STRAIN ENERGY-BASED FORMULATION FOR STRUCTURAL ANALYSIS

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Abstract

The increased use of composite materials in Aerospace in recent years is due to the weight reduction advantages. The drawback is that solid laminates and sandwich panels during manufacturing can be affected by a series of defects (skin depression, porosity, delamination, de-bonding…), which then reduce the static and fatigue performances of the component.
The Material Review Board (MRB) is responsible for repairing the component to restore its structure integrity in order to comply with the airworthiness requirements. Depending on the size , type and location of the defect ,a repair procedure and analytical tool are set up, both substantiated by a test database. Repairing the above listed defects involves mainly adding a secondary load path (doubler) or replacing the affected plies with new plies. The analytical approach presented here is covering the elastic, elastic-plastic range up to the failure of the bonded joint. This approach belongs to the Variational Method applied to the minimum strain energy formulation. The advantage of this method is that It doesn´t require the solution of differential equation of equilibrium or the stiffness matrix formulation.

1. Introduction

The development of an analytical approach became mandatory for FACC when the strength requirement of the bonded joint was extended beyond the limit load (linearity) up to the plastic range (ultimate load) and life estimation was welcome. Historically, the first analytical approach (Volkersen [1]) was simple and straightforward but limited to the adhesive linear behaviour and limited to double and single-lap joints. Hart Smith ([2],[3]) has extended the analysis thru plasticity up to failure and to step-lap joints. Both methods are based on differential equation of equilibrium. Models based on FEM approach are also available ([4]).
The strain energy method , presented on this note, is later on indicated as STRENE.

2. The Method

The analytical approach belongs to the family of Variational Methods. The basis of the algorithm consists on solving a function $\Phi(a_i)$ by "varying " the degree of freedom (dof : a_i) until the applicable constraints are satisfied , as symbolically shown in the Figure below.

$$\Phi(a_i) = \begin{Bmatrix} 0.0 \\ Y \\ \max \\ \min \end{Bmatrix}_{constrain}$$

$$\Delta(\Phi) = \Delta(W - E)_{\min}$$

where

$W = \sum_i P_i * a_i$ applied load work

$E = \frac{1}{2}\int \sigma\varepsilon \, dV$ strain energy

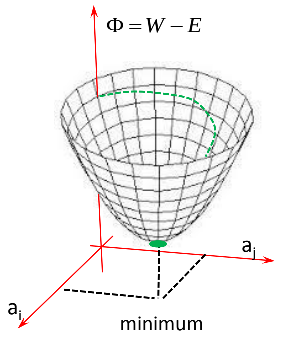
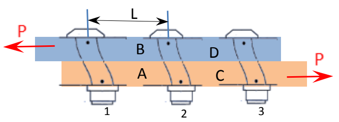


Figure 1. Algorithm description

The meaning of the function Φ depends on the unit of the degree of freedom (displacement, stress, forces, numerical values…). For the structural analysis, the function Φ is the strain energy and the variable a_i are the displacement and rotation. The principle behind this process is that the solution of static problem is unique and it coincides with the minimum of strain energy. This method´s first application was meant to simplify the analysis of the fastener joint as shown in the figure below dodging the compatibility and equilibrium equations.

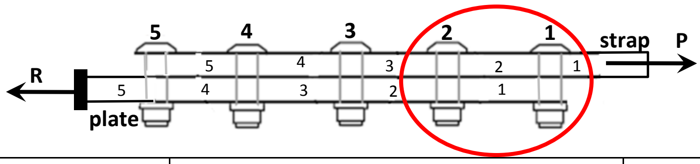


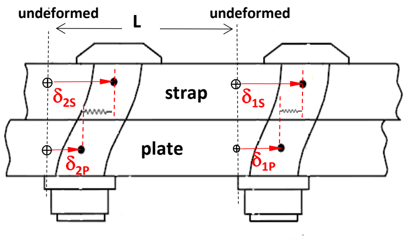
$K = {}^{L}\!/_{EA}$, F_e = bolt flexibility

R_i = BoltLoad Transfer

$$R_1 F_{e1} + R_1 K_A = (P - R_1) K_B + R_2 F_{e2}$$
$$R_2 F_{e2} + (R_1 + R_2) K_C = (P - R_1 - R_2) K_D + R_3 F_{e3}$$
$$R_1 + R_2 + R_3 = P$$

As an example of calculation let´s consider a 5 bolt fastener joint as shown below .





