PHYSICAL MICRO-FIELDS IN LARGE ASPECT RATIO FIBROUS COMPOSITES BY MESHLESS METHOD

V. Kompiš[a*], Z. Murčinková[b], M. Žmindák[c]

[a]*Faculty of Management Science and Informatics, University of Žilina, Univerzitná 1, 01026 Žilina, Slovakia.*
[b]*Faculty of Manufacturing Technologies, Technical University of Košice, Štúrova 31, 08001 Prešov, Slovakia.*
[c]*Faculty of Mechanical Engineering, University of Žilina, Univerzitná 1, 01026 Žilina, Slovakia.*
[*]*vlado.kompis@gmail.com*

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Abstract

The paper deals with modelling of physical micro-fields, mainly thermal and mechanical, in large aspect fibrous composite materials. Interactions of matrix and reinforcing fibres involve high gradients of physical micro-fields that caused difficulties in reliable numerical simulation of composite behaviour. The developed Method of Continuous Source Functions (MCSF) eliminates disadvantages of known numerical methods and reduces the solution considerably. It uses fundamental solution and its derivatives to simulate the interaction of large aspect ratio fibres with matrix. Material properties of both matrix and fibres are assumed to be homogeneous and isotropic. The results of numerical examples and micro-fields distribution are shown in paper.

1. Introduction

Fibre-reinforced composites have been widely used in engineering applications due to the superiority of their electro-thermo-mechanical properties over the single matrix. Understanding the physical behaviour of these fibre-reinforced composites is essential for structural design. The aim of this chapter is to show the Method of Continuous Source Functions (MCSF) developed by the first author [1] for composite materials reinforced by finite fibres. We will give the mathematical basis of the MCSF, some results and discussion to the toughening mechanism conductivity of composite materials reinforced by straight fibres of finite length.

2. Computational model

We assume all matrix materials and fibres are homogeneous and isotropic, the dimensions of the matrix are infinite (i.e. we will deal with the infinite matrix of material with homogeneous material properties) and models are restricted to linear behaviour. Primary variables can be scalar like the temperature field in heat conduction, or vector like the displacement field in deformation of elastic bodies by forces. All fields are split into two parts, the homogeneous part corresponding to the homogeneous problem of the matrix without fibres and the local (complimentary) part simulating the influence of interactions of fibres with the matrix. We will especially investigate the local fields in the matrix of the composite material.

Due to large aspect ratio of fibres, the toughening in elasticity and increase of conductivity in heat conduction of the composite material is realized especially by corresponding effect in the fibre axis direction. Because of this, the inter-domain compatibility between fibre and matrix can be simplified and to assume that the temperature, displacement, strain, stress, etc in all points of the cross-section are equal to each other. This property of the model is equivalent to assumption of zero bending stiffness of fibre, which is important to reduction of computational model and to correct simulation of composite materials reinforced by fibres with large aspect ratio as the nanotubes and similar fibres. The inter-domain compatibility are satisfied in discrete collocation points (Figure 1) on the fibre-matrix interface.

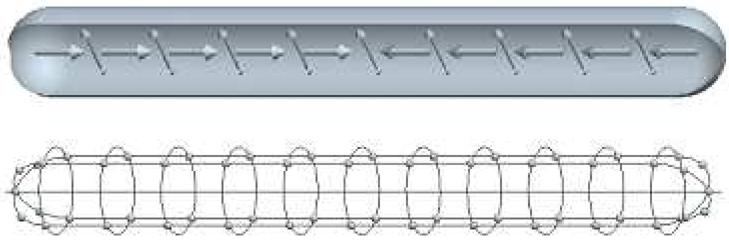


Figure 1. Distribution of source functions and collocation points.

The interaction of fibres with matrix is simulated in the MCSF by source functions, which are 1D-continuously distributed along the fibre axis. The source functions are fundamental solution of corresponding problem (heat sources in heat conduction and forces in elasticity) and their derivatives. The forces are directed in the fibre axis direction. These source functions, however, are not able to simulate correct the interaction of fibres with the matrix. In addition the derivatives of the source functions, heat dipoles and force dipoles and force couples are included along fibre axes in order to simulate correctly the large axial stiffness of fibres to negligible bending stiffness and also the interaction of fibre with other fibres. The dipoles and couples are derivatives of the source functions.

