A DAMAGE-BASED APPROACH TO THE FATIGUE OF COMPOSITE

P. A. Carraro, M. Quaresimin*

Department of Management and Engineering, University of Padova, Str.la S. Nicola 3, Vicenza, Italy
**marino.quaresimin@unipd.it*

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Abstract
In the present work a model is proposed for the prediction of the crack density evolution in multidirectional laminates subjected to cyclic loading. The multiple cracks initiation and propagation phases are treated separately and described by means of S-N and Paris-like curves, respectively. A multiscale strategy is adopted for the prediction of multiple cracks initiation, by means of the damage-based multiaxial fatigue parameters defined by the authors in a previous work. The statistical distribution of fatigue strength, damage accumulation and stress re-distributions after cracking are accounted for.

1. Introduction
The fatigue behaviour of multidirectional composite laminates is characterised by a progressive damage evolution from the beginning of fatigue life to the final failure. This damage scenario is characterised by initiation and through-the-width propagation of multiple matrix cracks in the off-axis layers, followed by the onset and propagation of delaminations [1-5]. In particular the accumulation of matrix cracks can lead to a considerable stiffness loss much before the laminate failure. In addition they act as stress concentrators for the 0° plies promoting delamination at the layers' interfaces and fibres failures which bring to the complete failure (i.e. separation) of a laminate. As a consequence it is fundamental to predict the evolution of the density of off-axis cracks during fatigue life. In spite of this, a general and well defined procedure is not still recognised by the scientific community.
Some interesting contributions can be found in the literature for predicting the crack density evolution under static loading (see Vigronadov and Hashin [6] and Huang et al. [7] among the others) .
Concerning fatigue loading, some empirical models were developed by several authors [8-10]. According to the authors' best knowledge, the only stress-based model for the prediction of the crack density evolution under cyclic loading in that proposed by Sun and co-authors which is valid for a cross-ply laminate [11]. They carried out a Monte Carlo simulation where the 90° layer of a cross-ply laminate was divided in small elements whose fatigue strength was assigned according to the Weibull distribution. They did not consider the transverse cracks propagation phase in view of the model validation on laminates with lay-ups in which the 90° plies thickness was high enough to promote the sudden propagation of the nucleated cracks.
Within this frame, the aim of the present work is to move one step further and develop a procedure for the prediction of the fatigue crack density evolution for symmetric laminates with a general lay-up, considering both the off-axis cracks initiation and propagation phases, which are, in general, distinct and can be described by means of different tools.
The presence of off-axis plies with generic orientations complicates the problem because of the multiaxial stress state, which changes during fatigue life because of stress re-distributions after cracking. For this purpose, a multiaxial criterion was proposed by the authors [12]. According to this criterion the S-N curves should be expressed by means an equivalent stress σ_{eq}, accounting for the multiaxial stress state. The equivalent stress was defined on the basis

of the damage mechanisms at the microscopic scale bringing to the initiation of a macro-crack. The Local Maximum Principal Stress (LMPS) and the Local Hydrostatic Stress (LHS) should be used as σ_{eq}, depending on the amount of in-plane shear stress σ_6 (see Ref. [12]).
This multiaxial criterion is included in the procedure proposed for the crack density prediction, allowing to deal with cracks initiation in off-axis plies.
The concepts and ideas at the basis of the procedure developed will be illustrated in the present works. In Ref. [13 a software tool implementing the procedure is presented.

2. Definition of crack density

According to its most trivial and common definition, the crack density is meant as the total number of initiated cracks divided by the observation length perpendicularly to the crack faces:

$$\rho = \frac{n}{L \cdot |\sin(\theta)|} \quad (1)$$

Where n is the total number of initiated cracks, L and θ are defined in figure 1.
It is proven that this definition of the crack density, which can be defined as the *total crack density* ρ, is not a suitable parameter to be correlated with the stiffness degradation [14]. In fact, if the cracks are not completely spanning through the width of a laminate their length becomes a variable of extreme importance to be accounted for a sound estimation of the stiffness degradation [1]. Accordingly, the weighted crack density, ρ_w, can be defined as the total number of cracks, each of them being weighted by its length c, divided by the observation length, as in equation (2):

$$\rho_w = \frac{\sum_{i=1}^{n} c_i}{w \cdot L} \quad (2)$$

where c_i is the length of the i-th crack, n is the number of initiated cracks and w is defined in figure 1.