Bolt deflection

$\delta_{1S}, \delta_{1P}, \delta_{2S}, \delta_{2P}$: bolt deflection

$\Delta_{strap} = \delta_{1S} - \delta_{2S}$: strap axial distorsion

$\Delta_{plate} = \delta_{1P} - \delta_{2P}$: plate axial distorsion

$\Delta_{bolt} = \delta_{1S} - \delta_{1P}$: bolt shear distorsion

Strain energy

$$\delta E_S = \frac{1}{2}\left(\frac{EA}{L}\right)_S * \Delta^2_{strap}$$

$$\delta E_p = \frac{1}{2}\left(\frac{EA}{L}\right)_p * \Delta^2_{plate}$$

$$\delta E_B = \frac{1}{2} K_B * \Delta^2_{bolt}$$

Figure 2 :Fastener joint geometry and strain energy calculation

The degree of freedom (dof) to minimize the strain energy and to satisfy the constraint are calculated with EXCEL SOLVER Plugin. The Figure below shows the algorithm and results comparison with [7].

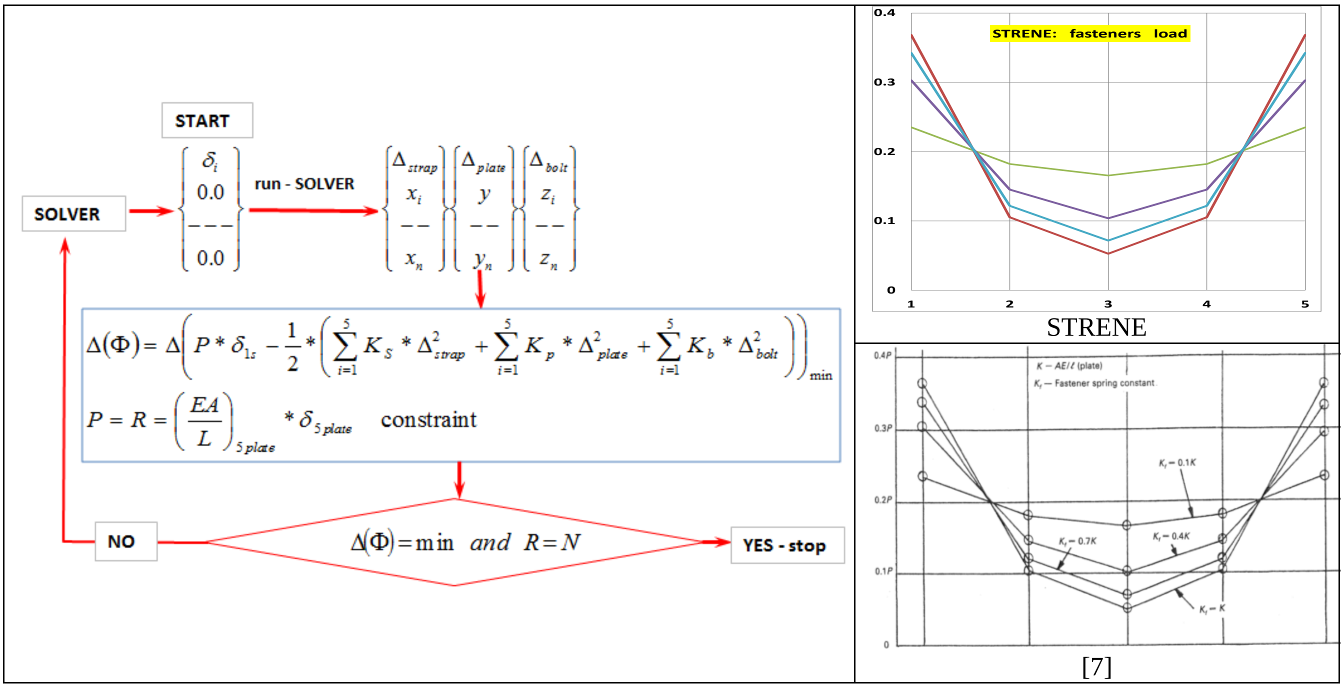


Figure 3. SOLVER flow chart and transfer load comparison (STRENE vs. [7])