Temperature field induced by a unit heat source acting in arbitrary point of infinite domain is the fundamental solution for heat problems and it is given by:

$$T = \frac{1}{4\pi r} \tag{1}$$

r is the distance of the field point *t* and source point *s,* where the heat source is acting at, i.e.

$$r = \sqrt{r_i r_i}, \quad r_i = x_i(t) - x_i(s) \tag{2}$$

with the summation convention over repeated indices.

Temperature field induced by a unit heat dipole in x_i direction is:

$$T = \frac{1}{4\pi}\left(\frac{1}{r}\right)_{,i} = -\frac{1}{4\pi}\frac{1}{r^2}r_{,i} = -\frac{1}{4\pi}\frac{x_i}{r^3} \tag{3}$$

where

$$r_{,i} = \frac{\partial r}{\partial x_i(t)} = \frac{r_i}{r} \tag{4}$$

Similarly displacement field in an elastic continuum caused by a unit force (upper index F denotes force) acting in the direction of the axis x_p is given by the Kelvin solution as it is known from BEM [2]:

$$U_{pi}^{\mathbf{F}} = \frac{1}{16\pi G(1-\nu)r}\left[(3-4\nu)\delta_{ip} + r_{,i}r_{,p}\right] \tag{5}$$

where G and ν are shear modulus and Poisson's ratio of the material of the matrix. δ_{ij} is the Kronecker's delta.

In the numerical evaluation we have to solve an integral equation, in which the intensity of source functions is approximated by 1D quadratic Non-Uniform Rational Basis Splines (NURBS) [3], which enable to define shape functions to have continuous first derivative over the whole integration path and non-equal distribution of nodal points. Basic variables are temperature in heat and displacement in elasticity problems.

The computations are performed on the homogeneous field of matrix material and boundary conditions (BC) are prescribed in collocation points along fibre boundaries. In the present models it is considered that the fibres are straight and parallel. As the BC are not known a priori, the problem is solved iteratively and it is assumed that the fibres are superconductors in heat and rigid in elasticity problems in the first iteration step. This is equivalent to the assumption of constant gradient of temperature and displacement in fibre axes direction, if the fibres are straight. This corresponds to constant heat flow in heat problem and constant strain in elasticity along a fibre. Finite heat flow and temperature distribution in the heat flow problem and strain and displacements in elasticity along fibres are computed in the next steps of iteration process.

The temperature/displacement change of the centre of each fibre by the interaction is not known a priori in both the heat and elasticity problem. It is obtained by energy-balance/equilibrium condition in each fibre. This is realized by including further r.h.s. (right hand side) by prescribing temperature/displacement in corresponding fibre centre equal to one and zero for the other fibres.

See [1,4] for more details about the computational model.

2. Computational results

As the material of both fibres and matrix is considered to be linear, all variables are dimensionless and so, it is supposed that material constants of the matrix, conductivity k_m and modulus of elasticity E_m, are equal to one. The fields in heat conduction and elasticity are similar, temperature correspond to displacement and heat flow to stress/strain. Because of restriction on the length of paper three examples are chosen to show the composite behaviour. In the examples, the radius of fibres R=1, the length of fibres L=1000 in the first (heat conduction) and L=100 in the last two (elasticity) problems. The fibres are regularly distributed, parallel to each other with overlay in fibres' direction x_3. The gaps in fibre direction x_3 is 40 in the first two examples and 5 in the third one and the distance between fibres in perpendicular direction is 100, 10 and 4, respectively in the examples (see Fig. 2). All quantities concerning fibres are in fibre direction x_3.

