INVESTIGATION OF PROGRESSIVE FAILURE BEHAVIOR OF NOTCHED COMPOSITE PLATES

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Abstract
In this study, the progressive failure behavior of notched laminates under in-plane loads is modeled to determine the ultimate failure load. Although numerous studies exist in the literature on this subject, correlations with the experimental results were examined qualitatively by comparing predicted and experimentally determined damage states. The final failure loads were also qualitatively determined by examining the predicted progression of damage state. In the present study, on the other hand, the final failure is quantitatively predicted. Successive finite element analyses of the part are carried out, while the load is incrementally increased. After each small load increment, failure of the material in each element is checked with the help of the maximum strain criterion. If the material fails for a given failure mode (fiber breakage, matrix cracking, or fiber-matrix shearing) according to the criterion, the respective material properties are degraded. Then, the overall stiffness of the part is calculated based on the deformation state of the part. A significant decrease in the stiffness is interpreted as final failure. In order to check the validity of the model presented here, the numerical predictions are compared with empirical results obtained on specimens having circular holes. Consistently good correlation is observed for different laminate configurations.

1. Introduction

There are mainly two approaches used to predict failure load of composite laminates: One is the first ply failure criterion [1, 2], according to which, failure occurs at the instant a damage initiates. This may be true for uniform plates under a uniform stress state. The other approach is the progressive failure approach. The progressive failure behavior of composite structures having notches has been studied extensively by many researchers in the literature [3-10,11,12,13,14,15,16,17, 18 19]. In these studies, various kinds of geometry and loading types were investigated such as beam under the three point bending load[3], the shear out and tension failure modes of a circular joint fastening the two composite plates[6], or the tensile failure of a composite plate including a notch in the form of circular opening[11]. The stress distribution in the composite laminate was computed with the help of classical laminated plate theory [11], higher order plate theories [13,14,15, 17, 18], or finite element method [11, 12, 13, 14, 15, 17]. Various types of loading conditions were considered including uniaxial in-plane loading [4, 5, 8, 9, 11, 12, 13, 15, 19], biaxial loading [4, 9], in-plane shear [5, 19], three point bending [3] and four-point bending [13]. In progressive failure models, loads are gradually increased and whenever failure is detected at a particular region by the failure criterion, the material properties in that region are degraded depending on the mode of failure, fiber breakage, matrix cracking, fiber-matrix shearing etc. This is done either by multiplying the strength and stiffness values with predefined constants found by trial and error [12,13,15] or by considering the scatter in the material properties and using statistical models such as Weibull distribution [11]. Chang [11], reduced the transverse modulus Ey and Poisson's ratio vy to zero in case of matrix cracking, however left the longitudinal Ex and the shear modulus of the layer unchanged. When fiber breakage and/or fiber-matrix shearing were predicted, the degree of property degradation in the damaged region depended on the predefined constants the value of which are related to the size of damage predicted by the fiber failure criterion; fiber failure, both Ey and vy were reduced to zero; but the longitudinal modulus Ex and the shear modulus Gxy were degenerated according to Weibull distribution. Seng [12] and Ozden [13] benefitted from the stiffness degradation factors whose values were calculated through a parametric study, then they were used to degrade the stiffness terms included in the laminate stiffness matrix. Degradation may not only be applied to in-plane strength and stiffness but also to interlaminar or intralaminar properties. For example, Goyal et al. [14] considered progressive delamination failure. As an alternative, micro damage models [16] were also developed for simulating the progressive damage in composite structures. In the previous studies, various failure criteria were used to detect local failure such as Tsai-Wu [4, 12, 18], Tsai-Hill [4, 6, 19], and Hashin [3, 6, 8, 10, 13, 14, 15]. Maximum stress, maximum strain, and Hashin criteria can also predict failure mode as opposed to Tsai-Wu and Tsai-Hill, and thus allow different degradations schemes for different failure modes. In the previous studies [6, 11, 12, 13], the final failure load is determined by inspecting the extent of the damaged region; In some of the studies [6, 8, 11, 12] the force defined on the model is increased in a step-wise manner until the damage propagates throughout the whole section. In the current study, the external force exerted on the work piece is raised at small intervals and then the stiffness of the structure is calculated. The loss in the stiffness is monitored, as it indicates the reduction in the total load bearing capacity of the model. When the predetermined ratio of stiffness loss is detected, the force is no longer incremented and its highest value is considered to be the failure strength of the part. This method of determining the ultimate failure load is used for the first time.