3. Bonded Joint Modelling Using STRENE

Bonded joint can be analysed using the same algorithm used for the fastener joint (see Par.2) with two major modifications:

1. Adhesive is simulated by using a continuous “virtual bolts”, shown in Figure 4.
2. The adhesive strain energy is calculated at each step via the stress-strain curve, shown in Figure 5.

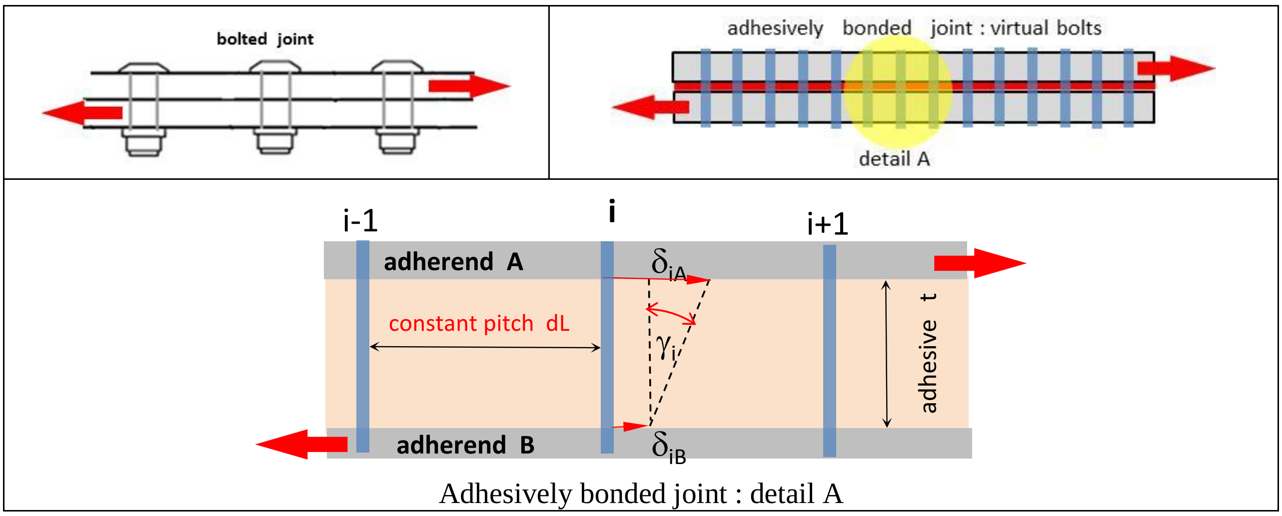


Figure 4. Bondline – Virtual bolt definition

Figure below shows the formulation of stress strain curve as reported on [6]. The adhesive strain energy E_i " at each virtual bolt" is calculated by integration of shear stress to γ_0 . Note that this energy calculation is effective even with a fully non-linear curve typical of adhesive at high temperature and the bi-linearization approximation used in [3] is not required.