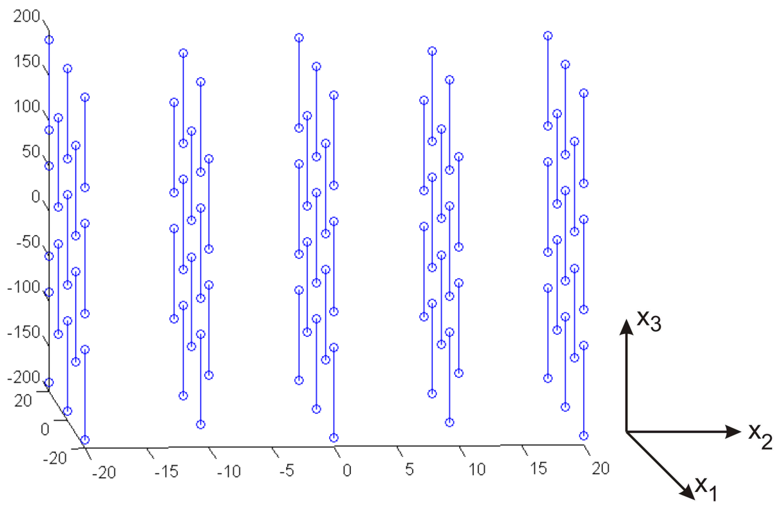


Figure 2. Fibre alignment in the matrix in example 2.

Heat flow in corresponding homogeneous cases will be q_3 = 1 and strain ε_{33} = 1. The material of fibres is close to superconductive material with coefficient of conductivity k_f = 50 000 in the first example and the modulus of elasticity is E_f = 1000 for elasticity problems. The fibres are arranged symmetrically in all directions.

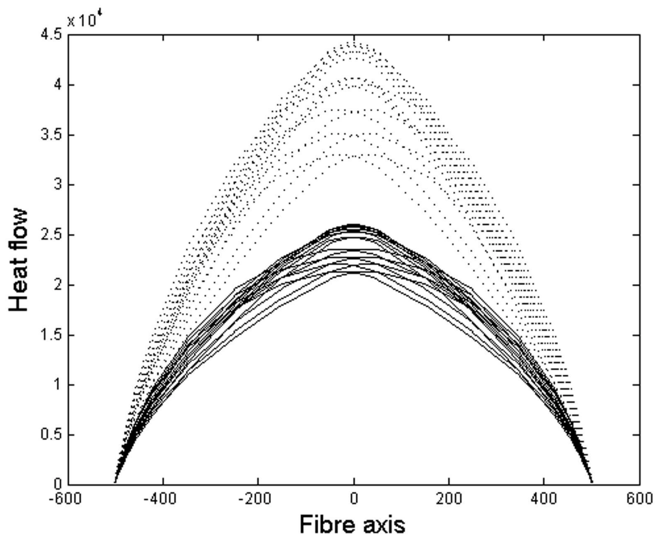


Figure 3. Heat flow along fibres.

The models contain finite number of fibres in infinite matrix (63 fibres, Fig. 2). Heat flow and temperature along fibres relative to the temperature in the fibre centres are shown in Figures 3 and 4, stress and displacement in fibre axes direction of points along fibre (also related the displacement in the centre of corresponding fibre) are presented in Figures 5 and 6. The results of the first iteration steps are shown by dashed lines, those of the last iteration step by full line. Temperature/displacement changes in the composite relative to their values for homogeneous material (given in parentheses) in the centre of fibres are given in the Tables 1 and 2.

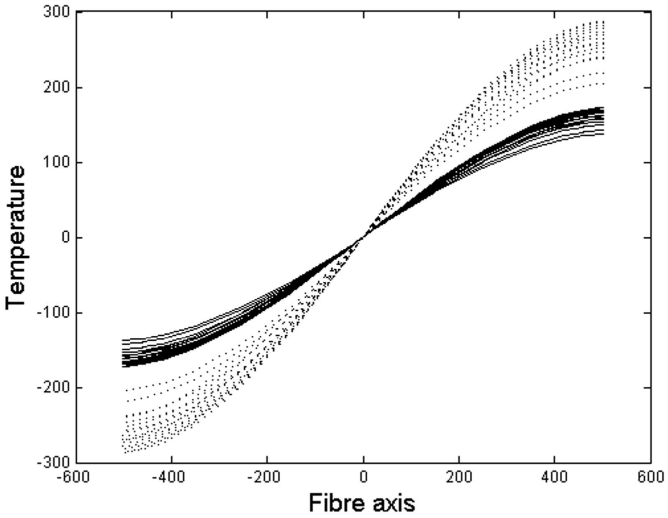


Figure 4. Temperature along fibers.

