

## A PRACTICAL APPROACH TO USE SINGULAR STRESS STATES IN THE DESIGN OF COMPOSITE ADHESIVE JOINTS

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### Abstract

*Composite to metal adhesively bonded joints generates critical points where geometry and material properties change abruptly. These points (multimaterial corners) are potential locations for failure initiation. There exist proposals predicting the initiation of failure at these multimaterial corners using a Fracture Mechanics approach, in which the Generalized Stress Intensity Factors (GSIFs) at the multimaterial corner control the failure initiation. The calculation of these GSIFs at anisotropic multimaterial corners involves not-straightforward calculations of the stress singularities and characteristic angular functions, numerical modelling of the joint and a careful postprocessing of the results. In this work a parametric Finite Element Analysis has been carried out allowing the generation of plots to calculate the GSIFs for unit values of the axial force, shear force and bending moment at one end of the overlap length. These results allow calculating the GSIFs at the multimaterial corners by a simply beam analysis of the joint, the use of these plots and the application of the superposition principle, for their use in the prediction of failure initiation by means of the singular parameters of the joint.*

### 1. Introduction

In previous works [1-6], the authors have studied the problem of failure initiation in adhesive joints with composite materials. The approach followed in these works lies in the idea that failure initiation in these type of joints is controlled by the presence of critical points, where material properties and geometry change abruptly (multimaterial corners).

In particular [1] characterizes the singularities of the stress state (stress singularity exponents and characteristic angular functions) in these multimaterial corners, under the assumption of generalized plane stress, and using the Stroh formalism [7] for anisotropic elasticity.

In [2] a simple and numerically robust procedure was developed and implemented for the calculation of the Generalized Stress Intensity Factors (GSIFs in what follows) in these corners, allowing several, real and complex, GSIFs to be determined simultaneously by means of the displacement and/or stress solution of a numerical model, using Finite or Boundary elements models (FEM/BEM). A comprehensive description of this stress characterization can be found in [3].

In [4,5] the allowable values of the GSIFs (the Generalized Fracture Toughnesses) were obtained by means of the proposal of a test, a modification of the Brazilian test, having the corner configuration included in the test specimen, see also [6] for a detailed explanation of the experimental procedure and results. The complete detailed description and definition of the entire procedure can be found in [8].

The previous outline of the entire procedure is intended to give an idea of the typical complexity of the procedure to characterize the local stress field, its applicability and use for designers being not straightforward. The present work is focused to give a more practical use to the mentioned approach, avoiding the calculation of all the intermediate complex steps.

The stress singularities can be characterized for typical multimaterial corners and it is not a complex task to create a list (or database) of common material configurations. The most complex task lies in the calculation of the GSIFs as it implies the generation of a detailed numerical model and the careful postprocessing of the results.

In particular, the objective of the present work is an alternative simplified procedure (based on the rigorous one, already developed and described previously) to determine the GSIFs. This objective is achieved performing a parametrical analysis (using FEM) of typical joint configurations, in which several geometrical parameters have been included.

Different overlap lengths, or adherent thickness, etc. have been studied and the full procedure has been used to determine the GSIFs of the corner. All this geometrical variations have been loaded using unit values at one end of the joint of the axial force, of the shear force, and of the bending moment. Then, a graphical representation is done in plots for unit values of these loadings, finally presenting all the GSIFs results for all of the geometrical parameters.

These plots allow, in a much more simply and direct way, to obtain the GSIFs in these multimaterial corners, by simply performing a beam analysis of the joint, obtaining the axial and shear forces and bending moment, using the plots and the superposition principle.

These plots have to be obtained for each material combination of materials in a particular adhesively bonded joint, but once the work is done, it is only necessary to perform a quick beam analysis of the joint.

Examples of these plots are presented in the present work for single- and double-lap joints between a composite material (unidirectional carbon fiber laminate) and a aluminum plate.

In Section 2 the parametrical model will be detailed and in Section 3, results for the selected material and geometrical configuration are presented.

## **2. Problem definition and parametrical study**

The problem under study is an adhesively bonded joint (single- and double-lap joints are considered) between a unidirectional carbon fiber reinforced laminate and an aluminum plate, using a structural epoxy adhesive.