When $\gamma \le \gamma_E$, $q = G\gamma$

and when $\gamma > \gamma_E$, $q = q_E + \left[\dfrac{\gamma - \gamma_E}{\dfrac{1}{G} + \dfrac{(\gamma - \gamma_E)}{q_A - q_E}} \right]$,

$\gamma \le \gamma_E$

$$E_i = \frac{1}{2}\gamma q * V = \frac{1}{2}\gamma^2 G * (t * dL)$$

$\gamma \ge \gamma_E$

$$E_i(\gamma_0) = \left[\frac{1}{2}\gamma_E^2 G + \int_{\gamma_E}^{\gamma_0} q * d\gamma \right] * V =$$

$$= \left[\frac{1}{2}\gamma_E^2 G + \int_{\gamma_E}^{\gamma_0} \left(q_E + \frac{\gamma - \gamma_E}{\dfrac{1}{G} + \dfrac{\gamma - \gamma_E}{q_A - q_E}} \right) d\gamma \right] * (t * dL)$$

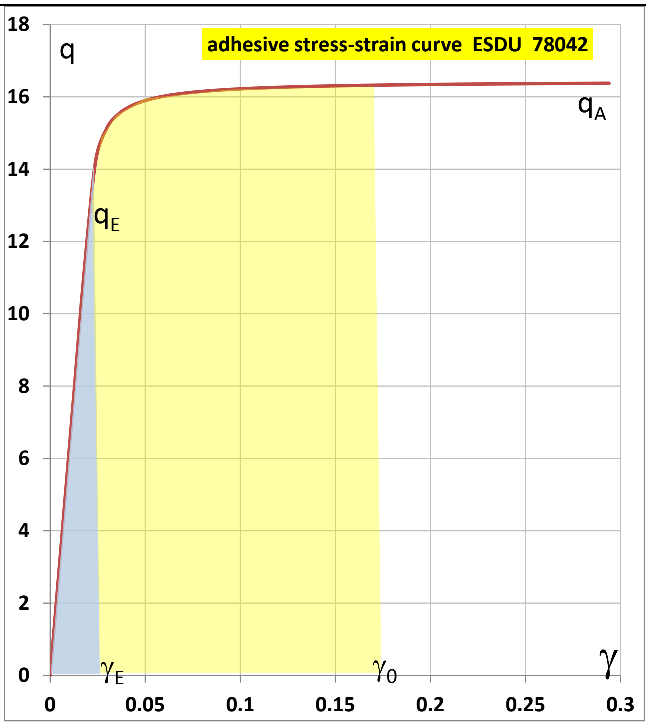
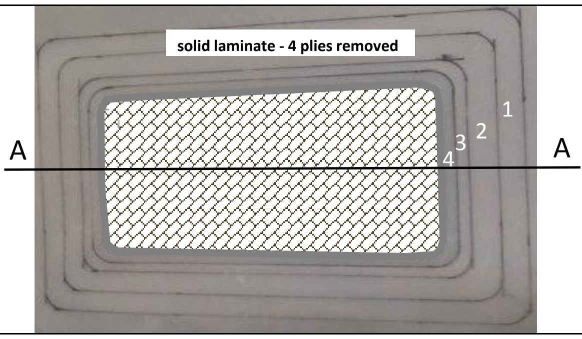


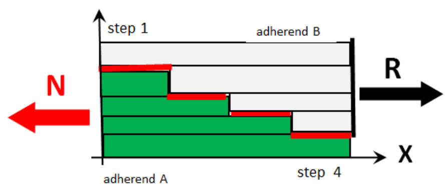
Figure 5. Stress-strain curve and strain energy calculation

3.1 Example of Calculation using STRENE

This paragraph reports a step-by-step procedure for calculation of ultimate bondline strength up to failure of a step-lap joint with 4 steps. Figure 6 shows the sanded area on a real part and a schematic sketch of the joint. MS Excel and the Solver Plugin is used to implement STRENE as it is user friendly and to make STRENE accessible for the broad public.

Step 1) Defect and Repair Definition





NDT inspection has detected a delamination on the sandwich panel face.
Defect was removed and a multistep repair was decided.

