MICRO-MECHANICAL MODELLING AND VALIDATION OF PROGRESSIVE ELASTO-PLASTIC DAMAGE OF SHORT WAVY STEEL FIBRE COMPOSITES

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Abstract

This paper describes modelling approaches for the prediction of the non-linear behaviour of short wavy steel fibre composites. The suggested modelling approach is based on the well-known Mean-Field homogenization (MF) techniques. A model is presented for the extension of mean-field solutions typically used for ellipsoidal inclusions to the case of short wavy fibres. Progressive damage models are developed for prediction of the non-linear behaviour of the elasto-plastic fibre and matrix. Finally, non-linear damage behaviour of short steel fibre composites is modeled with selective anisotropic degradation schemes.

1. Introduction

Short wavy steel fibre reinforced composites produced with injection moulding are new materials of interest, especially for the automotive industry due to their advantageous properties of combining high stiffness and ductility. Pressures developed during processing of the short steel fibre composites lead to complex micro-structures with development of stochastic wavy geometry of the steel fibres embedded in the polymer matrix and local entanglements of fibres.

Component size simulations of short steel fibre composites require a multi-scale approach, for which Eshelby based mean-field homogenization models provide a very cost-effective way of modelling local material behaviour at micro-scale. Nevertheless, these models in their known form are typically limited to the simple geometries of RVEs of straight fibres.

In this work, a novel modelling approach has been developed for extension of the mean-field approach to modelling of wavy fibres. The model is based on subdividing a wavy fibre into a number of smaller segments. Each segment is then modelled by an equivalent inclusion by assigning an effective aspect ratio of the equivalent ellipsoidal inclusion as a function of the local fibre curvature. The model is validated against FE simulations of the simulated wavy fibres as well as "real" geometries of wavy fibres extracted from micro-CT information. Combined elasto-plastic constitutive material behaviour is modelled reflecting weakening constraint of matrix and plasticity of steel fibres. Finally, a progressive damage model is developed reflecting the main damage modes observed in fractography analysis of the short steel fibre composite namely: fibre-matrix debonding and fibre breakage.

2. Mori-Tanaka Approach

In modelling fibre reinforced composites, a class of models exists which is based on Eshelby [1] solution for ellipsoidal inclusions, from which several mean field averaging schemes are derived for different types of materials. Among those schemes, the Mori-Tanaka method [2] is the most popular especially for the micro-mechanical modelling of sheet molded compounds (SMC), particle reinforced composites, and random fibre reinforced composites.

For straight fibre composite systems, fibres (inclusions) are approximated by generalized ellipsoids, for which the dilute Eshelby solution is known either analytically or numerically. Using the Mori-Tanaka formulation, the effective composite stiffness can be calculated as shown in Equation 1

$$C^{eff} = C^m + \sum_{\alpha=1}^{M} c_\alpha \left(C^\alpha - C^m\right) A^\alpha \tag{1}$$

Where C^{eff} is the composite effective stiffness, C^m and C^α are the stiffness of matrix and inclusion respectively, A^α is the strain concentration tensor which relates the strain inside of the inclusion to the macroscopic applied strain, c_α is the volume fraction of individual inclusions. The inclusion and matrix strain concentration tensor A^α and $A_m{}^\alpha$ can be calculated as in Equation 2, where $S^{\alpha\alpha}$ is the Eshelby tensor.

$$A^\alpha = A_m{}^\alpha \left(\sum_{\beta=1}^{M} c_\beta A_m{}^\beta \right)^{-1} \qquad A_m{}^\alpha = \left[I + S^{\alpha\alpha} \left(C^m\right)\left(C^\alpha - C^m\right) \right]^{-1} \tag{2}$$

Nevertheless, for more complex composite structures such as short random steel fibre composites, the fibres are curved (wavy) and the derivation of the Eshelby solution becomes mathematically complex. In this respect, a model is needed for the transformation of wavy fibres into an equivalent ellipsoidal inclusions system for which mathematical formulations are readily available.