Three main geometrical parameters have been included in the parametrical analysis:

a) The ratio of the overlap length vs adherent reference thickness ( $L/e$ ). Notice that in presence of two different adherents, there are two adherent thicknesses (one has to be chosen as the reference thickness).

b) Unbalance ratio of the joint (d). Measured as the ratio of total axial stiffness of the adherents:  $d=(E_1 \cdot t_1)/ (E_2 \cdot t_2)$  for the single-lap joint and  $d=(E_1 \cdot t_1)/ (2 \cdot E_2 \cdot t_2)$  for double-lap joint. Subindex 1 (or 2), in the case of double-lap joints, being associated to the inner (or outer) adherent.

c) The reference adherent thickness (e).

While ( $L/e$ ) and (d) are non-dimensional parameters, (e) has dimension of length.

The parametrical study consists on the analysis of 108 cases, corresponding to the product of:

- 2 configurations (single- and double-lap joint),
- 3 different unit loads (tensile force, shear force and bending moment),
- 3 ( $L/e$ ) values,  $L/e=10, 50, 100$ .
- 3 (d) values,  $d=0.5, 1, 2$ .
- 2 (e) values,  $e=2 \text{ mm}, 4 \text{ mm}$ .

In the numerical models of the joint, the unit values of the load are always applied at a distance of one overlap length, for standardization purposes. At the other side of the joint, zero displacements are imposed. For the sake of simplicity and clarity, figure 1 shows the unit load values at both ends of the joint.

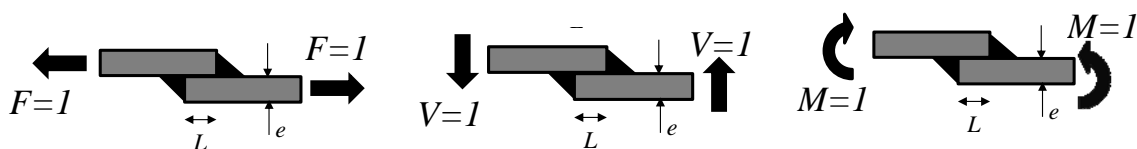


Figure 1. Unit load cases (for single-lap joint configuration).

### 3. Results

Generalized Stress Intensity Factors (GSIFs) have been calculated for the bimaterial corner at the end of the overlap of the joint. This is a closed corner (the two materials being perfectly bonded at the two common interfaces) created by the 90° solid wedge of the laminate end and the 270° solid wedge of the adhesive spew fillet (see figure 2). This corner has demonstrated to be a critical point where failure typically initiates [3].

For the mechanical properties of the materials:  $E=68670 \text{ MPa}$  (Young's Modulus),  $\nu = 0.33$  (Poisson's ratio) for aluminium 2024-T3;  $E_{11}=141300 \text{ MPa}$ ,  $E_{22}=E_{33}= 9580 \text{ MPa}$ ,  $\nu_{12}=\nu_{13}=0.3$ ,  $\nu_{23}=0.32$ ,  $G_{23}=3500 \text{ MPa}$ ,  $G_{12}=G_{13}=5000 \text{ MPa}$  for CFRP AS4/8552 unidirectional fibre laminate  $[0]_{12}$  (1 being the fibre direction) and  $E=3000 \text{ MPa}$ ,  $\nu=0.35$  for the adhesive FM<sup>®</sup>73 OST (toughened epoxy film, from Cytec). The local singular stress field has two singular terms.  $K_1$  and  $K_2$  being the so called Generalized Stress Intensity Factors (GSIFs).