2. Problem Statement

Symmetric 2-D multilayered laminates reinforced by continuous fibers are considered in this study. The plate contains a notch such as a hole (Figure 1) or fillet, or any structural feature causing stress concentration. The laminate is subjected to in-plane normal and shear loading as shown in Figure 1. Accordingly, bending or twisting of the plate is not considered.

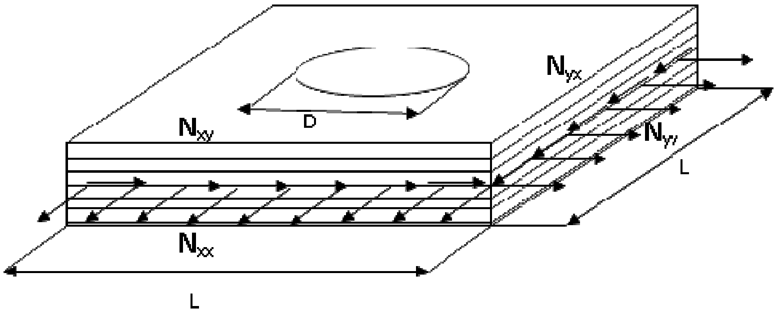

Figure 1. A scheme of the composite structure considered in this study.

The objective of this study is to develop a progressive failure assessment model to determine the ultimate failure load that the composite structure can sustain before it catastrophically fails

3. Methodology

3.1. Analysis of the Laminated Composite Plate

The structural analysis of the composite laminate is carried out using finite element method via commercial FEA software ANSYS. Considering that the plate is thin in comparison to its lateral dimensions, a layered shell element (SHELL181), based on the first order shear deformation theory, is used.

3.2. Static Failure Criterion

In order to determine whether the portion of a layer in a finite element has failed or not during an incremental increase of the applied load, a number of failure criteria proposed for composite laminates are tried. Among them, the maximum strain criterion predicts failure whenever one of the principal strain components exceeds its allowable strain limit. The failure envelope for a ply under in-plane normal and shear stresses can then be defined by the following inequalities:

$$\begin{aligned} &\varepsilon_1 < \left(\varepsilon_1^C\right)_{ult} \text{ and } \varepsilon_1 > (\varepsilon_1^T)_{ult} \text{ and } \varepsilon_2 < \left(\varepsilon_2^C\right)_{ult} \text{ and} \\ &\varepsilon_2 > (\varepsilon_2^T)_{ult} \text{ and } |\gamma_{12}| > \gamma_{12_{ult}} \end{aligned} \tag{1}$$

where $(\varepsilon_1^T)_{ult}$ and $(\varepsilon_2^T)_{ult}$ are the ultimate tensile strains in the fiber direction and transverse to it, respectively. $\left(\varepsilon_1^C\right)_{ult}$ and $\left(\varepsilon_2^C\right)_{ult}$ are the ultimate compressive strains in the fiber direction and transverse to it, respectively. $(\gamma_{12})_{ult}$ is the ultimate in-plane shear strain.