3. Modelling scheme for the prediction of the non-linear behaviour of short wavy steel fibre composites

3.1.The Poly-Inclusion Model (P-I)

The P-I model is a proposed model for the extension of Mori-Tanaka formulation to the case of wavy fibres composites. In the P-I model, each wavy fibre is subdivided into a sufficiently large number of small segments. The original yarn segments are replaced by equivalent ellipsoidal inclusions such that the average stress and strain states in both the original segment and the ellipsoid are the same. In that way, available mean field approaches based on Eshelby solution for ellipsoidal inclusions can be readily used. The aspect ratio of the equivalent ellipsoid is a function of the local fibre curvature.

Using a simple mathematical formulation, Huysmans et al. [3] presented the relationship between local curvature and equivalent ellipsoid aspect ratio as given by Equation 3.

$$b = \beta * R \tag{3}$$

Where b is the elongation (length) of the equivalent ellipsoid, R is the local radius of curvature of the segment (Figure 1). Factor β is the proportionality between both parameters.

Huysmans et al. applied the P-I model on knitted textile composites. They performed an analysis for the evaluation of the best values of the factor β by comparison of the predicted in-plane elastic constants upon the parameter β using the Mori-Tanaka approach with those found in experimental data. They found that the best correlation with experimental data was obtained with $\beta = \pi / 2$.

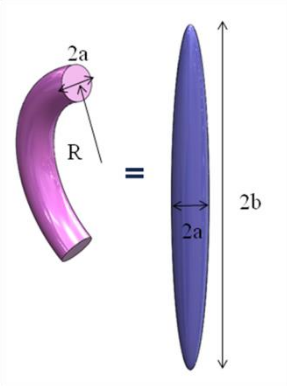


Figure 1. Definition of the equivalent ellipsoid replacing the original wavy fibre segment.

3.2.Plasticity model

The non-linear elasto-plastic behaviour of short steel fibre composites was modeled using the secant stiffness approach developed by Tandon and Weng [4]. Yielding of the matrix was applied using a von Mises criterion and an associated flow rule.

The stress and plastic strain of the polymer matrix and steel fibre is assumed to be represented by the modified Ludwik equation shown in Equation 4.

$$\sigma^* = \sigma_y + h \cdot (\epsilon^{p*})^n \tag{4}$$

Where σ^* is the von misses effective stress at the effective plastic strain ϵ^{p*}, σ_y, h, and n are the intial yield stress, the strength coefficient and the work-hardening exponent respectively.

In addition, the flow rule (Equation 5) is adopted.

$$\epsilon_{ij}^{p} = \frac{3}{2} \cdot \frac{\epsilon^{p*}}{\sigma^*} \cdot \sigma'_{ij} \tag{5}$$

σ'_{ij} being the deviatoric stress.

In the present model, the plasic matrix and steel fibre is replaced at every load step by linear elastic reference materials having the same secant properties as the original materials during this load step.

3.3.Progressive damage model

A partially debonded isotropic curved fibre segment is replaced by a perfectly bonded segment with degraded anisotropic stiffness properties degraded by the damage variables along the interface of debonded fibre and distinguishing between percentage of debonded surfaces loaded in tension or compression. To achieve that, three distinct micro-mechanical variables (d, γ, δ) are assigned to each inclusion. Where d represent the total percentage of debonded area along the inclusion equator, γ is the total amount of debonded surface loaded in tension, and δ is the relative amount of debonded surface loaded in compression (Figure 2).

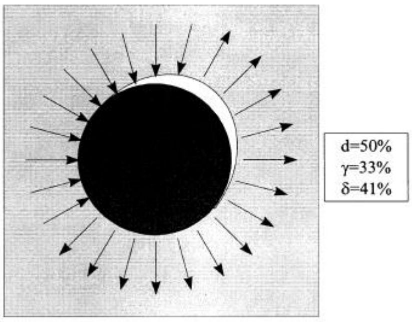


Figure 2. Example of a partially debonded yarn section with determination of the damage variables d, γ, δ [5-6].

During progressive loading, the damage parameters are easily obtained along the inclusion's equator and once they are updated, the segment stiffness components are degraded accordingly leading to an anisotropic degraded inclusion.

