NUMERICAL ANALYSIS OF FAILURES IN COMPOSITE LAMINATES USING MULTISCALE MODELLING

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Keywords: Composite Materials, Multiscale Modelling, Boundary Element Method, Representative Volume Element, Homogenization Theory, Failure Criterion,

Abstract
This paper presents a numerical analysis of failure in unidirectional composite laminates based on multiscale modelling. The approach assumes the material in two scales. In the global scale, the Boundary Element Method (BEM) for anisotropic plane elasticity is used to evaluate strain and stress fields in the lamina domain. In the local scale, equivalent to a material point of the structure (localization technique) and defined by a Representative Volume Element (RVE), the multi-domain BEM formulation is applied to resolve elastic problem in conjunction with the mean fields homogenization theory in order to locally converge to the global solution. The evaluation of failure in the RVE is performed by failure criteria of Tsai-Hill for matrix and maximum deformation criterion for fiber. Numerical example for a laminated with orientation of the fibers to $0°$ *is presented and compared to result obtained by the LARC03 failure criteria.*

1. Introduction

The use of composite materials in engineering applications, mainly in the field of aeronautics, automotive and naval industries, has been highlighted in the last decades. Thanks to the numerous benefits, achieved due to the combination among its microscopic constituents, whose purpose is to optimize their mechanical properties, rendering them superiors the same as that when the constituents are considered in isolation. Such optimized properties as light weight, strength and flexibility, make the material indispensable for the particular structural type of application. Because of this and due to the rapid development of computers, most advanced techniques, such as multiscale analysis of the mechanical behaviour of composite materials, has been gaining ground in the scientific community, mainly represented by work of authors such as [1, 2, 3, 4, 5].

Therefore, the multiscale approach of composite materials refers to simulating their behavior through multiple time and/or length scales; thereby, generating accurate results, as it captures the physical phenomena contained in smaller scales, also provides a saving computational time compared with the models that represent all scales solely [1].

So, for connecting the intrinsic results to the each scale, the homogenization techniques are used, which provide the properties or responses of a structure, macroscale, given the properties or responses of the structure's constituents, microscale, or conversely, the localization techniques are used, which provide the properties or responses of the constituents, given the properties or responses of the structure [6].

2. Multiscale Modelling

2.1. Representative Volume Element

The continuum mechanics is based on the concept of a homogeneous continuum, which can be repeatedly subdivided into infinitesimal subvolumes, each of which retains the properties of the bulk material. However, at some scale, all real materials are heterogeneous. The purpose of micromechanics is to account explicitly for a materials heterogeneous microstructure while allowing it to be treated as an effective continuum at a higher length scale (e.g., within a structure). To account for this microstructure, micromechanics relies on either a representative volume element (RVE), that be a unit cell, in cases of materials with periodic microstructure [6]. An RVE is a statistical representation of the material properties, that should contain enough information on the microscale e should be sufficiently smaller than the macroscopic structural dimensions.

In continuum micromechanics, each material point is considered as a finite volume of a homogeneous material which has zero structural dimensions from the macroscopic viewpoint, but which represents a volume of a finite microscopic dimension with a certain microstructure. For a non-periodic microstructure, the RVE is definided as a volume containing a very large number of elements on the microscale. This definition is valid only for the case of an ergodic material, i.e. the ergodic hypothesis implies that the heterogeneous material is assumed to be statistically homogeneous. This fact also implies that sufficiently large volume elements selected at random positions within the sample of the considered material have statistically equivalent components arrangements and contain the same averaged material properties. Such material properties are referred to as the effective material properties of the inhomogeneous material.Therefore, the volume in the homogenization/localization procedure should be chosen to be a proper RVE, with the sufficient size to contain all information necessary for describing the behaviour of the composite. Thus, such a choice largely determines the accuracy of the model of a heterogeneous material [2].