3.3. Progressive Failure Model

Load defined in the finite element model is increased in small increments starting from low levels at which no damage is detected. At each load level, each element in the finite element model is examined using the maximum strain failure criterion for every layer of the plate. When failure is predicted in a layer within a finite element, the stiffness properties of the layer pertaining to that element is modified; thereby failed parts of the elements are rendered ineffective and they contribute to the stiffness of the whole structure in a limited manner. Strength properties need not be changed, because there is no need to check the previously failed regions, considering that a damaged material cannot become sound and fail again.
Stiffness parameters of the damaged material are changed for the subsequent load increments as follows:

$$
\begin{aligned}
E_x^* &= D_f E_x \\
E_y^* &= D_m E_y \\
G_{xy}^* &= D_m G_{xy} \\
\nu_{xy}^* &= D_m \nu_{xy}
\end{aligned}
\tag{2}
$$

where E_x and E_y are the elastic moduli of the undamaged material along the fiber direction and transverse to it, respectively, G_{xy} is the in-plane shear modulus, and ν_{xy} is the in-plane Poisson's ratio; E_x^*, E_y^*, G_{xy}^*, and ν_{xy}^* are the corresponding terms for the damaged materials. Depending on the predicted mode of failure, different values are attributed to the factors, D_f and D_m, in the following way:

$$
\begin{aligned}
&D_f = 0.06,\ \ D_m = 0.13 \qquad \text{if} \quad \varepsilon_1 < \left(\varepsilon_1^C\right)_{ult} \ \text{or}\ \varepsilon_1 > (\varepsilon_1^T)_{ult} \\
&D_f = 1.00,\ \ D_m = 0.13 \qquad \text{if} \quad \varepsilon_2 < \left(\varepsilon_2^C\right)_{ult} \ \text{or}\ \varepsilon_2 > (\varepsilon_2^T)_{ult} \ \text{or}\ |\gamma_{12}| > \gamma_{12ult}
\end{aligned}
\tag{3}
$$

Here, if a matrix dominant damage occurs like matrix cracking and fiber-matrix shearing, fiber dominant stiffness terms are assumed to be not affected. If fibers break, the matrix is also assumed to be damaged. No theoretical or experimental method has been proposed in the literature to determine the values of factors D_f and D_m; only some statistical methods have been proposed. For this reason, suitable values are found by comparing the numerical results with experimental data for different sets of values as will be discussed in the results section.

At the beginning, the externally applied force on the structure is applied with a magnitude that will not cause any damage on the structure, P_{in}. The load is then increased in a step-wise manner at sufficiently small intervals $P_{i+1} = P_i + \Delta P$ and the structural analysis of the plate is carried out to determine the stress and strain states of the plate; the failure analysis of the plate is performed according to the chosen failure criterion. If the maximum strain criterion is used Eq. 1 is employed. If failure is predicted in any one of the elements, the material properties of this element is replaced with that of the damaged material in accordance with Eqs. 2 and 3. In the present study, unlike the previous studies, stiffness of the composite plate is monitored to determine its ultimate load bearing capacity rather than visually inspecting the propagation of damage through the plate as the load is increased. Before the force is increased, the stiffness of the structure is computed as follows

$$
k_i = \frac{P_i}{\delta_i} \tag{4}
$$

where δ_i is the deflection at the point of application of P_i in the same direction due to incremental increase in the load, ΔP; k_i represents the stiffness of the structure at the ith load increment. When a significant degradation occurs in stiffness, the structure is assumed to have catastrophically failed. The failure criterion can be stated as

$$
\text{if} \ \ k_i \leq 0.9 k_{in} \ \ \Rightarrow \quad \text{failure} \tag{5}
$$

If the stiffness after a certain load increment, k_i, becomes less than 90% of the initial stiffness, k_{in}, significant degradation is assumed to have occurred. The ultimate failure load is than taken to be equal to the current load level. As will be shown, if the load is continued to be increased after this failure criterion is satisfied, stiffness decreases at quite high rates.

4. Results and discussion

The finite element model of the structure is developed, loading state is defined, the structural analyses are carried out, failure analysis of each element is performed, degraded material properties are assigned, stiffness is calculated, and the ultimate load level is determined using a program developed in ANSYS parametric design language according to the aforementioned progressive failure assessment procedure.

