POST-BUCKLING ANALYSIS OF PIEZO-COMPOSITE LAMINATE WITH THROUGH-THE-WIDTH DELAMINATION BASED ON LAYERWISE THEORY

N.Chitsaz[a*], H.R.Ovesy[a], M.Kharazi[b]

[a] *Department of Aerospace Engineering, Amirkabir University of Technology, Tehran, 424 Hafez Avenue Iran*
[b] *Department of Mechanical Engineering, Sahad University of Technology, Sahand new Town, Tabriz, Iran*

**chitsaz.nasim@gmail.com*

Keywords: Piezo-composite, Delamination, Post-buckling, layerwise method

Abstract

In this paper, an investigation analytically of the post-buckling behavior of rectangular cross-ply piezo-laminate with through-the-width delamination is performed. The Rayleigh–Ritz method has been adopted and displacement fields are obtained by incorporating polynomial series. The equations, which are achieved through the application of the principle of Minimum Potential Energy, are solved by employing the Newton–Raphson iterative procedure. Some interesting results are obtained and compared with those achieved by the FEM analysis and is shown that this method is capable to predict post-buckling analysis of delaminated piezo-composite material. Piezo layers acts as actuator in this study. A good agreement is seen to exist between the results.

1. Introduction

Recently, there has been an increasing interest in using piezoelectric materials in advanced structures to transform them into "smart" or adaptive structures. The piezoelectric effect has been widely employed in a large number of applications, ranging from ultrasonic transducers and wave filters to strain gauges and MEMS (micro-electro-mechanical systems). Mitchell and Reddy have developed plate theories for composite laminates with embedded and/or surface mounted piezoelectric sensors and actuators [1]. Numerous finite element studies have also been conducted [2-5]. Varelis and Saravonas developed an 8 node nonlinear plate element for coupled buckling and post buckling analysis of piezoelectric laminate and shown reducing mechanical buckling and post buckling with the piezo-actuator [6]. Kapuria Developed a benchmark 3-D piezoelasticity exact solution based on a transfer matrix for buckling of simply-supported laminate hybrid plates with embedded or surface bonded piezoelectric layers and showed that the electric boundary conditions plays an important role in buckling as previously shown [7]. Similar research could be found at [8-11].

Delamination has been a subject of major concern in engineering application of composite laminates. Many delamination related studies have primary focused on prediction of the buckling load and post-buckling behavior. In 2008, Kharazi and Ovesy investigated the compressive stability behavior of composite laminates with through-the-width delamination by developing an analytical method based on Rayleigh–Ritz approximation technique by consideration of CLPT requirements. The method could handle both local buckling of the delaminated sublaminate and global buckling of the whole plate [12]. Ovesy and Kharazi studied the compressional stability behavior of composite plates with through-the-width and embedded delamination by using first order shear deformation theory (FSDT) and its formulation is developed on the basis of the Rayleigh–Ritz approximation technique [13]. Ovesy et al. investigated the post-buckling behavior of composite laminates containing embedded delamination with arbitrary shape. The nonlinear equilibrium equations of the delaminated plates were obtained by using the Rayleigh–Ritz approximation technique and the higher order shear deformation theory was implemented in the formulation[14]. In 2014, Kharazi et al. presented a novel layerwise theory based on FSDT has been developed to evaluate the buckling load of delaminated composite plates with through-the-width and rectangular embedded delamination. The Rayleigh–Ritz method has been adopted and displacement fields are obtained by incorporating polynomial series. to investigate the Buckling analysis of delaminated composite plates using a novel layerwise theory [15].

In this paper, post-buckling analysis of piezo-composite laminate with through-the-width delamination is presented. The analytical method is based on a new layerwise theory which presented in [15]. The main benefit of this method is considering zigzag displacement field through the thickness of piezo-composite and material discontinuity.

The finite element analysis is also carried out using ANSYS5.4 commercial software for results verification. A 3D FEM plate has been modeled in Ansys software and post-buckling analysis of composite plate with two piezo layers at top and bottom of plate has been performed. The agreement between the results is very good.