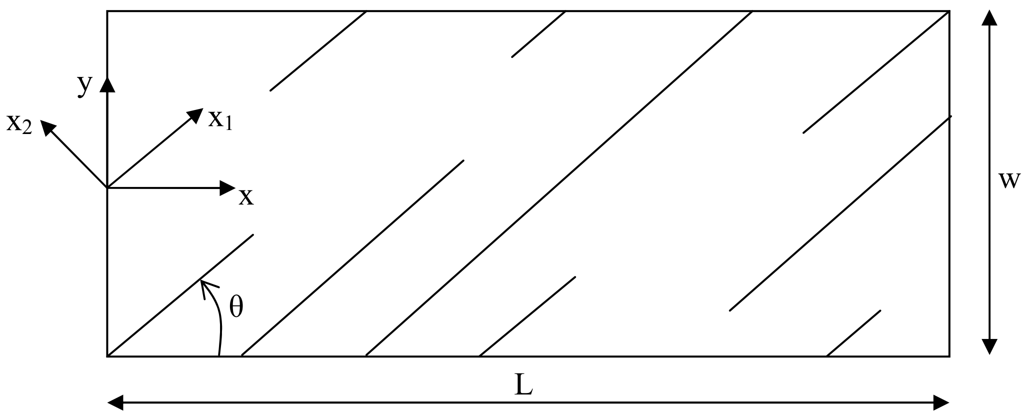

Figure 1: geometry of a cracked layer and global reference system

According to these observations, it is fundamental to be able to predict the weighted crack density, which depends on both the fatigue crack initiation and propagation.

3. Multiscale strategy

In this section the procedure for the prediction of the weighted crack density is briefly described. Because of the inhomogeneous nature of composite materials damage evolution is always multiscale, as highlighted by Talreja [15]. In addition, "*length scales of damage and their hierarchy are not fixed but are subject to evolution*" [15]. In fact the fatigue behaviour of a UD lamina is characterised by a progressive damage evolution by means of irreversible mechanisms at the microscopic scale, such as fibre-matrix debonding or matrix micro-cracking, as pointed out also by the present authors [1,12]. The evolution of these damage mechanisms, consisting for instance in the coalescence of debonds and micro-cracks, leads to the formation of a macroscopic crack, spanning through the thickness of a lamina and propagating along its width. Therefore the length scale changes from the micro-scale (the length scale of the fibre diameter and the inter-fibre spacing) to the macro-scale, related to the crack spacing and the laminate width. Accordingly, the initiation of a macro-crack is controlled by the evolution of damage at the microscopic scale. It has been shown by the authors that two local parameters, LHS and LMPS, are a sound representation of the driving forces for the micro-damage evolution in the presence of a negligible or high enough in-plane shear stress, respectively [12]. LHS and LMPS can be used as equivalent stress, σ_{eq}, for a successful representation of the lamina S-N curves in two single scatter bands, accounting for the influence of multiaxial stress states. The bridge between the macro-stresses in the material coordinates system (σ_1, σ_2, and σ_6) and the micro-stresses is represented by stress concentration factors computed by means of a micromechanical analysis, as reported by the authors in Ref. [12]. The macro-stresses can be uniform along the transverse direction, if the ply is undamaged, or non uniforms because of stress redistribution due to cracking. The stress redistribution can be treated as a macro-scale phenomenon, how it is commonly done in the literature, and it can be computed by means of several analytical and semi-analytical methods or by means of Finite Element (FE) analyses of cracked laminates. Once the initiation of cracks has been predicted, it is fundamental to describe their through-the-width propagation, which is treated as a macroscopic phenomenon. It has been shown (Ref. [1]) that the crack propagation can be successfully described by means of a Paris-like law relating the Crack Growth Rate (CGR) to the Energy Release Rate (ERR). A schematic of the multiscale procedure for the prediction of the weighted crack density evolution is shown in figure 2, and it consists in the simulation of fatigue life by progressively increasing the number of cycles by steps ΔN.

For a given step of fatigue cycles ΔN, and for every off-axis layer, the analysis is divided in the following points:

1) Calculation of ply stresses in each layer, with the possible presence of off-axis cracks, and therefore considering the stress redistribution.