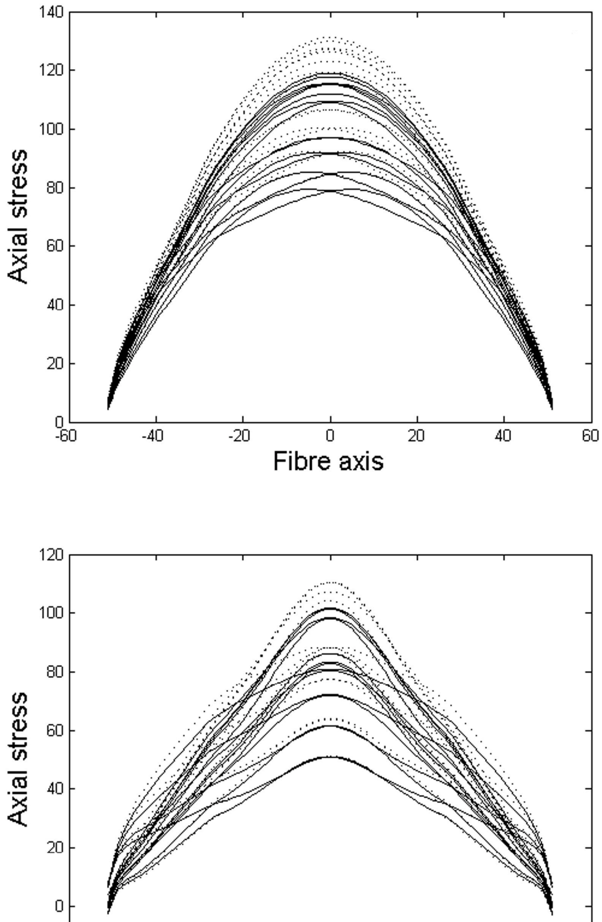
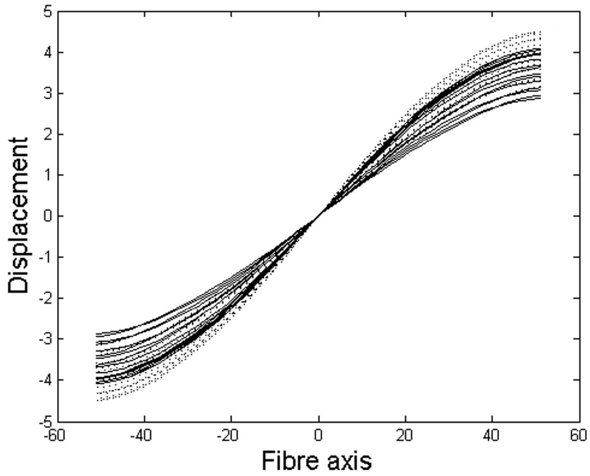


Figure 5. Axial stress along fibres for examples 2 and 3.



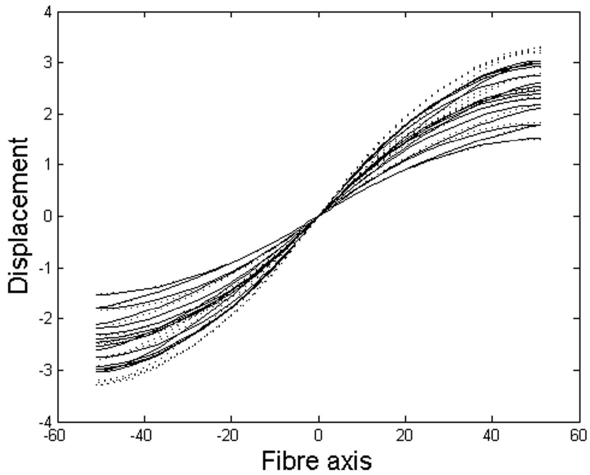


Figure 6. Displacement in axis direction along fibres for examples 2 and 3.