2. Modeling of the through-the-width delamination

In this section, analytical model of piezo-composite laminate based on layer-wise theory has been described. It mentioned below that layer-wise theory is based on ref [15]. In this theory, delaminated composite is divided into different plates in two general steps. First, delaminated plate is divided into four regions. Second, for modeling the zigzag behavior of in-plane displacement through the thickness, each region is divided into a number of plates through the thickness. It should be mentioned that considering computational time and accuracy, the number of division through the thickness could be varied from one plate to number of layers. The first order shear deformation theory (FSDT) has been considered for each plate displacement field. The FSDT are expressed as:

$$u(x,y,z) = u_0(x,y) + z\phi_x ; v(x,y,z) = v_0(x,y) + z\phi_y \tag{1}$$
$$w(x,y,z) = w_0(x,y)$$

Where u_o, v_o, and w_o are displacements at the reference plane z = 0 (or mid-plane) and ϕ_x and ϕ_y are the rotations of the transverse normal plane about the y and x axis, respectively. Strain vector could be followed as:

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\frac{\partial u}{\partial x}+\frac{1}{2}(\frac{\partial w}{\partial x})^2\\ \frac{\partial v}{\partial y}+\frac{1}{2}(\frac{\partial w}{\partial y})^2\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+(\frac{\partial w}{\partial x})(\frac{\partial w}{\partial y})\end{Bmatrix},\psi=\begin{Bmatrix}\psi_{xx}\\ \psi_{yy}\\ \psi_{xy}\end{Bmatrix}=\begin{Bmatrix}\frac{\partial \phi_x}{\partial x}\\ \frac{\partial \phi_y}{\partial y}\\ \frac{\partial \phi_x}{\partial y}+\frac{\partial \phi_y}{\partial x}\end{Bmatrix},\gamma=\begin{Bmatrix}\gamma_{xz}\\ \gamma_{yz}\end{Bmatrix}=\begin{Bmatrix}\frac{\partial w}{\partial x}+\phi_x\\ \frac{\partial w}{\partial y}+\phi_y\end{Bmatrix} \tag{2}$$

Moreover, constitutive law for piezo-composite layers could be expressed as:

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0\\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45}\\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}-\begin{bmatrix}0 & 0 & 0 & \bar{e}_{14} & \bar{e}_{15}\\ 0 & 0 & 0 & \bar{e}_{24} & \bar{e}_{25}\\ \bar{e}_{31} & \bar{e}_{32} & \bar{e}_{36} & 0 & 0\end{bmatrix}^T\begin{Bmatrix}E_x\\ E_y\\ E_z\end{Bmatrix} \tag{3}$$

Where $[\sigma],[\tau]$ and $[Q]$ are the components of the stress tensor and transformed reduced stiffness coefficient. Based on equation (3), stress result is governed from two mechanical and electrical loads. Thus, for layers with no electrical load, second term of constitutive law would be considered zero. The constitutive equations can be compute using of equation (2) and (3) and appropriate integration through the thickness. These equations can be of a very general form which includes general anisotropy and full coupling between in plane and out of plane behavior equation 4.

$$\begin{Bmatrix}\{N\}\\ \{M\}\\ \{Q\}\end{Bmatrix}=\begin{bmatrix}[A] & [B] & 0\\ [B] & [D] & 0\\ 0 & 0 & [As]\end{bmatrix}\begin{Bmatrix}\{\varepsilon^0\}\\ \{\psi\}\\ \{\gamma^0\}\end{Bmatrix}-\begin{Bmatrix}\{N^P\}\\ \{M^P\}\\ \{Q^P\}\end{Bmatrix},(A_{mn},B_{mn},D_{mn})=\int_{-\frac{t}{2}}^{\frac{t}{2}}\overline{Q_{mn}}(1,z,z^2)dz;\mathrm{m,n}=1,2,6 \tag{4}$$

$$(As_{mn})=K_m\int_{-\frac{t}{2}}^{\frac{t}{2}}\overline{Q_{mn}}dz;\mathrm{m}=(4;5),\{N^P\}=\sum_{K=1}^{N}\int_{z_k}^{z_{k+1}}\left[\bar{e}\right]^{(k)}\{E\}^{(k)}dz,\{M^P\}=\sum_{K=1}^{N}\int_{z_k}^{z_{k+1}}\left[\bar{e}\right]^{(k)}\{E\}^{(k)}zdz,$$

Where K_m = 5/6 is shear correction factor and t is the thickness of laminate. For the plate, subjected to end shortening ε and continuous electrical field V, the total strain energy and external work could be followed as:

$$U=\frac{1}{2}\int_V\sigma^T\varepsilon dV=\frac{1}{2}\int_V(\varepsilon^TQ\varepsilon-\varepsilon^Te^TE)dV,W=\frac{1}{2}\int_V(E^TD)dV=\frac{1}{2}\int_V(E^TbE+E^T\bar{e}\,\varepsilon)dV \tag{5}$$

