. [12] and Mantič et al. [5], where the influence of a secondary transverse load in the debond onset and propagation between fibre and matrix was also studied.

4. Conclusions

A new computational procedure combining the FFM and LEBIM has been developed. It opens new possibilities to study the onset and propagation of cracks along interfaces and adhesive layers using realistic values of strength, fracture toughness and in particular layer stiffness, which can be significantly higher than in the original LEBIM (corresponding to $\mu = 1$). It is interesting to observe that for the present fibre-matrix system the predictions of the crack onset and propagation obtained by FFM and LEBIM differ only slightly from those obtained by the original LEBIM, which indicates only a moderate dependence of these predictions on the interface stiffness. The failure curves obtained show that the presence of a secondary compressive load makes easier the debond onset.

Acknowledgements

This work was supported by the Spanish Ministry Economy and Competitiveness (Project MAT2012-37387), the Junta de Andalucía and the European Social Fund (Projects P08-TEP-04051 and P12-TEP-1050).

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