INTERPRETING DATA FROM FOUR-POINT BENDING TESTS OF THIN ANGLE PLY LAMINATES – TEST SET-UP, EDGE AND WIDTH EFFECTS

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Abstract

The response of composite laminates to combined in-plane and out-of-plane static and fatigue loading can be evaluated using bending tests, but data obtained from bending tests needs to be carefully evaluated before being applied to specific design cases. There may be significant differences in the internal stress states that develop when different physical test configurations and ply angles are used with identical load cases. Flexural stiffness values obtained from bending tests cannot be used to predict component deflections unless the influence of the test configuration is accounted for. Data obtained from bending tests is dependent on interlaminar stresses and edge effects as they can influence static and fatigue failure modes and failure loads. The current work uses three-dimensional finite element simulations to predict the internal stress state when pure bending is applied to thin angle ply laminates. Three different methods of load application are considered; a directly applied moment and four-point bending using two different support conditions. The induced internal loads are quantified in relation to the externally applied loads using the classical laminate theory (CLT). Finite element modeling is used to predict interlaminar and edge stresses, and the effects of varying ply angles and stacking sequence on the resulting internal loads and the out-of-plane deflection are investigated.

1. Introduction

One potential solution for reducing development costs and improving reliability in the design of composite structures is to focus initial testing on details and small subcomponents where the intended application and expected loading conditions may result in structural responses unique to composites. Carefully designed detail and subcomponent testing can generate data that are useful for design but not readily available from typical tension, compression and shear tests performed during material characterization, particularly for composite materials. Bending tests are a good alternative to tension, compression and shear testing for understanding composite material behaviours [1,2]. Bending tests are particularly useful for predicting the behavior of thin laminates such as those commonly used for composite aircraft skin-stringer panels that are critical in compression instability rather than in tension or shear. The response of composite laminates to combined in-plane and out-of-plane static and fatigue loading can be evaluated using bending tests. Bending tests have an advantage over tension and compression tests because they use smaller coupons and tabs are not required to prevent failure at the grips. A four-point configuration has an advantage over three-point bending because the maximum internal loads are generated in-between the load introduction points rather than under the loading nose as is the case with the three-point test set-up. Static bending tests can be used to generate flexural stiffness data and stress-strain curves and to estimate compression allowables for thin laminates in axial loading with limited out-of-plane loads. Fatigue tests in bending can be used to estimate laminate fatigue behavior under multi-axis loading. Problems associated with the successful application of four-point bending tests include failure under the loading nose, data scatter and hysteresis behavior due to friction [3]. However, data obtained from bending tests needs to be carefully evaluated before being applied to specific design cases.

When designing composite structures, it is often only the in-plane stresses that are considered to be of importance. In many cases, however, failure is preceded by delamination at the free edges of a component and may be caused by the presence of interlaminar stresses [4,5,6,7]. As the material allowable is much lower in the out-of-plane direction, it is possible that lower margins may result when interlaminar stresses are considered instead of in-plane stresses. Another consideration is that the overall stress state within a component is typically determined using only the externally applied loads. Internal loads due to thermal and manufacturing effects are often accounted for, but what is often not considered is the internal load state that is generated within a laminate as a result of structural boundary conditions interacting with the bending-torsion or extension-torsion coupling behavior unique to composite materials. In some cases, and particularly in thin composite structures, these internal loads can be significant and a methodology to quantify them and evaluate the consequences with respect to static and fatigue design allowables could lead to a more efficient design process. In this paper, these two issues are explored in terms of predicting interlaminar stresses and internal loads using finite element simulation.