2) Calculation of micro-stresses (or local stresses) by means of a fibre matrix unit cell subjected to periodic boundary conditions (see the stress concentration factors defined in Ref. [13]).

3) Calculation of the fatigue parameters LHS and LMPS.

4) According to the multiaxial condition, calculation of the total density of nucleated cracks, by using the LHS of LMPS master curves for the material considered. The statistical distribution of fatigue strength and the sequence effect, due to the stress redistribution, must be accounted for.

5) Analysis of the crack propagation phase to compute the length of all the nucleated cracks and calculation of the weighted crack density.

6) Updating the crack spacing, increasing the number of cycles of a further step ΔN and repeating steps 1-6.

Steps 4 and 5 will be treated more in detail in the next sections. It s important to say that, at present, cracks are considered to initiate only at the edges of a laminate, but the concepts can be extended for considering also more general cases.

4. Prediction of multiple crack initiation

As already mentioned, the basic idea is that the initiation of cracks is controlled by the fatigue resistance of the material considered, represented by the S-N curves expressed in terms of an equivalent stress σ_{eq} (LHS or LMPS). The S-N curves are expressed by means of the typical power law relating the equivalent stress to the number of cycles to failure N_f:

$$\sigma_{eq} = K \cdot N_f^a \tag{3}$$

The exponent *a* is an input of the model and it can be obtained by a series of experimental data on an off-axis lamina with generic orientation. More precisely, two values of *a* should be introduced, a_{LHS} and a_{LMPS}, to be used whether the fatigue behaviour is LHS- or LMPS-controlled, respectively.