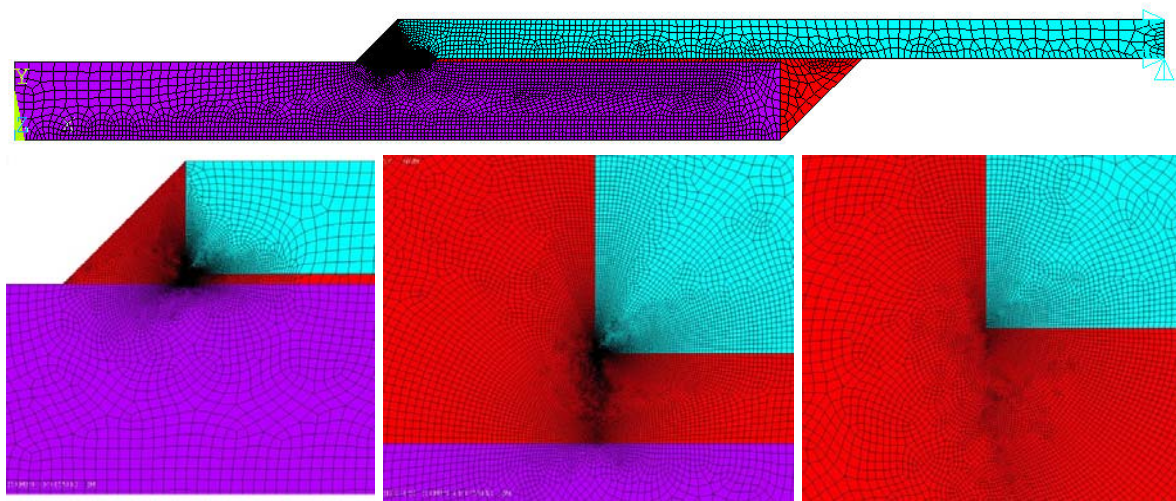
$$\sigma_{\alpha\beta}(r, \theta) \cong \frac{K_1}{r^{0.236764}} f_{\alpha\beta}^{(1)}(\theta) + \frac{K_2}{r^{0.110611}} f_{\alpha\beta}^{(2)}(\theta) + \dots \quad (1)$$

The previous works of the authors have been used to make the calculations of the parameters of the local singular stress field [1,2]. For the sake of brevity and simplicity, the details of the calculations are not addressed in this work, all details being available in the mentioned references.

All models, whatever the parametric values are, share the same local corner geometry configuration at the end of the overlap, the particular values of each parameter not affecting the already detailed local corner configuration.

Thus, the calculations of the stress singularities and characteristic angular functions are unique for each global selection of adherents/adhesive system, as they only depend on the local geometrical and local material configuration. The GSIFs calculation has to be carried out for each parameter value (the GSIFs depend on the global loading and geometrical characteristics), by the least squares procedure proposed in [2] and this is the time consuming part of the parametric analysis.

Although not completely necessary, as explained in [2], a fine mesh has been used at the corner (see figure 2, for a particular set of parameters in a single-lap joint configuration).



**Figure 2.** Detail of the mesh for the GSIFs calculation.

The mesh refinement used in the calculations is not practical for design purposes, that is the reason why this work is carried out. Making the calculations only once, for a set of different geometrical parameters and representing the results in a plot to be used in ulterior calculations, avoids the necessity of repeating tedious numerical modellings and manipulation of results.

An example of the results is shown in figures 3, 4 and 5, where GSIFs results for unit values of axial load (figure 3), shear load (figure 4) and unit bending moment (figure 5) are plotted. In these figures, the two GSIFs,  $K_1$  and  $K_2$ , associated to the two singular terms of the asymptotic series expansion of the stress representation at the corner neighborhood, have been included, see [4] for further details of the singular stress field at this particular corner.

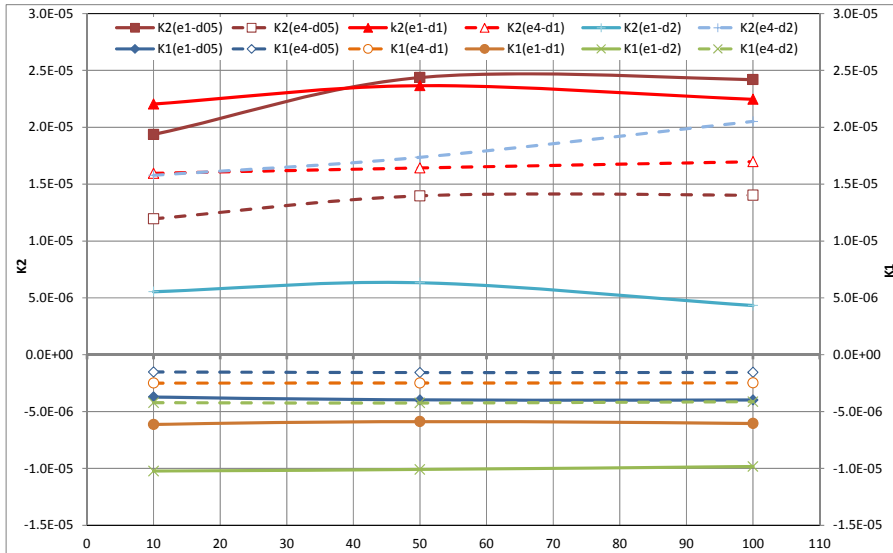


Figure 3. Results for GSIFs ( $K_1$ ,  $K_2$ ) for the unit axial load case.