Level x_1, x_2, x_3	**2, 1, 1**	**0, 1, 1**	**1, 1, 2**	**0, 0, 2**	**2, 0, 2**	**2, 2, 2**
temperature	(520)-44.8	(520)-106.8	(1040)-244.4	(1040)-261.1	(1040)-224.5	(1040)-194.5

Table 1. Temperature change in the fibre centres

Level x_1, x_2, x_3	**2, 1, 1**	**0, 1, 1**	**1, 1, 2**	**0, 0, 2**	**2, 0, 2**	**2, 2, 2**
Displacement in example 2	(70) - 21.54	(70) - 24.82	(140) - 54.75	(140) - 58.76	(140) - 49.84	(140) - 42.41
Displacement in example 3	(52.5) - 36.03	(52.5) - 40.58	(105) - 82.21	(105) - 86.34	(105) - 76.60	(105) - 68.15

Table 2. Displacement of the fibre centres

3. Discussion and conclusions

In the paper we present the use of MCSF for computational simulation of the mechanism of increase conductivity and toughening by short fibres in composite materials. The MCSF is a quasi-meshles method using 1D continuous distribution of source functions along fibre axes so, that the interdomain compatibility between fibres and matrix is satisfied in collocation points. Present models simulate the interaction of matrix with a patch of finite number of fibres regularly distributed in an infinite matrix. Material of the fibres is supposed to be considerably better conductor and considerably stiffer than the matrix. As the model has some inaccuracies and restrictions, it will help to understand the composite material properties from the influence of fibres, their configuration and topology.

Although the temperature (scalar) field in heat conduction and displacement (vector) field in elasticity are described by integral equations with different intensity of the singularity (stronger singularity is in the elasticity problem), the behaviour of both problems is similar. Equivalent fields in both problems are temperature/displacement and heat flow/strain (stress).

As for the presented models simulate the local reinforcement of the matrix, one can observe that the fields in middle fibres and those on the boundaries of the reinforcement behave differently and the fields are not symmetric along fibres. Similar effect is to be expected on the boundaries of composite of finite dimensions.

The ends of fibres influence strongly the fields not only in the matrix, but in neighbour fibres as well. This effect is not so clear from the presented figures as all fibres are shifted so that their centres have coordinate $x_3=0$. One has to imagine the configuration of fibres in the space, the overlap and gaps in fibre direction in order to understand better the figures. This and previous studies [5,6] on the fields near fibre ends document the importance of detail studies of these fields for evaluation of interaction of closest fibres and matrix for understanding the total composite behaviour and defining macroscopic material properties from microscopic fields.

The results document that the finite number of fibres in the infinite matrix simulates partial reinforcement of the matrix. This is similar to the composite inclusion in a homogeneous matrix. It is clear that also other BC (the form of the finite domain, interface with part of different material properties, etc.) are parameters influencing the resulting material properties. The present models simulate straight fibres, however, the fibres in real material can be curved with random configuration. The MCSF can be used also for such cases, but further generalization of models will be necessary (to simulate cross sectional changes of temperature and displacements in fibres by BC on fibre-matrix interface to local direction of fibre axis, etc.). Also the RVE for homogenization has to contain larger number of fibres for more general configuration in order to represent the material properties in macro-dimension.

In present models we solve problem for infinite matrix with BC along fibre boundaries and the finite material fibre properties are included iteratively. This is because the BC are not known a priori and the corrections are obtained integrating results from the preliminary steps. If the material properties of fibres and matrix are too different and fibres are not too close to each other, the solution is obtained in one or two iterations. In problems like those presented here, the convergent results are received in not more than 8 steps.

Present models do not include nonlinear effects in all material behaviour of matrix and fibres, large deformations, etc. We have to realize that the micro-fibres will be often curved and the curvature can change with deformation and this will contribute to nonlinear behaviour of composite, too.

The models were programmed in MATLAB and they are suitable for parallel computing and both versions of the programs are available.

We would like to emphasize that the models can be used to simulate also composite materials reinforced by nanofibres. Although the nanofibres have cross-sectional dimensions in nanometres and so, the methods of continuum mechanics cannot be used to model such structure, if however the aspect ratio of the fibres is large, then the length dimensions satisfy conditions when continuum mechanics can be used for the simulation.

One can obtain also other fields from the models, especially those from elasticity, like shear stresses in the fibre-matrix interface, maximal stresses in the matrix, etc. [5], which are important for damage and fracture analysis of this kind composite materials. It is expected that all kind of problems will be important for design and optimization of the composite materials. In this way one can obtain functionally graded materials (FGM) with properties suited to the form and special conditions of use of the structure.

MATLAB codes for parallel and serial computations of the models can be obtained from and any questions will be answered by vlado.kompis@gmail.com.

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