2.2. Average Theorems and Boundary Conditions

In multiscale modeling techniques that use the homogenization process, the fields of stress and strain of the heterogeneous microstructure have direct influence in the macroscale behaviour, through their volumetric averages. This way, the average tensors of the fields of stress and strain that describe the state of the macroscopic material point, are calculated from theorems of the theory of average fields, as follows:

$$\left\langle \sigma_{ij} \right\rangle = \frac{1}{V} \int_V \sigma_{ij}(x) dV \tag{1}$$

$$\left\langle \varepsilon_{ij} \right\rangle = \frac{1}{V} \int_V \varepsilon_{ij}(x) dV \tag{2}$$

where V is the volume of the RVE and the x argument indicates that fields of stress and strain in the microscale are spatially dependent. Thus, it is postulated which the macroscopic infinitesimal tensors of stress $\bar{\sigma}_{ij}$ and strain $\bar{\varepsilon}_{ij}$ are equal the spatial averages of the tensors of stress and strain in a RVE, conversely, through of the average theorems of the stresses e strains. Therefore, to calculate the averages of the fields of stress and strain of a RVE, is necessary apply properly the homogeneous boundary conditions in their boundary Γ. This boundary conditions are basically divided in, boundary condition of linear displacement ($u_i(\Gamma) = \bar{\varepsilon}_{ij} x_j$) and boundary condition of uniform traction ($t_i(\Gamma) = \bar{\sigma}_{ij} n_j$).

Therefore, to calculate the average strains in a composite material, it is necessary to resolve the elasticity problem of the RVE subject to homogeneous boundary condition of linear displacement, and so:

$$\left\langle \varepsilon_{ij} \right\rangle = \frac{1}{2V} \int_\Gamma (u_i n_j + u_j n_i) d\Gamma \tag{3}$$

where n_j is the outward normal vector on boundary Γ of the RVE. Similarly, the average stresses in a composite material are calculated. The difference is in the boundary conditions applied in the RVE, i.e. it is considered the homogeneous boundary conditions of uniform traction [6]:

$$\left\langle \sigma_{ij} \right\rangle = \frac{1}{V} \int_\Gamma \sigma_{ik} x_j n_k d\Gamma \tag{4}$$

In the present paper, the homogeneous boundary condition of linear displacement are used for the direct homogenization procedure. This technique is known as deformation driven [7], i.e. the analysis is realized of the follows: given the deformation tensor $\left\langle \varepsilon_{ij} \right\rangle$, it is calculated homogenized stress tensor $\left\langle \sigma_{ij} \right\rangle$ based on microstructure elastic response.

3. Numerical Modelling

3.1. BEM for Macroscale

The macroscale the structural model used is a unidirectional composite laminate, that the by definition presents special orthotropic behavior . Thus, the numerical modelling of the laminate uses the formulation of the BEM for anisotropic elasticity, using codes developed in the programming environment of the software MATLAB. Therefore, the basic equation of the BEM is given by [8]:

$$c_{ij} u_j(x') + \int_\Gamma T_{ij}(x', x) u_j(x) d\Gamma(x) = \int_\Gamma U_{ij}(x', x) t_j(x) d\Gamma(x) \tag{5}$$

where u_i and t_i are respectively the displacement and the traction, U_{ij} and T_{ij} are the fundamental solutions of displacement and traction respectively, x' and x denote the collocation point and the integration point respectively and c_{ij} are the values of the influence coefficients, which depend of the location of the point x'. Thus, for the anisotropic elasticity the fundamental solutions of displacements and traction, are given respectively by [9]:

$$U_{ji}(x', x) = 2Re\left[q_{i1}A_{j1}\ln(x_1 - x'_1) + q_{i2}A_{j2}\ln(x_2 - x'_2)\right] \tag{6}$$

$$T_{ji}(x', x) = 2Re\left[\frac{1}{(x_1 - x'_1)}g_{i1}(\mu_1 n_1 - n_2)A_{j1} + \frac{1}{(x_2 - x'_2)}g_{i2}(\mu_2 n_1 - n_2)A_{j2}\right] \tag{7}$$

where q_{ik} is the complex parameters matrix, A_{ik} are the complex constants, μ_k the complex roots of the anisotropic characteristic equation and g_{ik} is the matrix which contain the complex roots combined.