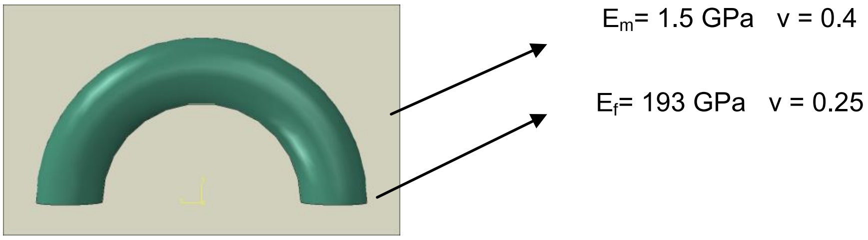
Fibre breakage occurs when the stress state in the equivalent inclusion reaches its ultimate strength in which case, in the present model, the broken fibre is replaced by 2 segments each with half the original length.

4. Methodology

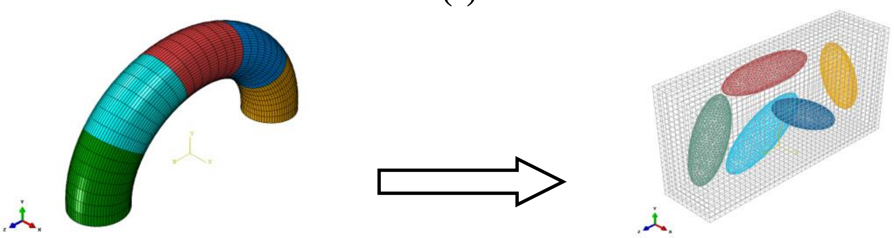
The P-I model has been only validated by Huysmans et al. for prediction of the overall macroscopic elastic constants against experimental values. Nevertheless, the model is based on the assumption that the stress state in the curved fibre segments is simulated by the stress state in the equivalent ellipsoids. This assumption has not been validated. Moreover, damage modelling requires accurate preditions of the local stress state. For this aim, a simple geometry of a single wavy fibre RVE with circular curvature pattern was considered. In this work validation of the P-I model was performed on the basis of the overall homogenized elastic constants as well as the detailed local stress state in inclusions against full FEA. The next step is the validation of the P-I model for a "real" assembly of short wavy steel fibres extracted from micro-CT scanned samples. For further modelling of the non-linear and damage behaviour of short steel fibre reinforced composites, the above described plasticity model and damage models (i.e. debonding, fibre breakage) are investigated and investigated first for simple cases of single short steel fibre composites. Finally, the proposed methodology for modelling of the non-linear elasto-plastic deformation and damage behaviour of short steel fibres is applied on an RVE of steel fibe reinforced polyamide composite and validated against experimental data.

5. Results and discussion

Figure 3 (a) shows the single wavy fibre RVE under consideration. The fibre is given a simple circular curvature. The fibre and matrix materials were given typical values for Polypropylene and Steel Fibres respectively. Figure 3 (b) shows a schematic of the application of the P-I model.



(a)

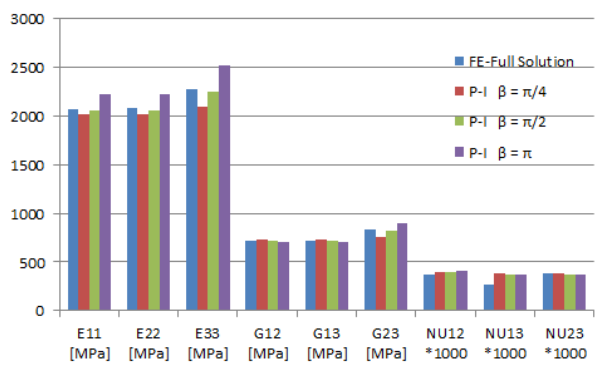


(b)

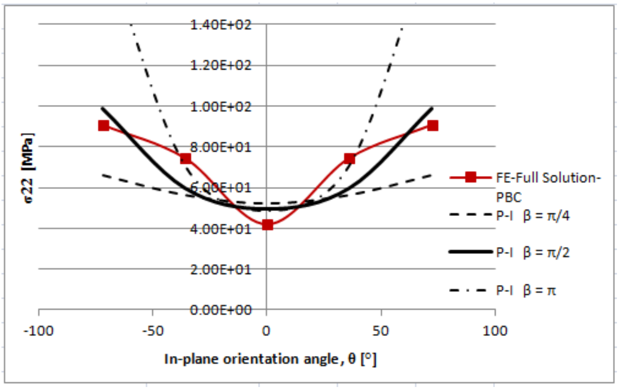
Figure 3. Validation of the P-I model. (a) RVE of a single wavy fibre composite with a simple half cylindrical curvature (b) Schematic representing the application of the P-I model.