2. Finite Element Model Description and Validation

2.1 Model description

The finite element model used in the current study corresponds to a flat coupon with dimensions of 100 mm x 15.88 mm and an overall thickness of 0.59 mm. The dimensions of the model are chosen to represent the 4-point bending test specimen shown in Figure 1, and for which experimental data is readily available to this study [8]. A macroscopic approach is taken, where each ply is explicitly defined as a homogenous, transversely orthotropic material with the following properties: E_{11} = 150GPa, $E_{22} = E_{33}$ = 8GPa, $G_{12} = G_{23} = G_{13}$ = 4GPa, and $\nu_{12} = \nu_{23} = \nu_{13}$= 0.25. Three layups were considered in order to produce a wide range of interlaminar stresses; (A) $[(0/90)_4]_s$, (B) $[0/45/90/-45/0/45/90/-45/0]_s$, and (C) $[0/65/90/-65/0/65/90/-65/0]_s$. The coupon was simply supported at its ends, and loaded across its width at a distance of 25mm from each end. The supports prevented twisting of the coupon about its longitudinal axis. The applied load was varied between each layup to produce a longitudinal strain of 0.001 on the top surface. Each ply was meshed with at least two elements through its thickness and smaller elements were used near the free edges of the coupon.

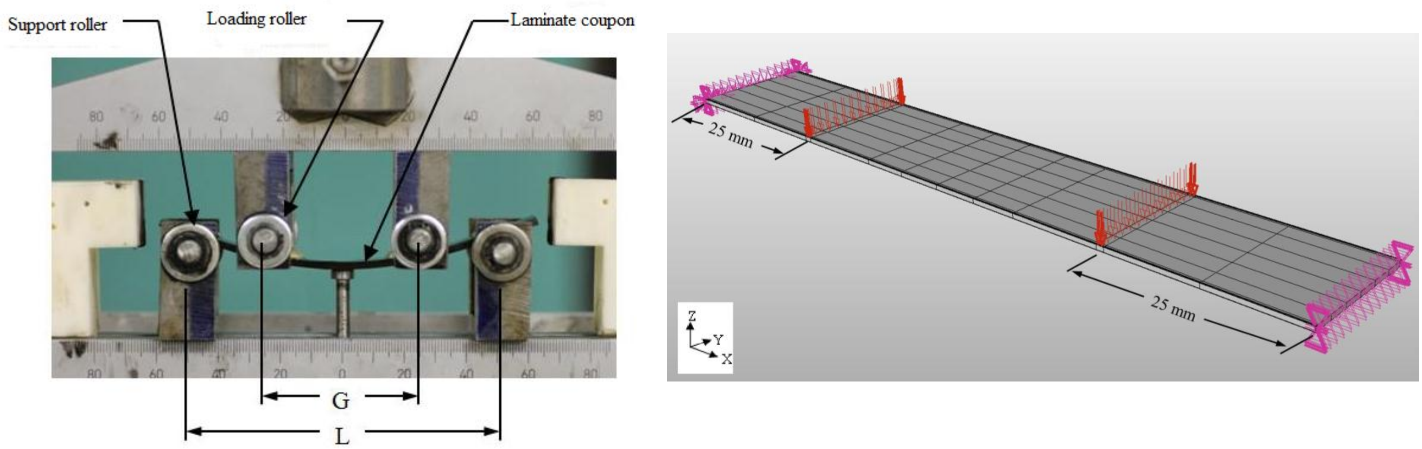

Figure 1. (a) Experimental 4-point bending test fixture and coupon and (b) finite element representation.

2.2. Model validation

The finite element model was validated against analytical results obtained using classical laminate theory, against data from the literature for tension-tension tests, and was compared to experimental data for the four-point bending case. A 4-ply laminate coupon was modeled using data from the study by Pipes and Pagano [4] and subjected to the same tension-tension loading used in that study. Results for both in-plane and out-of-plane interlaminar stresses obtained from the model agreed well with those of Pipes and Pagano. The model was then extended to include the four-point bending boundary conditions and loading as shown in Figure 1(b). The in-plane stresses were compared to analytical results from the classical laminate theory and the interlaminar stress field at the free-edges compared to the results from the tension-tension tests done by Pipes and Pagano. A 16-ply $[(0/90)_4]_s$ lay-up was then modeled to represent the test specimen shown in Figure 1(a). Displacement results from the finite element model were validated against experimental data in order to confirm that the model correctly represented a four-point bending test.