4.1. Comparison with Experimental Results

In order to check the validity of the developed method, the predictions of the failure model for ultimate load are compared with experimental results. Irvine and Ginty [20] conducted experiments on specimens containing a circular hole to determine their failure loads. The Plies with a thickness of 0.127 mm were used along with the material properties of E11 = 137.9 GPa, E22 = 10.3 GPa, G12 = 6.55 GPa, ν12 = 0.21 , XT = 1793 MPa, XC = −1448 MPa, YT = 44.82 MPa, YC = −172.4 MPa, S = 62.05 MPa.
The plate having a uniform rectangular shape including a circular hole of a diameter 6.35 mm has a width of 50.8 mm. Figure 2 shows a comparison of the experimental results and the predictions of the progressive failure model obtained using various failure criteria, maximum stress, maximum strain, Hashin, Tsai Wu. The values of the degradation factors, $\mathbf{D_f}$ and $\mathbf{D_m}$, are chosen to be 0.05 and 0.15, respectively, which are within the range of values suggested in previous studies [12,13]. The results obtained based on the maximum strain criterion are better in terms of the predicted trend and the least-square approximation.

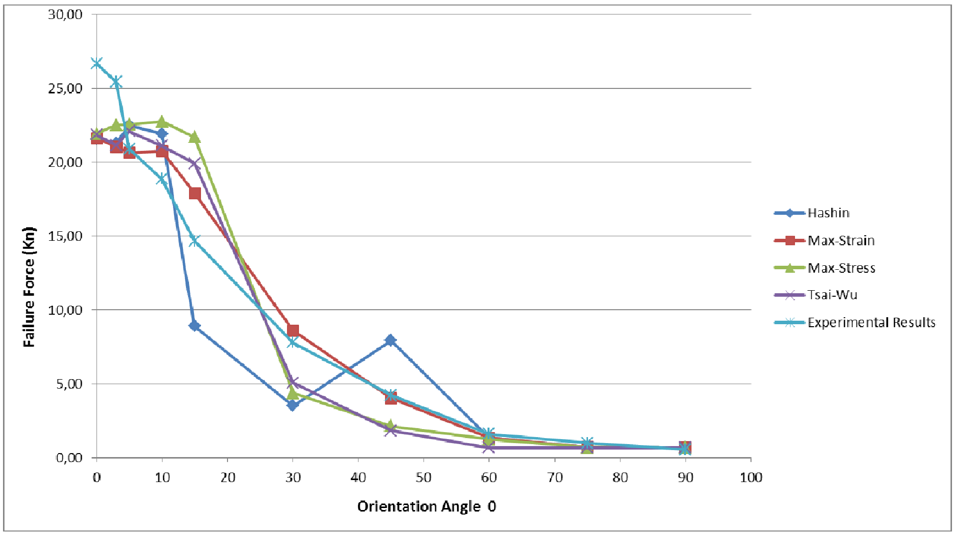

Figure 2. Comparison of the experimental results and the predictions of the progressive failure model based on various failure criteria.

The values of the degradation factors, $\mathbf{D_f}$ and $\mathbf{D_m}$, are optimized to obtain a better correlation. The values of 0.06 and 0.13, respectively, are found to provide the best correlation. Table 1 and Figure 3 show the comparison for the optimized values.

Table 1.Experimentally determined values for ultimate failure loads [20] and the values predicted based on the maximum strain criterion with optimized values of degradation factors, D_f and D_m.

Fiber Orientation Angle	Measured Ultimate load (kN)	Predicted Ultimate load (kN)	Error (%)
0	26.69	22.64	15
+/-3	25.44	22.45	12
+/-5	20.91	21.99	-5
+/-10	18.86	21.88	-16
+/-15	14.68	19.97	-36
+/-30	7.78	9.51	-22
+/-45	4.23	4.53	-7
+/-60	1.60	1.48	7
+/-75	0.98	0.82	16
+/-90	0.53	0.71	-33