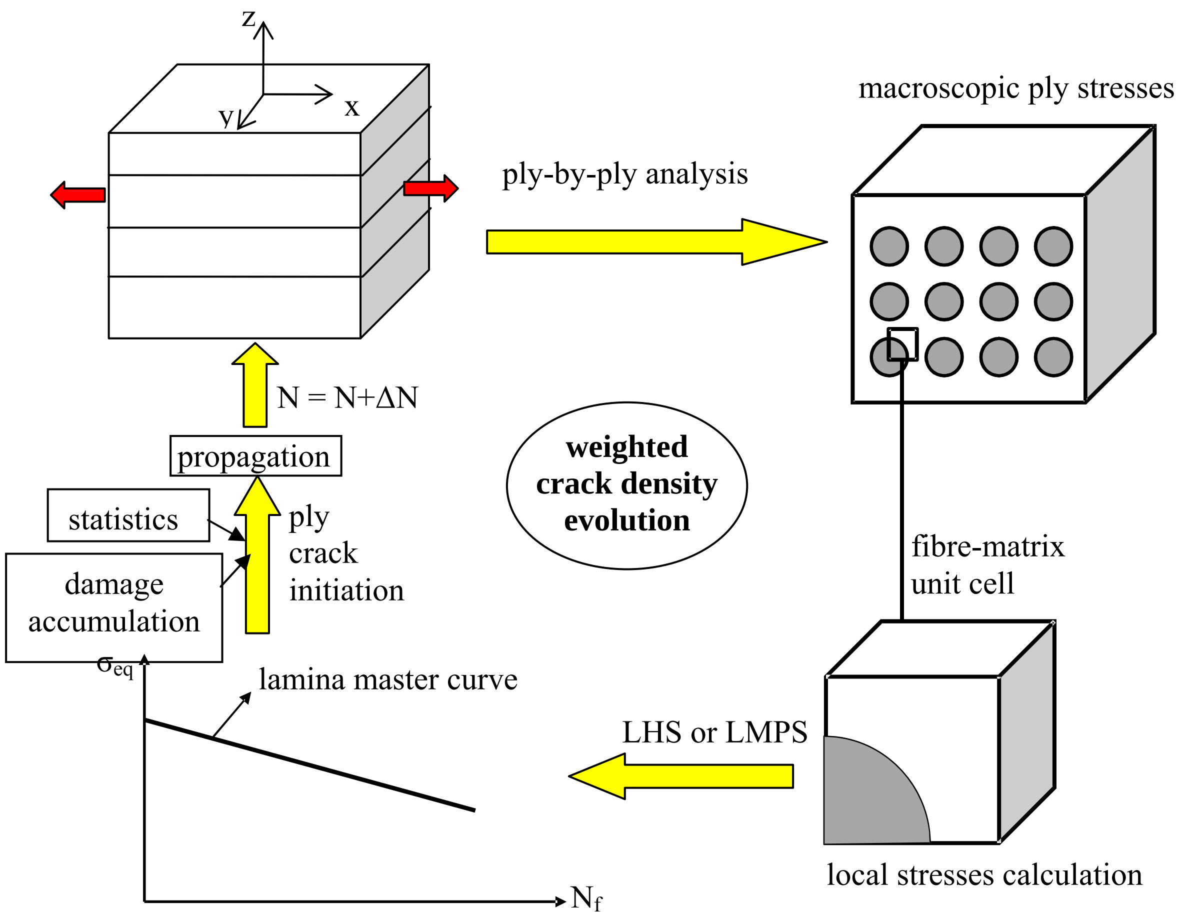

Figure 2: schematic of the procedure for the prediction of the weighted crack density

A fundamental hypothesis of the model is that the scatter band describing the fatigue life to crack initiation is characterised by a Weibull distribution of the coefficient K, the cumulative probability P(K) being expressed by equation (4).

$$P(K) = 1 - \exp\left[-\left(\frac{K}{K_0} \right)^m \right] \tag{4}$$

m and K_0 are respectively the shape and scale parameters of the Weibull distribution. They are input variables that can be computed as explained in [13]. Expressing K from equation (3) and substituting in equation (4) the probability to form cracks at a number of cycles equal to N, P(N) yields

$$P(N) = 1 - \exp\left[-\left(\frac{\sigma_{eq}}{K_0 \cdot N^a} \right)^m \right] \tag{5}$$

Equation (5) holds if the local cyclic stress remains constant during the entire fatigue life. This condition is verified only at the beginning of fatigue life, when the laminate is undamaged or when the initiated cracks are far enough and thus non interacting. This condition is defined as "Non-Interactive regime" and it holds until the crack density is lower than a limit value, ρ_{NI}, after which the initiated cracks start interacting causing stress redistributions. This is defined as "Interactive regime", and it is characterised by the variability of σ_{eq} with the number of cycles and the x_2 coordinate. As a consequence, the prediction of crack initiation in the Non-Interactive and Interactive regimes must be treated separately in the procedure.

In the non-interactive regime the density of the initiated cracks can be simply calculated from equation (5). If the crack density at the edges of the *k*-th ply is higher than ρ_{NI}, the cracks in the *k*-th layer start interacting with each other causing the redistribution of the stress components σ_1, σ_2, σ_6, and thus of the equivalent stress σ_{eq}, which are now functions of the transverse coordinate x_2. Therefore each point of the ply is subjected to a different stress state, both in terms of modulus and degree of multiaxiality. Obviously the cyclic values of these stress components vary proportionally to the external applied load. At this point it is essential to make the simplifying assumption that the crack spacing at the edges, S_e, is uniform and equal to $1/\rho_e$, ρ_e being the edge crack density. Of course this is not exactly representative of real phenomena, mainly in the earlier stages of fatigue life. However considering the crack spacing distribution is not possible in the present case since the edge crack spacing does not depend only on the cracks initiated at the edge but also on those coming from the opposite edge and fully propagated.

Since at every step ΔN_i of the simulation a new set of cracks initiates, each point along the x_2 direction is subjected to a locally variable amplitude fatigue loading. As a consequence a damage accumulation model is needed to account for this phenomenon. Neither results nor models are available in the literature concerning the effect of variable amplitude fatigue on the matrix-dominated fatigue behaviour of UD composites. In the present work a simple linear damage accumulation rule is used to define the entity of damage D under variable amplitude fatigue. Accordingly, the entity of damage after N_i cycles is

$$D_i = \sum_{j=1}^{i} \left(\frac{\Delta N_j}{N_{f,j}} \right) \tag{6}$$

5. Crack propagation

As already mentioned, the propagation of the initiated cracks is assumed to be governed by a Paris-like law, relating the CGR to the ERR. Since the propagation occurs in mixed I + II mode conditions, an equivalent ERR, G_{eq}, should be used, according to equation (7).