3. Results

3.1 Predicting interlaminar stress distribution (edge effects)

The four-point bending finite element model was used to compare the maximum interlaminar stresses at the free-edges of the three laminates; (A) $[(0/90)_4]_s$, (B) $[0/45/90/-45/0/45/90/-45/0]_s$ and (C) $[0/65/90/-65/0/65/90/-65/0]_s$, and are presented in Table 1. Stress values within 0.1mm of the free edge were excluded due to the stress singularity resulting from the macroscopic representation of the material interface between the lamina [9,10,11]. Of the three configurations studied, the $[0/45/90/-45/0/45/90/-45/0]_s$ laminate had the largest values for all out-of-plane interlaminar stresses (σ_z, τ_{yz} and τ_{xz}). The increase in the interlaminar shear stresses τ_{xz} and τ_{yz} near the edge of the coupon is shown in Figure 2 for all three laminates.

Laminate	A	B	C
τ_{xz} (MPa)	0	53.6	10.2
τ_{yz} (MPa)	8.2	52.1	22.2
σ_z (MPa)	1.3	18.2	11.3

Table 1. Maximum out-of-plane interlaminar free-edge stresses for three laminates; (A) $[(0/90)_4]_s$, (B) $[0/45/90/-45/0/45/90/-45/0]_s$, and (C) $[0/65/90/-65/0/65/90/-65/0]_s$ under 4-point bending.

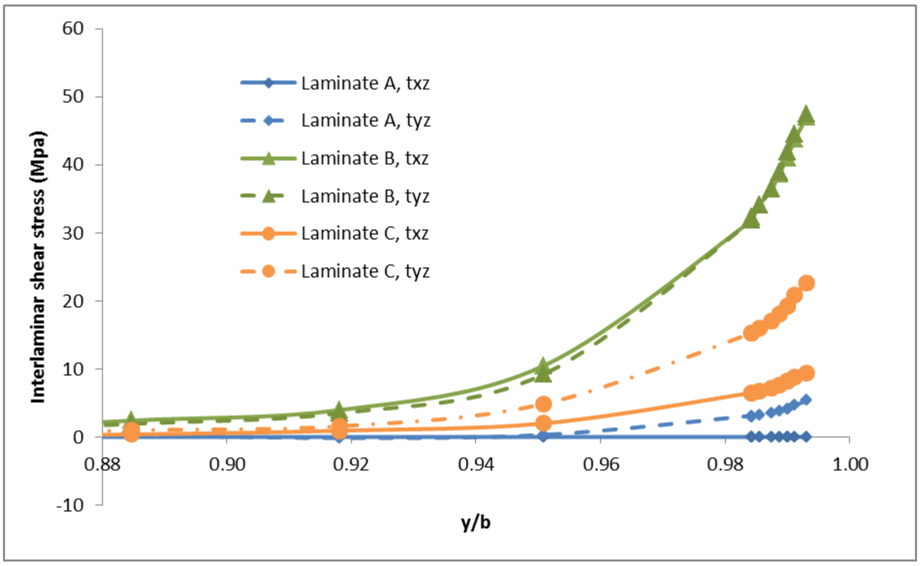

Figure 2. Out-of-plane interlaminar shear stress between plies 1 and 2 for laminate A and between plies 2 and 3 for laminate B and C. "y/b" represents the location across the width of the coupon relative to the total coupon width of 15.88mm.