Where D, E, b and e are the electrical displacement filed, electrical field, electric permittivity matrix and piezoelectric coupling coefficient matrix, respectively. Electrical field is followed as:

$$E=-\partial V/\partial Z \tag{6}$$

Where, V is applied voltage and Z is the direction of polarization of piezo layers. It should be noted that, where no electric field is applied, external work is zero. In other word, external work is just only dependent on the electrical field. Finally, the total potential energy function is given by:

$$\Pi = U - W, \Pi = \frac{1}{2}\int_V (\varepsilon^T Q\varepsilon - \varepsilon^T e^T E)dV - \frac{1}{2}\int_V (E^T bE + E^T \bar{e}\ \varepsilon)dV \qquad (7)$$

The plate equilibrium equations are obtained by applying the principle of minimum potential energy. For this purpose, displacement filed is required for delaminated piezo-composite layers. It is obvious that for delaminated layers, different displacement field is required. In this study, laminate is divided in to 18 plates (Figure 1).

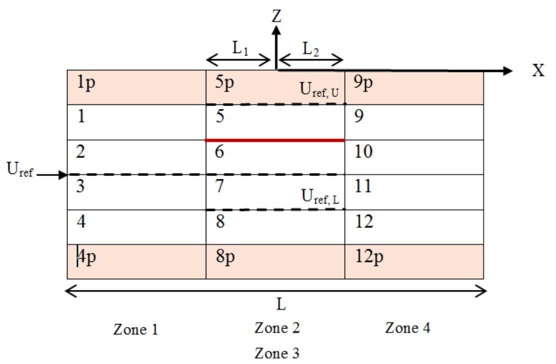

Figure 1. Division of Delaminated Piezo-composite region

2.1. Model Description and Boundary Condition

In the previous section, it was expressed that displacement field is required for the post-buckling analysis of piezo-composite laminates based on layer-wise theory. Displacement field should be defined to fulfill boundary condition at four regions and between different plates. In this study, polynomial relation has been considered for displacement field. Although, there is no restriction in this analytical method, clamped boundary condition on the edge of the plates has been selected for analysis. Moreover, piezo layers act as actuator and subjected to continuous electrical field. These boundary conditions are:

$$\begin{aligned} &W_{(1)}\Big|_{x=(-L/2)} = W_{(4)}\Big|_{x=(L/2)} = 0, \phi_{(1)}\Big|_{x=(-L/2)} = \phi_{(4)}\Big|_{x=(L/2)} = 0 \\ &(Voltage)_{-t/2} = -(Voltage)_{t/2} = V \end{aligned} \qquad (8)$$

The subscript (i=1,4) is the region's number. The requirement of the continuity of the displacements and the rotations at the boundaries of different region is satisfied as follows:

$$\begin{aligned} &(U,V,W)_{(1)}\Big|_{x=L_1} = (U,V,W)_{(2)}\Big|_{x=L_1} = (U,V,W)_{(3)}\Big|_{x=L_1} \\ &(U,V,W)_{(4)}\Big|_{x=L_2} = (U,V,W)_{(2)}\Big|_{x=L_2} = (U,V,W)_{(3)}\Big|_{x=L_2} \\ &(\phi_x,\phi_y)_{(1)}\Big|_{x=L_1} = (\phi_x,\phi_y)_{(2)}\Big|_{x=L_1} = (\phi_x,\phi_y)_{(3)}\Big|_{x=L_1} \\ &(\phi_x,\phi_y)_{(4)}\Big|_{x=L_2} = (\phi_x,\phi_y)_{(2)}\Big|_{x=L_2} = (\phi_x,\phi_y)_{(3)}\Big|_{x=L_2} \end{aligned} \qquad (9)$$

Where L_1 and L_2 are defined at Figure 1 and the subscript (i=1,2,3,4) is region's number. In this study, the composite plates and the individual sub-laminates are assumed to be mid-plane symmetric, thus the in-plane and out-of-plane coupling stiffness coefficients (B_{ij}) are zero.

Moreover, due to in-plane geometrical symmetry, the displacement fields of plates are the same at region 1 and 4.