Figure 4 shows the results of the validation of P-I model for a single fibre RVE with simple half-circular waviness pattern. Figure 4 (a) shows the predicted overall elastic constants by the P-I model with different β values against FEA, Figure 4 (b) shows the comparison of the predicted local stress states in equivalent ellipsoids of the P-I model against FEA results of stress states in original segments.

Figure 4 (a) shows that the predictions of elastic constants by P-I model showed very good agreement against FE. $\beta = \pi/2$ yielded the most accurate results. Figure 4 (b) showed that the P-I model with $\beta = \pi/2$ gives very good agreement with FEA for accurate prediction of the local stress state in the wavy segments. This provides very good means for precise modelling of local damage of wavy fibre RVEs. The value of β is the same as that suggested by Huysmans et al. for textile composites.



(a)



(b)

Figure 4. Comparison of P-I model and Full FEA for single fibre RVE of half circular wavy fibre. (a) Comparison of overall macroscopic elastic constants (b) comparison of local stress state in equivalent inclusions against stress in original segments.

Figure 5 shows the assembly of short wavy steel fibre composites used for the validation of the P-I model. The assembly of fibres is extracted from micro-CT scans of a real short wavy fibre reinforced composite sample. The figure shows the high degree of waviness of the steel fibres embedded in the matrix.

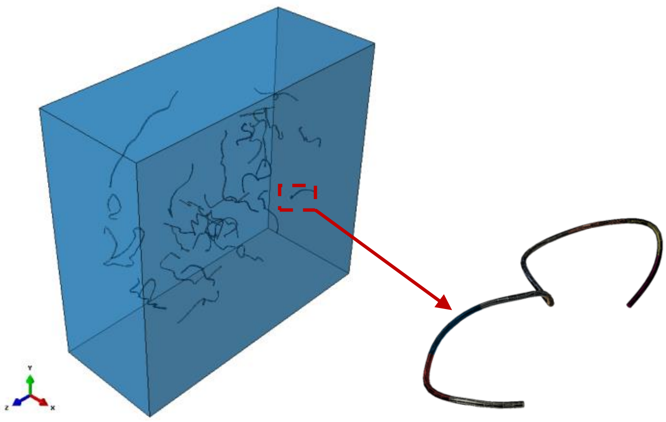


Figure 5. Validation of the P-I model for an assembly of "real fibres" extracted from micro-CT samples.

Figure 6 shows the results of the validation of P-I model on an assembly of short wavy fibres against full FEA. The figure shows the local stress state of a random fibre within the assembly predicted by the P-I model against the corresponding FE stress state. The figure indicates very good agreement between the predictions of the P-I model and the real stress state calculated from full FE analysis, which provides very strong basis for further predictions of damage behaviour of short wavy steel fibre composites.

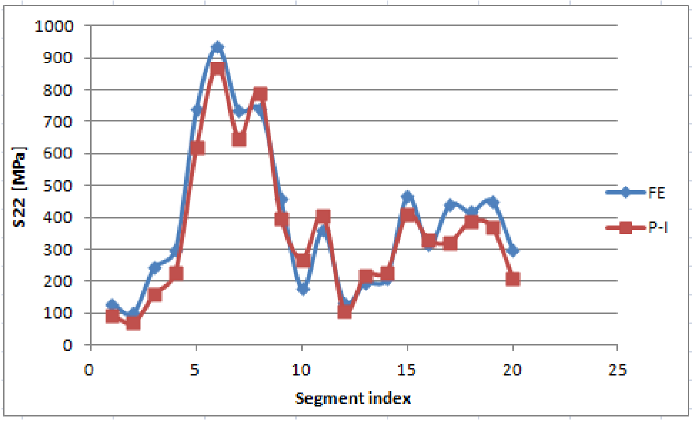


Figure 6. Validation of the P-I model for an assembly of "real fibres" extracted from micro-CT samples.

Figure 7 presents the predictions of the proposed plasticity model applied for a single short steel fibre polyamide composite with different fibre volume fraction c1. The figure indicates the ability of the developed model for the prediction of the onset of yielding of the composite as well as the non-linear composite plastic deformation. To verify the implementation of the model, the predicted stress-strain curve of the composite with c1=0 is compared to the experimental stress-strain curve of the polyamide matrix. The predictions showed excellent agreement with the experimental curve suggesting that the model can further be used for the prediction of the onset of yielding and the plastic deformation behaviour of more complex assemblies of inclusions.