3.2 Margin of safety evaluation for free-edge stresses

The value to the designer of being able to determine the free-edge stresses in composite laminates lies in being able to determine a margin of safety for the component with respect to the material properties in the out-of-plane direction. Out-of-plane stresses, even when much lower than fibre-direction stresses need to be considered in the design of composite structures subject to out-of-plane loads because the out-of-plane critical stresses for layered composites are substantially lower than those for the in-plane, fibre direction. Table 2 compares the margins of safety in compression for the (A) $[(0/90)_4]_s$ and (B) $[0/45/90/-45/0/45/90/-45/0]_s$ laminates in the fibre direction (σ_1) with margins for the three interlaminar stresses which refer to the global coordinate system. The values in Table 2 assume that the properties of the matrix in transverse shear are the same as those for the out-of-plane direction. There is some question surrounding what allowable to use for the interlaminar shear, and future experimental testing will hopefully shed some light on this. The results presented demonstrate that the margins of safety for the interlaminar shear stresses at the free-edge are of the same order of magnitude as the critical in-plane stress (in this case the 1-direction compression allowable), suggesting that these stresses, which are not considered in the widely used classical laminate theory in-plane analysis, may not be negligible in the analysis of composite structures subject to bending loads.

Laminate A				**Laminate B**			
	Maximum stress (MPa)	Material allowable (MPa)	Margin of safety		Maximum stress (MPa)	Material allowable (MPa)	Margin of safety
σ_1	-1600	-1744	0.09	σ_1	-1600	-1744	0.09
σ_z	1.3	90	68.2	σ_z	18.2	90	3.9
τ_{yz}	8.2	90	10.0	τ_{yz}	53.6	90	0.7
τ_{xz}	-	90	-	τ_{xz}	52.1	90	0.7

Table 2. Margins of safety for (A) $[(0/90)_4]_s$ and (B) $[0/45/90/-45/0/45/90/-45/0]_s$ laminates under 4-point bending.

4. Prediction of Internal Loads Generated During 4-point Bending Tests

In addition to the free edge stresses that arise during four-point bending test as a result of the transverse discontinuities in material properties, there may be significant differences in the internal stress states that develop when different physical test configurations and ply angles are used to apply identical load cases [12,13]. For example, in the four-point bending of angle-ply laminates, a test fixture in which the coupon is supported by rollers does not produce a stress state representative of a pure moment as expected. Instead, a complex stress state is induced due to the presence of shear coupling associated with the ply angles and the boundary conditions associated with the physical means of load introduction. Flexural stiffness values obtained from bending tests cannot be used to predict component deflections unless the influence of the test configuration is accounted for.

4.1. Model Description

A 4-ply, angle-ply $(+/-)_s$ layup with overall dimensions of 100 x15.88 mm and an overall thickness of 2.36 mm was used for the internal load prediction of a 4-point bend test. Two load cases were considered; a simply supported case where twist about the longitudinal axis of the coupon is prevented (LC2), and a contact case where the coupon is free to lift off the supports (LC3). The contact case incorporated all four rollers into the simulation and is the most realistic representation of the physical test set-up for a 4-point bend test as shown in Figure 1. The simply supported case would be more common in panels that are attached to a supporting structure (such as skin-stringer panels in an aircraft). Results from these two methods of load introduction were compared to the idealized case of an unconstrained coupon subject to a pure moment (LC1). The mesh and material properties were the same as those used for the interlaminar free-edge stress distribution study described in the previous section.

4.2. Effects of load application method

The vertical deflection at the centre of the coupon is plotted in Figure 3 for all three load cases and ply angles between 0° and 90°. Ply angles of 0° and greater than 60° show minimal difference in the central deflection between a pure moment and the four-point bend cases while the maximum difference occurred at a ply angle of 20°. For all ply angles, the contact model predicted larger vertical deflections than the simply supported model as a slight twist was allowed to develop in the coupon.