In this article, polarization is considered through the thickness. So, E_x and E_y are zero. PZT5A has been selected for this investigation. Figure 2 illustrates schematic view of described model. It should be noted that, governing results are affected by direction of the electrical field.

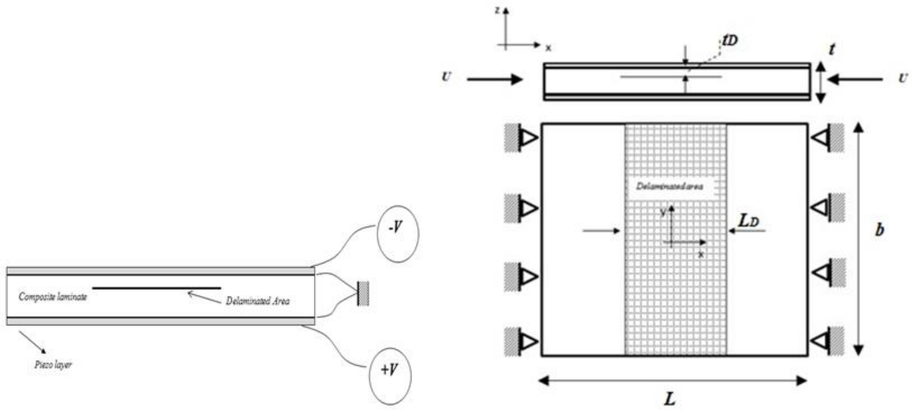

Figure 2. Geometric and boundary condition of delaminated piezo-composite

2.2. Displacement field

For simply supported boundary condition by consideration of FSDT requirements, the assumed out-of-plane displacements are:

$$
\begin{aligned}
&\mathrm{W}^{(1)} = \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-\frac{L}{2}\right)\left(x-\frac{L}{2}\right)W_{mn}^{(1)}x^m y^n \\
&\mathrm{W}^{(1\mathrm{p})} = \mathrm{W}^{(2)} = \mathrm{W}^{(3)} = \mathrm{W}^{(4)} = \mathrm{W}^{(4\mathrm{p})} = \mathrm{W}^{(1)} \\
&\mathrm{W}^{(5\mathrm{p})} = \mathrm{W}^{(5)} = \mathrm{W}^{(1)} + \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-L_1\right)\left(x-L_2\right)W_{mn}^{(5)}x^m y^n \qquad (10) \\
&\mathrm{W}^{(6)} = \mathrm{W}^{(7)} = \mathrm{W}^{(8)} = \mathrm{W}^{(8\mathrm{p})} = \mathrm{W}^{(2)} + \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-L_1\right)\left(x-L_2\right)W_{mn}^{(6)}x^m y^n \\
&\mathrm{W}^{(9\mathrm{p})} = \mathrm{W}^{(9)} = \mathrm{W}^{(10)} = \mathrm{W}^{(11)} = \mathrm{W}^{(12)} = \mathrm{W}^{(12\mathrm{p})} = \mathrm{W}^{(1)}
\end{aligned}
$$

In order to satisfy continuity between each layers, a reference value has been defined which is shown by U^{ref}. It should be noted that that this reference value could be placed between each two layers. In this article, the reference value is assumed to be in the same depth as delamination. For the same reason, $U^{ref,L}$ and $U^{ref,U}$ are defined in region 3 and region 2 respectively. The assumed in-plane displacement functions in x-direction by consideration of FSDT requirement could be followed as:

$$
\begin{aligned}
&\mathrm{U}^{ref} = -\varepsilon \times x + \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-\frac{L}{2}\right)\left(x+\frac{L}{2}\right)U_{mn}^{ref} x^m y^n \\
&\mathrm{U}^{\mathrm{ref,\,u}} = (\mathrm{U}^{(1)} + \mathrm{h}^{(1)} \times \varphi_x^{(1)}) + \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-L_1\right)\left(x-L_2\right)U_{mn}^{ref,u} x^m y^n \\
&\mathrm{U}^{\mathrm{ref,\,L}} = (\mathrm{U}^{(2)} - \mathrm{h}^{(2)} \times \varphi_x^{(2)}) + \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-L_1\right)\left(x-L_2\right)U_{mn}^{refL} x^m y^n \\
&\mathrm{U}^{(1\mathrm{p})} = \mathrm{U}^{(9\mathrm{p})} = \mathrm{U}^{(1)} + \mathrm{h}^{(1)} \times \varphi_x^{(1)} + \mathrm{h}^{(1\mathrm{p})} \times \varphi_x^{(1\mathrm{p})} \\
&\mathrm{U}^{(1)} = \mathrm{U}^{(9)} = \mathrm{U}^{ref} + \mathrm{h}^{(1)} \times \varphi_x^{(1)}, \mathrm{U}^{(2)} = \mathrm{U}^{(10)} = \mathrm{U}^{ref} - \mathrm{h}^{(2)} \times \varphi_x^{(2)} \\
&\mathrm{U}^{(3)} = \mathrm{U}^{(11)} = \mathrm{U}^{(2)} - \mathrm{h}^{(2)} \times \varphi_x^{(2)} - \mathrm{h}^{(3)} \times \varphi_x^{(3)}, \mathrm{U}^{(4)} = \mathrm{U}^{(12)} = \mathrm{U}^{(3)} - \mathrm{h}^{(3)} \times \varphi_x^{(3)} - \mathrm{h}^{(4)} \times \varphi_x^{(4)} \\
&\mathrm{U}^{(4\mathrm{p})} = \mathrm{U}^{(12\mathrm{p})} = \mathrm{U}^{(4)} - \mathrm{h}^{(4)} \times \varphi_x^{(4)} - \mathrm{h}^{(4\mathrm{p})} \times \varphi_x^{(4\mathrm{p})}, \mathrm{U}^{(5\mathrm{p})} = \mathrm{U}^{(5)} + \mathrm{h}^{(5)} \times \varphi_x^{(5)} + \mathrm{h}^{(5\mathrm{p})} \times \varphi_x^{(5\mathrm{p})} \\
&\mathrm{U}^{(5)} = \mathrm{U}^{\mathrm{ref,\,u}} - \mathrm{h}^{(5)} \times \varphi_x^{(5)}, \mathrm{U}^{(6)} = \mathrm{U}^{\mathrm{ref,\,L}} + \mathrm{h}^{(6)} \times \varphi_x^{(6)} \\
&\mathrm{U}^{(7)} = \mathrm{U}^{\mathrm{ref,\,L}} - \mathrm{h}^{(7)} \times \varphi_x^{(7)}, \mathrm{U}^{(8)} = \mathrm{U}^{(7)} - \mathrm{h}^{(7)} \times \varphi_x^{(7)} - \mathrm{h}^{(8)} \times \varphi_x^{(8)} \\
&\mathrm{U}^{(8\mathrm{p})} = \mathrm{U}^{(8)} - \mathrm{h}^{(8)} \times \varphi_x^{(8)} - \mathrm{h}^{(8\mathrm{p})} \times \varphi_x^{(8\mathrm{p})}
\end{aligned}
\tag{11}
$$

The assumed rotation functions by consideration of FSDT requirements are:

$$
\begin{aligned}
&\varphi_x^{(1\mathrm{p})} = \varphi_x^{(9\mathrm{p})} = \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-\frac{L}{2}\right)\left(x+\frac{L}{2}\right)\varphi_{x_{mn}}^{(1\mathrm{p})} x^m y^n, \varphi_x^{(1)} = \varphi_x^{(9)} = \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-\frac{L}{2}\right)\left(x+\frac{L}{2}\right)\varphi_{x_{mn}}^{(1)} x^m y^n \\
&\varphi_x^{(2)} = \varphi_x^{(10)} = \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-\frac{L}{2}\right)\left(x+\frac{L}{2}\right)\varphi_{x_{mn}}^{(2)} x^m y^n, \varphi_x^{(3)} = \varphi_x^{(11)} = \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-\frac{L}{2}\right)\left(x+\frac{L}{2}\right)\varphi_{x_{mn}}^{(3)} x^m y^n \\
&\varphi_x^{(4)} = \varphi_x^{(12)} = \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-\frac{L}{2}\right)\left(x+\frac{L}{2}\right)\varphi_{x_{mn}}^{(4)} x^m y^n, \varphi_x^{(4\mathrm{p})} = \varphi_x^{(12\mathrm{p})} = \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-\frac{L}{2}\right)\left(x+\frac{L}{2}\right)\varphi_{x_{mn}}^{(4\mathrm{p})} x^m y^n \\
&\varphi_x^{(5\mathrm{p})} = \varphi_x^{(1\mathrm{p})} + \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-L_1\right)\left(x-L_2\right)\varphi_{x_{mn}}^{(5\mathrm{p})} x^m y^n, \varphi_x^{(5)} = \varphi_x^{(1)} + \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-L_1\right)\left(x-L_2\right)\varphi_{x_{mn}}^{(5)} x^m y^n \\
&\varphi_x^{(6)} = \varphi_x^{(2)} + \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-L_1\right)\left(x-L_2\right)\varphi_{x_{mn}}^{(6)} x^m y^n, \varphi_x^{(7)} = \varphi_x^{(3)} + \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-L_1\right)\left(x-L_2\right)\varphi_{x_{mn}}^{(7)} x^m y^n \\
&\varphi_x^{(8)} = \varphi_x^{(4)} = \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-L_1\right)\left(x-L_2\right)\varphi_{x_{mn}}^{(8)} x^m y^n, \varphi_x^{(8\mathrm{p})} = \varphi_x^{(4\mathrm{p})} = \sum_{n=0}^{N}\sum_{m=0}^{M}\left(x-L_1\right)\left(x-L_2\right)\varphi_{x_{mn}}^{(8\mathrm{p})} x^m y^n
\end{aligned}
\tag{12}
$$