3.2. BEM for Microscale

On account of the RVE to be composed of an isotropic matrix and an transversely isotropic fiber, the numerical modelling of the microscale uses the multi-domain BEM formulation. Here, for the elastic solution of the transversely isotropic fiber, is used the same formulation of the BEM for anisotropic elasticity used on the solution of problems orthotropic laminas of composite materials, because, in plane stress, the constitutive equations transversely isotropic materials are the same for orthotropic materials. Thus, for isotropic materials have the fundamental solutions of displacement and traction, given by:

$$U_{ij}(x', x) = \frac{1}{8\pi G(1-\upsilon)}\left[(3-4\upsilon)\ln(\frac{1}{r})\delta_{ij} + r_{,i}r_{,j}\right] \tag{8}$$

$$T_{i,j}(x', x) = -\frac{1}{4\pi(1-\upsilon)r}\left\{\frac{\partial r}{\partial n}[(1-2\upsilon)\delta_{ij} + 2r_{,i}r_{,j}] - (1-2\upsilon)(r_{,i}n_j - r_{,j}n_i)\right\} \tag{9}$$

where G is the shear modulus, υ Poisson's coefficient, r length of the vector connecting x' to x and δ_{ij} Kronecker's delta. Thus, using the fundamental solution corresponding to each phase of the RVE, the resultant system of the multi-domain BEM is given by:

$$\begin{bmatrix} H_E^1 & H_I^1 & -G_I^1 & 0 \\ 0 & H_I^3 & G_I^3 & H_E^3 \end{bmatrix} \begin{Bmatrix} u_E^1 \\ u_I \\ t_I \\ u_E^3 \end{Bmatrix} = \begin{bmatrix} G_E^1 & 0 \\ 0 & G_E^3 \end{bmatrix} \begin{Bmatrix} t_E^1 \\ t_E^3 \end{Bmatrix} \tag{10}$$

where E represents the elements that are not on the interface and I represents the interface elements, where are used the relations of equilibrium of forces $\{t_I\}_{region} = -\{t_I\}_{region+1}$ and compatibility of displacements $\{u_I\}_{region} = \{u_I\}_{region+1}$.

4. Microscale Failure

The Tsai-Hill criterion is used to predict failure in the matrix of the RVE:

$$\frac{\sigma_{11}^2 + \sigma_{22}^2 - \sigma_{11}\sigma_{22}}{Y_{mt}^2} + \frac{\tau_{12}^2}{T_m^2} = d_m^2, \quad \sigma_{22} > 0 \tag{11}$$

where the mode is of failure under tensile loading. Similarly, for the mode of failure under compression loading:

$$\frac{\sigma_{11}^2 + \sigma_{22}^2 - \sigma_{11}\sigma_{22}}{Y_{mc}^2} + \frac{\tau_{12}^2}{T_m^2} = d_m^2, \quad \sigma_{22} < 0 \tag{12}$$

where Y_{mt} and Y_{mc} are respectively, the matrix transverse strength in tension and matrix transverse strength in compression, and T_m is the matrix shear strength. Matrix failure occurs when $d_m \geq 0$ [3].

The maximum deformation criterion is used to predict failure in the fiber of the RVE:

$$\frac{\varepsilon_{11}}{\varepsilon_{ft}^U} = d_f^2, \quad \varepsilon_{11} > 0 \tag{13}$$

where the mode is of failure under tensile loading. Similarly, for the mode of failure under compression loading:

$$\frac{\varepsilon_{11}}{\varepsilon_{fc}^U} = d_f^2, \quad \varepsilon_{11} < 0 \tag{14}$$

where ε_{ft}^U and ε_{fc}^U are respectively, the tensile ultimate fiber strain and the compressive ultimate fiber strain. Fiber failure occurs when $d_m \geq 0$ [3].

5. Results

5.1. Macroscopic Model and Geometry and Size RVE

The macroscopic structural model examined was a unidirectional laminate AS4 3501-6 epoxy of 100*mm*x100*mm* with central hole of 15*mm*, orientation of the fibers the $\theta = 0°$, thickness of 1 *mm*, subjected the a positive displacement along the axis x in the right edge of 0.20 *mm* with constraints of displacements in the directions x and y in the left edge and with extern boundary and the central hole discretized from 7 discontinuous boundary elements and with 52 internal points, according as is illustred in the Fig.1.

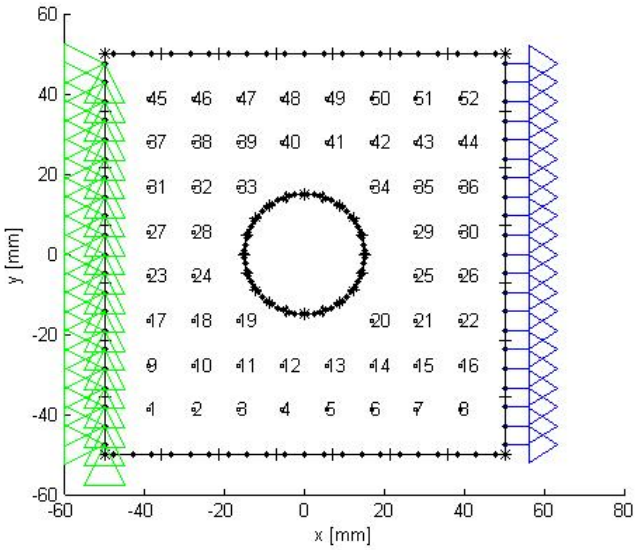

Figure 1. Discretization and boundary conditions of the laminate with fibers $\theta = 0°$.