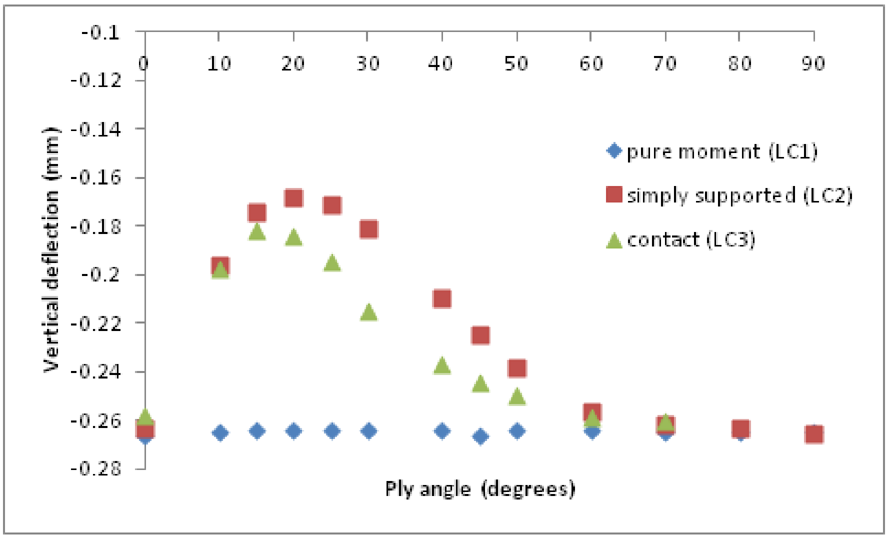

Figure 3. Total vertical deflection as a function of ply angle for $[+/-]_s$ laminates in four-point bending and varying boundary conditions representing test fixture design variations.

One of the reasons that four-point bend tests are often preferred over three-point bend tests is the assumption that the central portion of the specimen is free of transverse shear loads. This is true of isotropic materials and cross-ply laminates (0 and 90 degree plies). In the case of angle-ply composite laminates in four-point bending, this assumption is not true. As shown in Figure 4, shear stresses develop in the out-of-plane direction for all of the load cases and ply angles between 10° and 60°. These out-of-plane shear stresses are a result of the internal torsional load that is generated when the 4-point bend fixture physically prevents the angle-ply coupons from twisting about their longitudinal axes.

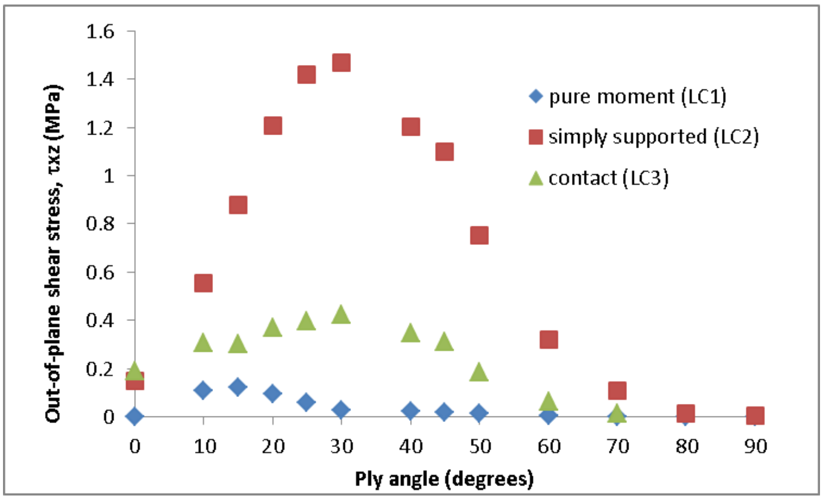

Figure 4. Out-of-plane shear stress as a function of ply angle for $[+/-]_s$ laminates in four-point bending and varying boundary conditions representing test fixture design variations.