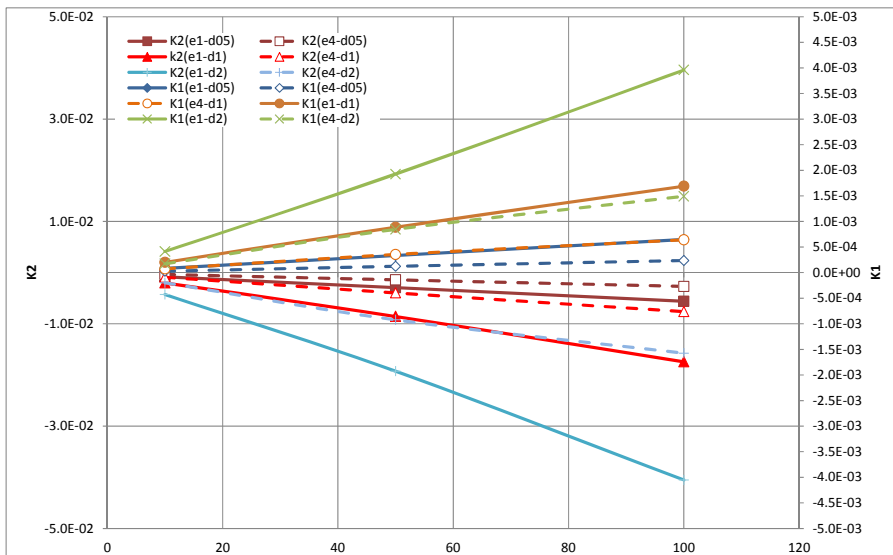


Figure 4. Results for GSIFs ( $K_1$ ,  $K_2$ ) for the unit shear load case.

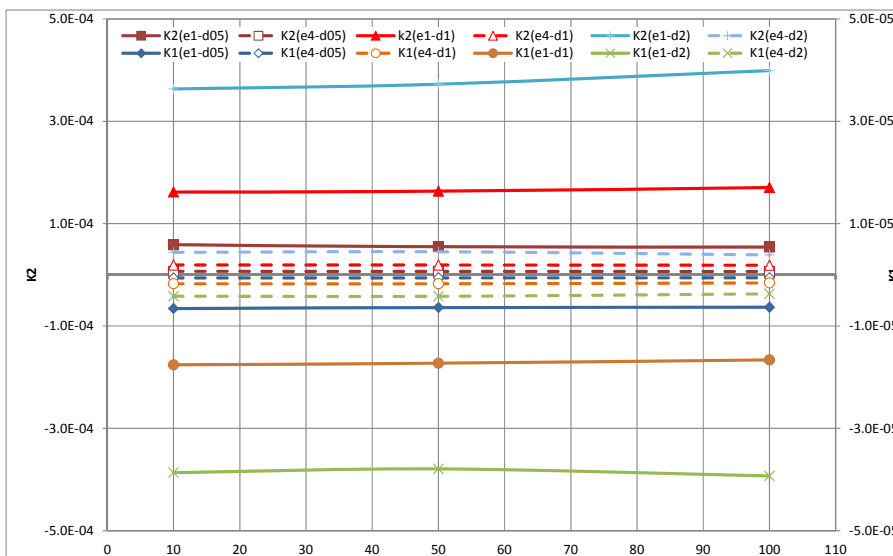


Figure 5. Results for GSIFs ( $K_1$ ,  $K_2$ ) for the unit bending moment load case.

#### 4. Concluding remarks

In the present work, the drawbacks (from a practical point of view in terms of design) associated to the use of a Fracture Mechanics approach have been solved performing a parametrical analysis of the problem and presenting the results in a graphical form. It has been applied to the problem of adhesively bonded joints between metallic and composite materials, in both single- and double-lap joint configurations.

In particular, the proper analytical and numerical steps have been used to accurately calculate the Generalized Stress Intensity Factors (GISFs) at the most critical corner in the adhesive joint, appearing at the end of the overlap area. These calculations have been done for unit value loading cases and for a set of three geometrical parameters, covering a wide range of geometrical configurations.

All results have been presented in graphical form, allowing future designers to simply perform a beam analysis of the joint to calculate the axial force, shear force and the bending moment at the end of the overlap and then applying superposition using the unit value results plots obtained in the present work.

As a future work, other material systems (adherents and adhesives) can be explored to create a database for design purposes.

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