The suitable geometry of the RVE was determined according with the orientation of the fibers in the laminated, i.e., for the laminated with orientation of the fibers of $\theta = 0°$ the RVE chosen faithfully copy the microstructure of the laminated. By Fig.2, it can be seen clearly that the AS4 fiber (dark region) is oriented the 0° of the axis x, as required by combination of the materials in the laminated.

Figure 2. Geometry of the RVE for a laminated with orientation of the fibers to the $\theta = 0°$.

To determine the size of the RVE, it was used a similar methodology to the used by [10], where a property of the material was employed as convergence criterion, i.e., of representativeness of the volume element. Here, the constitutive property was utilized, i.e., the components of the stiffness matrix of the laminated. Thus, the size of the RVE was found used the boundary condition of linear displacement, divided into two cases: Boundary condition of linear displacement in x, which corresponds the uniform displacement $\langle \varepsilon_{11} \rangle$; Boundary condition of linear displacement in y, which corresponds the uniform displacement $\langle \varepsilon_{22} \rangle$. The Tab.1 presents the values of the sizes of some RVE, for each case of linear displacement:

RVE	Size linear displacement in x (mmxmm)	Size linear displacement in y (mmxmm)
28	0.21x0.21	0.12x0.12
33	0.24x0.24	0.15x0.15
40	0.26x0.26	0.20x0.20

Table 1. Values of the size RVE.

5.2. Validation

Thus, after specified all sizes of the RVE, the evaluating the failure was performed and then compared with reviews of failures made by use of the LaRC03 criterion, performing the modelling only in the scale structural laminated. For this was used the same procedure of [11], which solved the elastic problem through of the ABAQUS/CAE finite element software, using customized routines through the programming language PYTHON, designated of *script*.

Fig.3 shows the graph with the values of the failure indices obtained by the two analyzes. Thus, compatibility was observed the values obtained by it, which validates the multiscale model presented here, showing to be very efficient for evaluating failure in composites laminates.

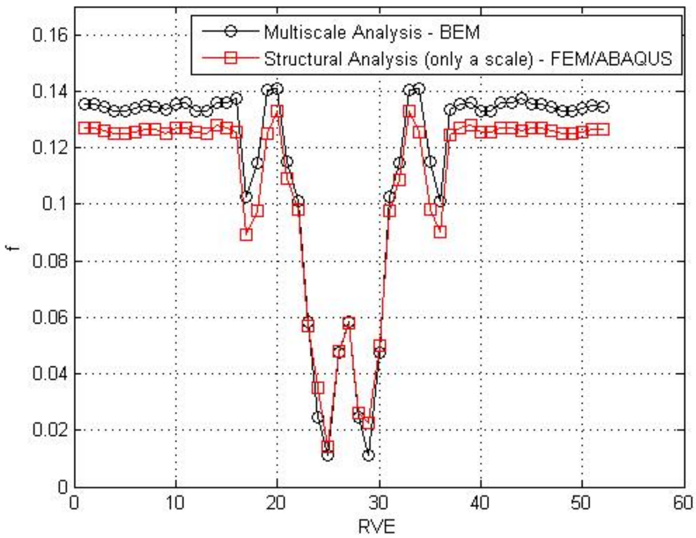

Figure 3. Values of the failure indices.

6. Conclusions

The comparison of the multiscale analysis failure with the failure analysis performed only in the structural scale of the composite laminated has been developed. The failure criterions of Tsai-Hill and maximum deformation was used in the multiscale analysis, to detect respectively the failed matrix and fiber, whereas the LaRC03 failure criterion was used in the structural analisys, and the comparison between them has proved that they both present similar results. Therefore, the capacity of the multiscale models in simulate the heterogeneous materials behaviour is highlighted, also taking the advantage of identifying the physical failure mechanisms.

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