4.3. Predicting the "induced" internal loads

The internal torsional load that is induced in angle-ply laminates subjected to 4-point bending can be predicted by examination of the laminate "ABD" compliance matrix used in the context of the Classical Laminate Theory (CLT). The 16 term in the "D" matrix describes how much twisting deformation occurs when an external moment in the fibre direction, M_x, is applied to a laminate. The 66 term defines the amount of torsional loading required to twist a laminate, or for the case of 4-point bending, the torsional loading required to "untwist" a laminate back to its horizontal position. This relationship can be used to calculate the internal torsional loads generated as a result of the physical restraints of the four-point test fixture. The classical laminate theory can be used to find the mid-plane strains and curvatures associated with the application of a bending moment Mx. The internal torsional moment can then be determined by setting the resulting twist (κ_{xy}) to zero for a simply supported assumption (LC2) and solving for Mxy. The value of Mxy represents the internal loads generated by the physical restraints of the testing fixture. This methodology was employed to study the internal loads as a function of the D16/D66 ratio for a number of different angle-ply and quasi-isotropic lay-ups. Figure 5 shows that, for *angle*-ply layups between θ = 0 and 50°, and for three different quasi-isotropic laminates, there is a linear relationship between the ratio D16/D66 and the internal torsional moment predicted using the CLT for the simply supported four-point bend test (LC2). These results follow the same trend as when the internal torsional moment is extracted directly from the corresponding finite element simulations.

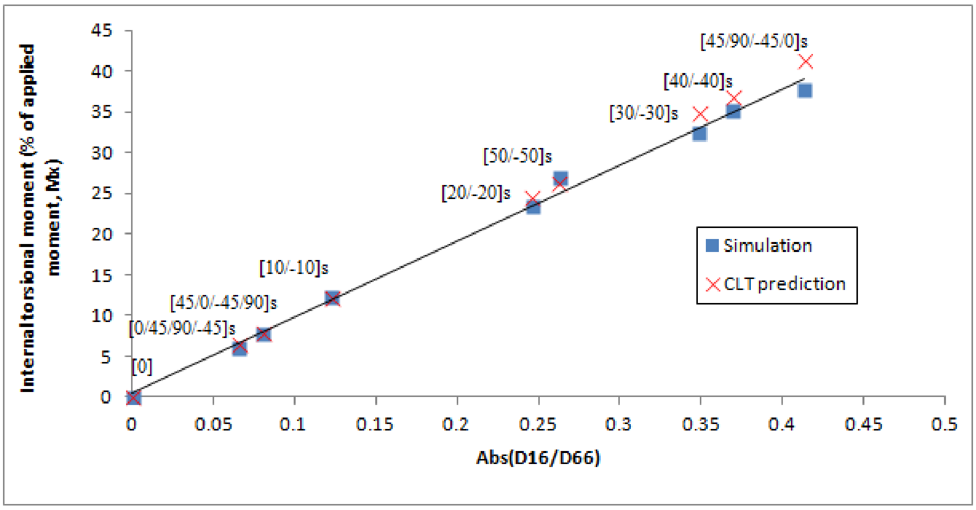

Figure 5. Internal torsional loading as a function of absolute D matrix ratio for angle-ply and quasi-isotropic laminates subject to four-point bending tests.

5. Conclusions

Predictions of the out-of-plane interlaminar stresses for coupons in four-point-bending show a sharp rise in magnitude when approaching the free edge. The margins of safety based on the maximum interlaminar stresses cannot be considered negligible, and are of the same order of magnitude as the margin for the in-plane stress in the fibre direction. Angle ply laminates in four-point bending do not produce the same internal stress state as a pure moment would achieve, and the resulting out-of-plane deflection is reduced. The internal torque that is generated due to the physical constraint of the test apparatus has a direct relationship with the D16/D66 ratio for the case of a simply-supported four point bend test. These results highlight that with thin angle-ply laminates, it is not always correct to assume a state of plane stress when in four-point-bending. Edge effects and induced internal loading can create out-of-plane stresses that may not be negligible when considering margins of safety for design purposes.

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