$h^{(i)}$is defined as half of the i th plate thickness.

3. Finite element analysis

The FEM analysis has been employed in order to investigate the validation of the results obtained by the method developed in the current study. For this purpose, Ansys 5.4 software has been employed. Eight-node solid46 and solid5 have been considered for modeling of composite laminate and piezoelectric-layers respectively. In addition, contact element has been placed in the delaminated region.

It is obvious that for post-buckling analysis, an initial imperfection is required. For this purpose, first an Eigen-buckling analysis performs, and then geometry is updated according to the first mode shape of buckling (Scale factor is 0.0001 in this study). Finally non-linear analysis is performed for post-buckling analysis of piezo-composite laminate. Figure 3 shows typical finite element mesh of piezo-composite model.

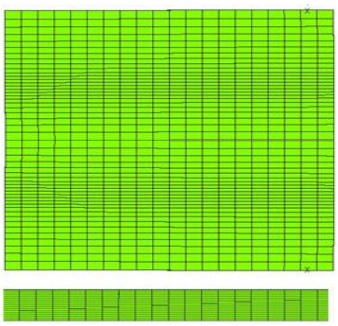

Figure 3. Typical mesh for laminate with a thorough the width delamination

4. Result and discussion

Figure 4.a shows non-dimensional out-of-plane displacement versus non-dimensional in-plane displacement for plate with clamped boundary condition for various applied voltage. The stacking sequence of the sub-laminate and base-laminate are $[0_p/0/90/90/0]$ and $[(0/90/90/0)_3/0_p]$ respectively which subscript p indicates piezo layers. Delamination is located at 0.25 depths from above. The total thickness of plate is 2.2 mm and the L_D/L is 0.58. It should be noted that Ux_{cr0} is the critical end shortening corresponding to the laminate without delamination and voltage equal zero. Ux_{cr0} is 0.097 for this. It is clear that, out-of-plane displacement is significantly reduced with increasing applied voltage at specific end-shortening. Figure 4.b shows non-dimensional membrane loading (Nx) versus non-dimensional in-plane displacement. Nx_{cr0} is membrane loading at buckling load for plate with no delamination and voltage is equal zero. It is interesting that membrane loading is decreasing with increasing applied voltage. It is also seen in figures that the agreement between the results of the presented method and those of FEM are very good.

5. Conclusion

Post-buckling analysis of delaminated piezo-composite plate based on novel layerwise theory has been performed and demonstrated that this method is capable to consider both zigzag displacement field and material discontinuity through the thickness. FEM analysis has been implemented for result verification and showed that results are in good agreement.