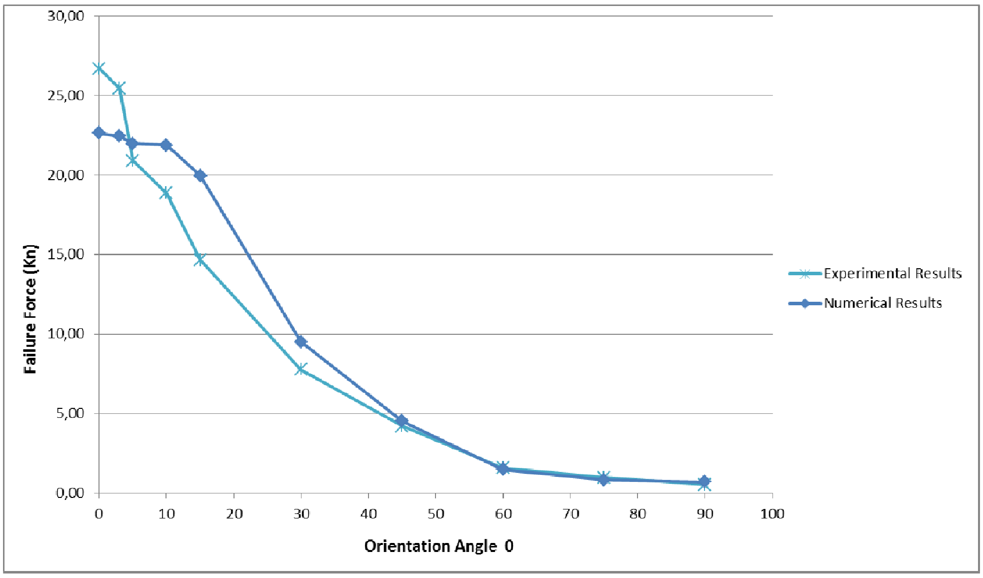

Figure 3. Comparison of the experimental results and model predictions based on the maximum strain criterion using the optimized values for the degradation factors.

Figure 4 shows the failed and sound elements of $[+75/-75]_s$ laminate at instant the algorithm detects more than 10% degradation in stiffness, indicating the ultimate failure. Because the stress distribution is not uniform throughout the plate and severe only around the notches, damage starts around the notch, then spreads towards the edges of the plate. Ultimate failure occurs when the damage reaches to the edges. For this configuration, only matrix failure occurs, because stresses in the direction transverse to the fibers are significant.

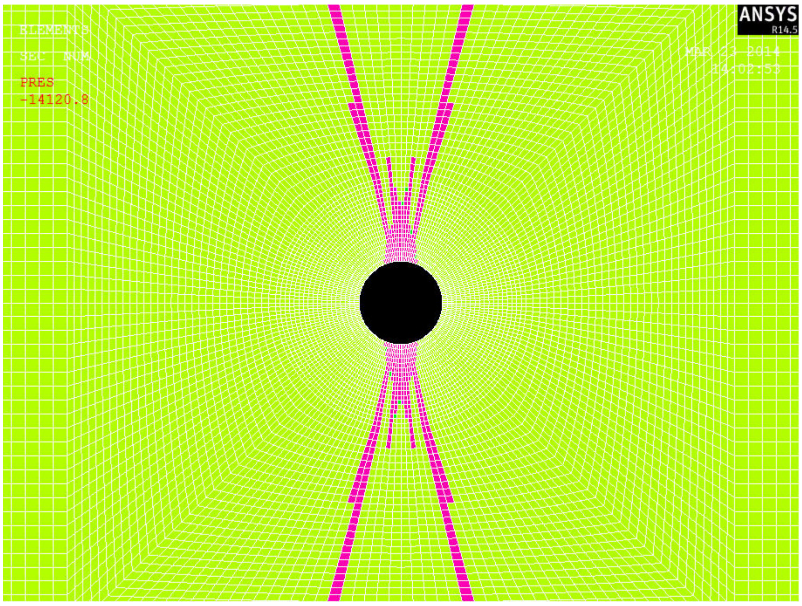

Figure 4. Failure state of $[+75/-75]_s$ laminate at the instant the load is increased to its ultimate level.

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