$$CGR = C \cdot G_{eq}^{d} \tag{7}$$

The coefficients C and *d* have to be calibrated with experimental results. Several phenomenological expressions have been proposed in the literature for the G_{eq} to be used in equation (7). A physically based criterion is not available, according to the authors' best knowledge. However, an experimental investigation carried out by the authors [1] revealed that, when the Mode Mixity MM = G_{II}/G_{tot} is low, i.e. the loading condition is mode I dominated, the propagation is controlled only by the mode I contribution G_I. This is because the presence of the fibres does not allow the deviation of the crack propagation direction which would be the effect of the mode II contribution. Conversely, when the loading condition is mode II dominated, but not pure mode II, it can be observed that the Paris-like curves are well collapsed in terms of the total ERR $G_{tot} = G_I + G_{II}$, as shown in figure 3.

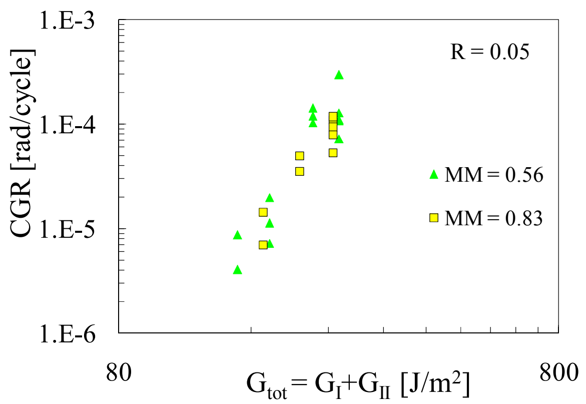

Figure 3: Paris-like curves in terms of G_{tot} for transverse cracks in tubes [1]

According to these observations the equivalent ERR can be defined as follows:

$$G_{eq} = \begin{cases} G_I, \; if \;\; MM < MM^* \\ G_{tot,} \; if \;\; MM^* < MM < 1 \end{cases} \tag{18}$$

where MM* is the transition point between the G_I and the G_{tot} driven propagation. Therefore, the coefficients of Paris-like curves C_I, d_I and C_{tot}, d_{tot} have to be provided to be used when the propagation is G_I or G_{tot} controlled, respectively.
It is remarked that, however, G_{tot} cannot be used for pure mode II loading. In addition, the use of G_{tot} for MM > MM* is only a phenomenological criterion, of which the general validity should be checked.
A tunnelling crack within an off-axis ply surrounded by tougher layers is characterised by a constant value of the ERR components if the crack length *c* is higher than two times the layer thickness [16]. This causes a Steady State (SS) propagation of tunnelling cracks. However, the ERR components are functions of the crack density. In the procedure developed here they are computed step by step on the basis of the predicted crack density by means of a shear lag analysis of the cracked laminate.

6. Crack density calculation

After the procedure described in section 5, the length of all the initiated cracks can be calculated at the cycle N_i. Thus the weighted crack density $\rho_w(N_i)$ can be computed as follows.

$$\rho_w\left(N_i\right) = \frac{\left|\sin(\theta)\right|}{w} \cdot \sum_{s=1}^{i} \left[\Delta\rho\left(\Delta N_s\right) \cdot c\left(s, N_i\right)\right] \tag{22}$$

where $\Delta\rho(\Delta N_s)$ is the increase of total crack density at the *s*-th step.
Now the edge crack density has to be calculated on the basis of both the initiated and fully propagated cracks. On the basis of the edge crack density the stress redistributions between cracks at the edges can be calculated by means of any analytical or numerical method, considering the simplifying assumption that the crack spacing is uniform along the entire edge. The entire procedure, at the *i*-th step, has to be applied to all the layers oriented off-axis with respect to the external loads, so that the total, weighted and edge crack densities can be calculated for all of them.
All the steps can be repeated until the desired number of cycles is reached, providing the evolution of the weighted crack density in each layer with the number of cycles. These results can then be used as input for a model to predict the stiffness degradation of general symmetric damaged laminates.

6. Conclusions

An analytical procedure for the prediction of the total and weighted crack density in multidirectional laminates under fatigue loading has been developed. The procedure adopts a multiscale strategy to predict the initiation of multiple cracks and then considers the propagation phase in order to estimate their length which is essential for the computation of the *weighted* crack density. The model considers the statistical distribution of fatigue strength, the stress re-distribution and shielding effect after crack initiation and eventually damage accumulation due to variations of the local stresses during fatigue life.

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