Figure 6. Typical defect repair (left: sanded area; right: schematic joint sketch)

Step 2) INPUT Data:

STEP		GEOMETRY	sta		MATERIAL and MECHANICAL PROPERTIES							
		VIRTUAL BOLT	X	dL	adherend A			adherend B			adhesive	
ID	length		mm	mm	E	t	K	E	t	K	G	t
1	12.7 mm	1	0.000	0.6684	44911	0.79248	53246	60949	0.19812	18065	624	0.2921
		-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------
		19	12.032	0.6684	44911	0.79248	53246	60949	0.19812	18065	624	0.2921
2	12.7 mm	20	12.700	0.9071	37663	0.59436	24677	44911	0.39624	19617	624	0.2921
		-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------
		33	24.493	0.9071	37663	0.59436	24677	44911	0.39624	19617	624	0.2921
3	12.7 mm	34	25.400	0.9071	44911	0.39624	19617	37663	0.59436	24677	624	0.2921
		-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------
		47	37.193	0.9071	44911	0.39624	19617	37663	0.59436	24677	624	0.2921
4	12.7 mm	48	38.100	0.6684	60949	0.19812	18065	44911	0.79248	53246	624	0.2921
		-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------
		66	50.132	0.6684	60949	0.19812	18065	44911	0.79248	53246	624	0.2921
		67	50.800								624	0.2921

Table 1. STRENE input data

The following note apply:

1. Virtual bolt : 1 step (19) , 2 step (14) , 3 step (14) , 4 step (19)
2. Moduli E for adherend A and B are calculated via Classical Laminate Theory (CLT)
3. For both adherend the stiffness K=(E*t)/dL

STEP 3) Results via SOLVER

1. Applied load : N = 534. [N]
2. Set all adh A and B displacement δi = 0.0 and then start SOLVER.
3. Run SOLVER for (E-W) to minimum and to satisfy the constraint (N=R)

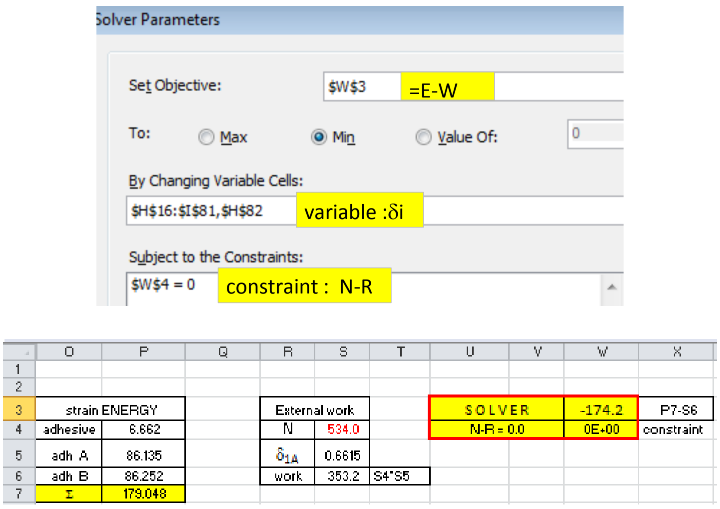


Figure 7. SOLVER : minimum energy and constraint check

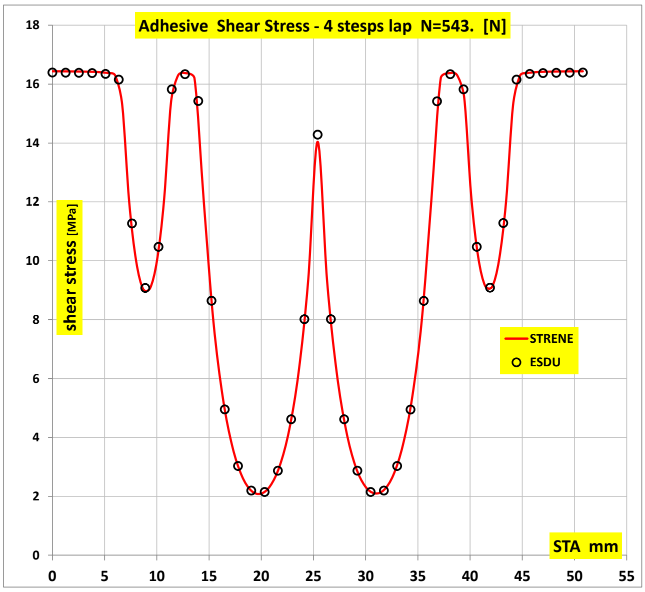
Table below reports the results:

STEP		GEOMETRY		VIRTUAL BOLT DISPLACEMENT		ADHEREND TENSION LOAD and ADHESIVE SHEAR STRESS									
		VIRTUAL BOLT	sta X	adh A	adh B	adherend A			adherend B			ADHESIVE			
ID	length		mm	[1] displ	displ	$\delta_{i+1}-\delta_i$	N_{axial}	δE_A	$\delta_{i+1}-\delta_i$	N_{axial}	δE_B	$\delta_{Ai}-\delta_{Bi}$	γ	τ	[4] δE_B
1	12.7 mm	1	0.000	0.66	0.595	0.00993	529	2.6239	0.00031	5.5	0.001	0.066729	0.228445	16.4	0.34497
		-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------
		19	12.032	0.51	0.503	0.00669	356	1.19133	0.00981	177.3	0.870	0.009532	0.032632	16.3	0.06211
2	12.7 mm	20	12.700	0.51	0.493	0.01390	343	2.38378	0.00969	190.2	0.922	0.012657	0.043331	16.4	0.11344
		-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------
		33	24.493	0.35	0.341	0.01107	273	1.51114	0.01328	260.4	1.729	0.004363	0.014936	9.3	0.01845
3	12.7 mm	34	25.400	0.33	0.328	0.01328	261	1.72967	0.01107	273.1	1.511	0.006572	0.022500	14.0	0.04186
		-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------
		47	37.193	0.18	0.170	0.00972	191	0.92583	0.01394	344.1	2.398	0.008518	0.029161	16.2	0.06934
4	12.7 mm	48	38.100	0.17	0.156	0.00984	178	0.87488	0.00670	357.0	[3] 197	0.012745	0.043632	16.4	0.11457
		-------	-------	-------	-------	[2] -------	-------	-------	-------		-------	-------	-------	-------	-------
		66	50.132	0.07	0.010	0.00030	5	0.00079	0.01003	534.0	2.678	0.057013	0.195183	16.4	0.58321
		67	50.800	0.07	0.000				0.0000000			0.066746	0.228502	16.4	0.34507

(1) Displacement of the load application point N.
(2) Centerline point . It is constraint (=0.0) for symmetry.It doesn´t take part in the solver prcess.
(3) Reaction at vitual bolt 67. N=R (see Figure 6)
(4) The strain energy is calculated using the stress-strain curve of the material (see Figure 5)

Table 2. STRENE results

Diagram below reports the graph relevant to adhesive shear stress and strain. Shear strain is reported on the material stress-strain curve. Results are compared with the analysis performed using the algorithm on [5]. Red circle in stress-strain diagram shows identical values for plastic strain.



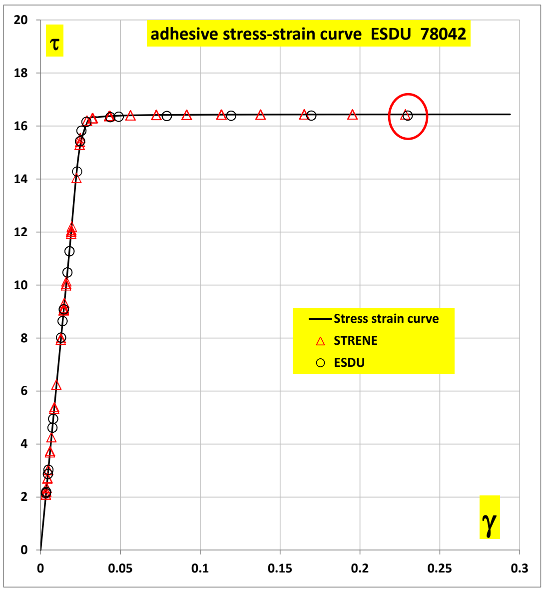


Figure 8. 4 steps repair analysis – shear stress and strain diagram

3.2 Bonded joint calculation moduli

The following type of bonded joint are at present moment covered by STRENE analysis. All the algorithm have the same structure of the lap bonded joint described on Par. 3.1. The only modification on the Input Data are relevant to the constraint location.