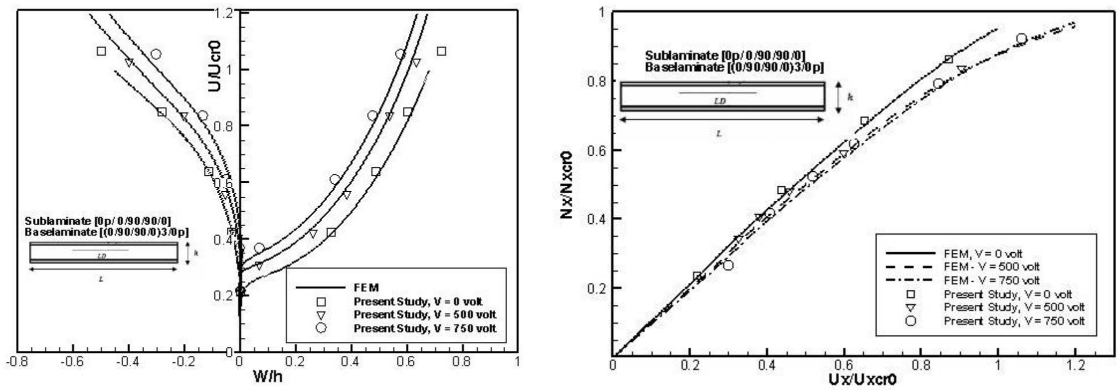

Figure 4. (a) Out-of-plane displacement. (b) Membrane loading of a delaminated clamped piezo-composite with a central through the width delamination for various applied voltage (L = 100, L_D/L = 0.58, h = 2.2 mm)

References

[1] Mitchell, J.A. and J.N. Reddy, *A refined hybrid plate theory for composite laminates with piezoelectric laminae.* International Journal of Solids and Structures, 1995. **32**(16): p. 2345-2367.

[2] Allik, H. and T.J.R. Hughes, *Finite element method for piezoelectric vibration.* International Journal for Numerical Methods in Engineering, 1970. **2**(2): p. 151-157.

[3] Robbins, D.H. and J.N. Reddy, *Analysis of piezoelectrically actuated beams using a layer-wise displacement theory.* Computers & Structures, 1991. **41**(2): p. 265-279.

[4] Ha, S.K., C. Keilers, and F.-K. Chang, *Finite element analysis of composite structures containing distributed piezoceramic sensors and actuators.* AIAA Journal, 1992. **30**(3): p. 772-780.

[5] Batra, R.C. and T.S. Geng, *Comparison of Active Constrained Layer Damping by Using Extension and Shear Mode Piezoceramic Actuators.* Journal of Intelligent Material Systems and Structures, 2002. **13**(6): p. 349-367.

[6] Varelis, D. and D.A. Saravanos, *Coupled buckling and postbuckling analysis of active laminated piezoelectric composite plates.* International Journal of Solids and Structures, 2004. **41**(5–6): p. 1519-1538.

[7] Kapuria, S. and G.G.S. Achary, *Exact 3-D piezoelasticity solution for buckling of hybrid cross-ply plates using transfer matrices.* Acta Mechanica, 2004. **170**(1-2): p. 25-45.

[8] Shen, H.-S., *Postbuckling of laminated cylindrical shells with piezoelectric actuators under combined external pressure and heating.* International Journal of Solids and Structures, 2002. **39**(16): p. 4271-4289.

[9] Kapuria, S. and G.G.S. Achary, *Nonlinear coupled zigzag theory for buckling of hybrid piezoelectric plates.* Composite Structures, 2006. **74**(3): p. 253-264.

[10] Datchanamourtya, B. and G.E. Blandford, *Nonlinear Buckling Analysis of Piezothermoelastic Composite Plates*, 2008.

[11] Faria, A.R.d. and M.ı.V. Donadon, *The use of piezoelectric stress stiffening to enhance buckling of laminated plates.* Latin American Journal of solids and structures, 2010. **7**: p. 17.

[12] Kharazi, M. and H.R. Ovesy, *Postbuckling behavior of composite plates with through-the-width delaminations.* Thin-Walled Structures, 2008. **46**(7–9): p. 939-946.

[13] Ovesy, H.R. and M. Kharazi, *Compressional stability behavior of composite plates with through-the-width and embedded delaminations by using first order shear deformation theory*. Computers & Structures, 2011. **89**(19–20): p. 1829-1839.

[14] Ovesy, H.R., M. Taghizadeh, and M. Kharazi, *Post-buckling analysis of composite plates containing embedded delaminations with arbitrary shape by using higher order shear deformation theory*. Composite Structures, 2012. **94**(3): p. 1243-1249.

[15] Kharazi, M., H.R. Ovesy, and M. Asghari Mooneghi, *Buckling analysis of delaminated composite plates using a novel layerwise theory.* Thin-Walled Structures, 2014. **74**(0): p. 246-254.