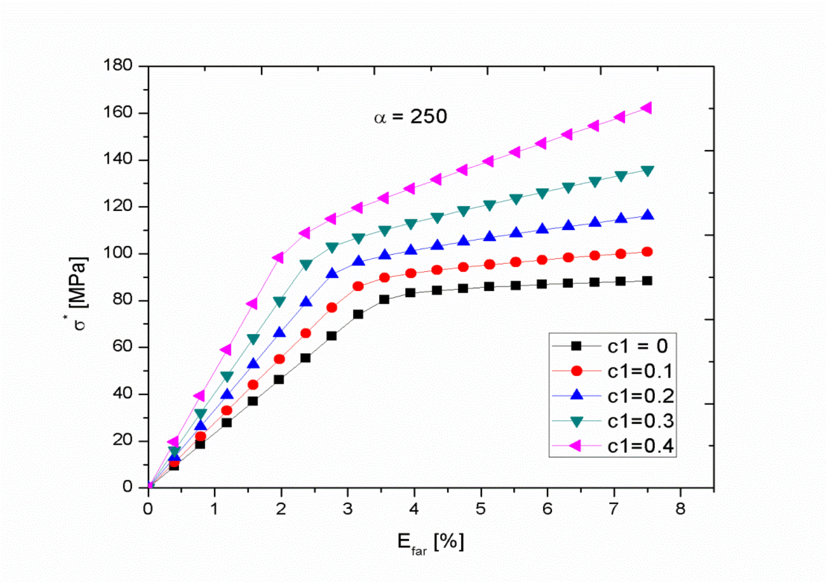


Figure 7. Non-linear stress-strain curve predicted for a single short steel fibre reinforced composites predicted by the proposed plasticity model.

Figure 8 shows the results of the implementation of the debonding model for the extreme case of a single steel fibre polyamide composite subjected to transverse loading. The figure shows the prediction of progressive debonding of the steel fibre until complete separation from the matrix.

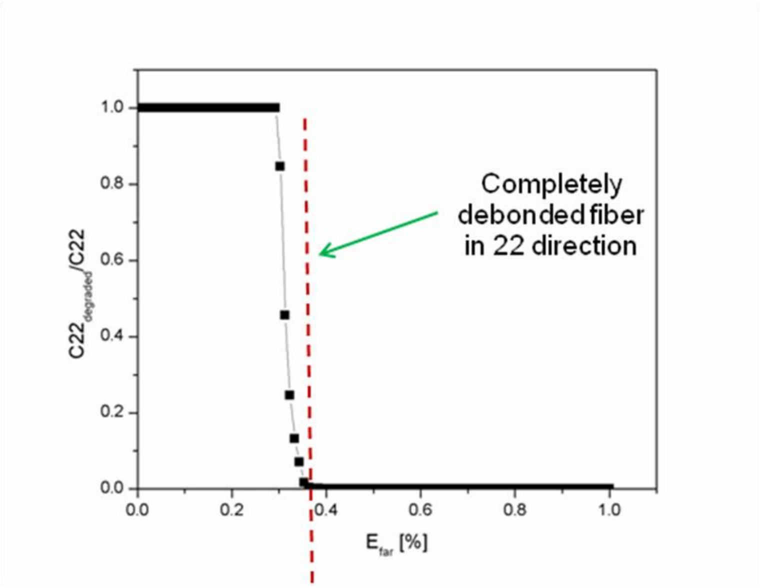


Figure 8. Progressive debonding of a single short steel fibre as predicted by the developed debonding model.

6. Conclusions

A modelling methodology is presented for prediction of the overall non-linear tensile behaviour of short steel fibre composites, starting by a model for extension of mean-field solutions to short wavy fibres followed by progressive damage modelling accounting for non-linear behaviour of elasto-plastic fibre and matrix, fibre-matrix debonding modelled with a selective anisotropic degradation scheme and finally fibre breakage. The proposed methodology provides means for modelling the mechanical behaviour of a wide range of way fibre composite systems exhibiting non-linear deformation behaviour.

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