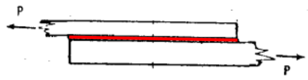
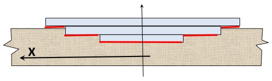
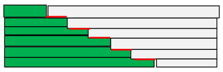
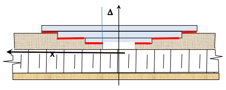
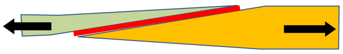
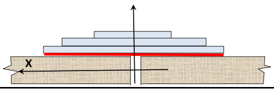
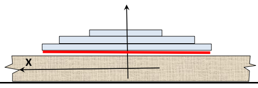
Type	Definition	Par.	Type	Definition	Par.
	Single lap joint Tension-shear	4.1		Precured patch	Not included on paper
	Stepped joint	3.1		Stepped lap joint with filler	4.2
	Scarf	Not included on paper		Splice	4.2
	Doubler	Not included on paper		Shear and Peel stress	Not included on paper

Table 3. Type of repair covered by STRENE approach

4. Validation

This paragraph reports some comparison between STRENE and the program presented on ESDU 79016 ([5]) which is an implementation of Hart-Smith's [3].

4.1. Single lap joint : tension[P] and shear [Q]

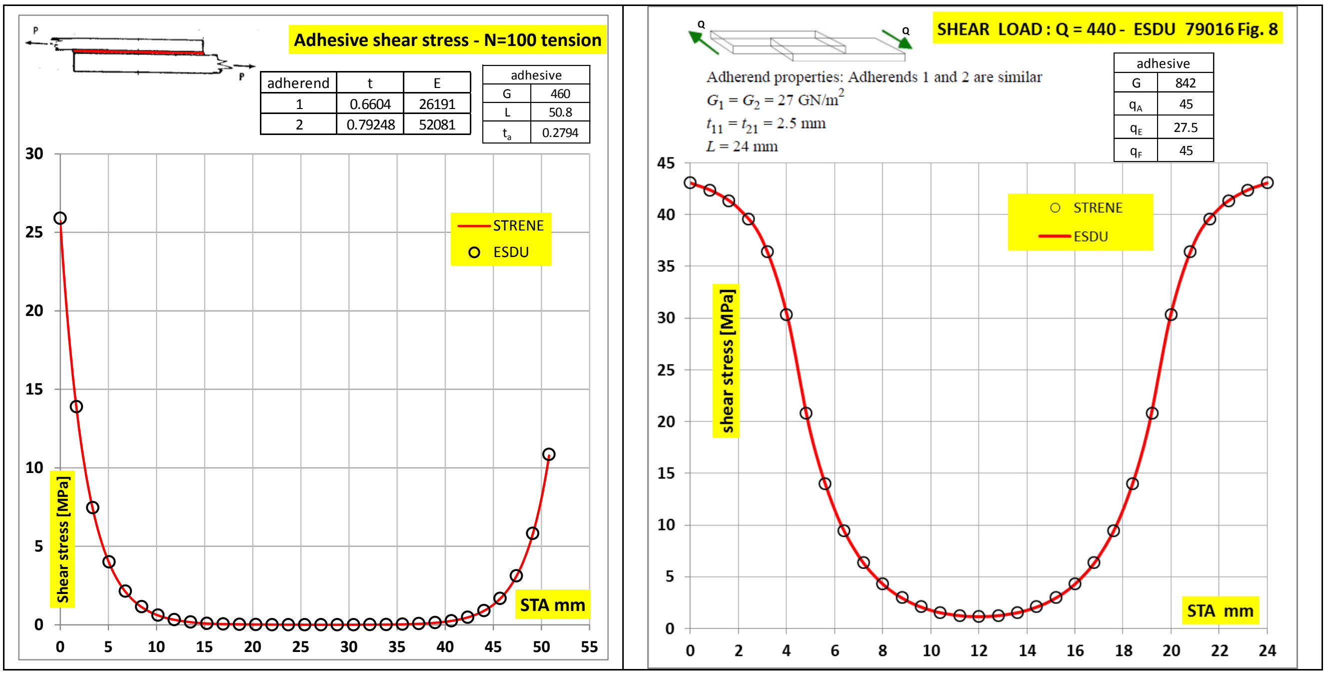


Figure 9. Single lap : adhesive stress diagram

4.2. Stepped Bonded and Splice

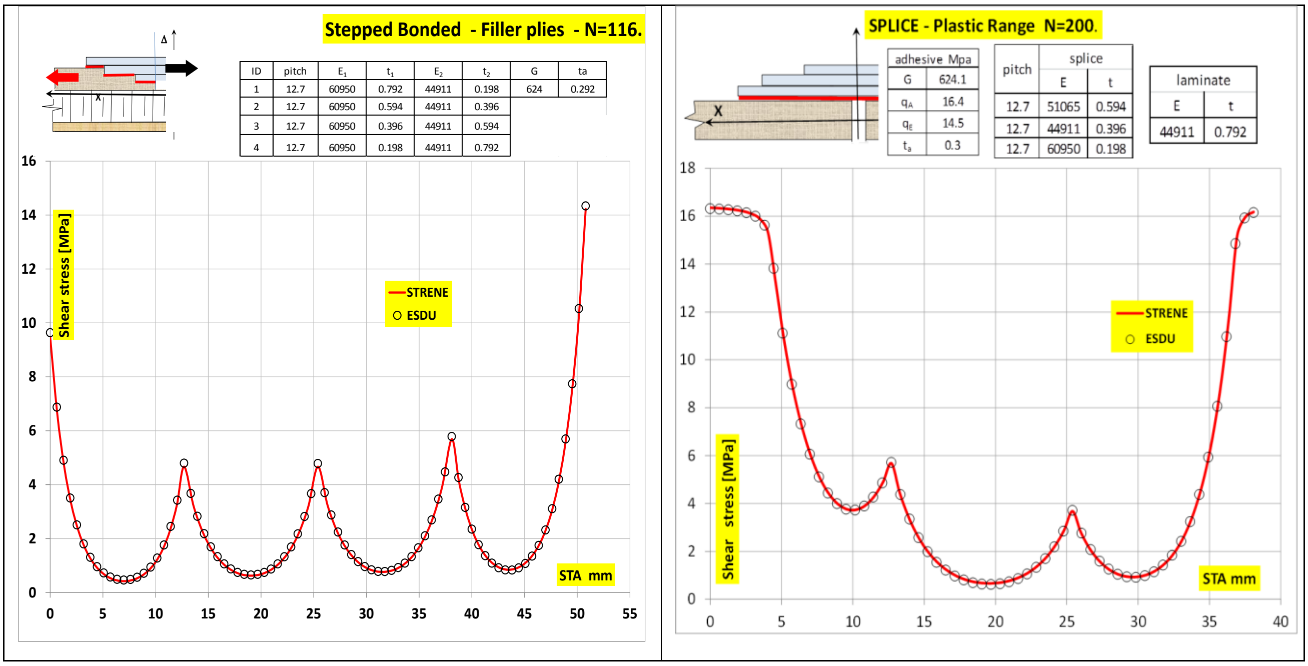


Figure 10. Stepped bonded and Splice : adhesive shear stress diagram

5. Conclusions

The STRENE algorithm based on strain energy principle and the variational method reveals to be simple and effective to analyse bonded joints in different configurations. STRENE is implemented in MS Excel Platform and the Solver Plugin to be user friendly and to make STRENE accessible for the broad public. Some additional calculation moduli, not related to composite structure, are available for other structural analysis : 2D and 3D bolt group, Beam on elastic foundation, Socket analysis , Pin joint analysis, Frame analysis.

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