DAMAGE DEVELOPMENT AND STIFFNESS REDUCTION IN LAMINATES IN OUT-OF-PLANE LOADING

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Abstract

Simple approach based on Classical Laminate Theory (CLT) and effective stiffness of damaged layer is suggested for bending stiffness determination of laminate with intralaminar cracks in surface 90-layers. The effective stiffness of layer with cracks as a function of crack density is back-calculated comparing in-plane stiffness of laminates with and without damage. The accuracy of the CLT and effective stiffness approach is demonstrated comparing with bending stiffness results from FEM simulated 4-point bending test on laminate with damage. Analytical model for damaged laminate stiffness is presented which gives similar values for effective stiffness as FEM calculations for unit cell. Effect of local delaminations initiated from transverse cracks is analyzed.

1. Introduction

The most typical and earliest damage mode in laminated composites is intralaminar cracking in layers. The crack plane is usually transverse to the laminate middle-plane and the crack runs parallel to fibers in the layer covering the whole thickness of the layer and the width of the specimen. Intralaminar cracking is the first mode because the transverse strain to failure of unidirectional polymeric composites is lower than other failure strain components. Due to statistical variation of transverse and shear strength values along the specimen, the first intralaminar crack is created in the weakest position. With increasing applied load the number of cracks characterized by crack density increases. High shear stress concentration at the tip of the intralaminar crack may lead to local delaminations.

An inevitable consequence of microdamage progression is degradation of macroscopic thermo-mechanical properties of the laminate. Constitutive models for in-plane behavior of damaged laminates have been historically developed using two basic approaches, see review for example in [1]: a) continuum damage mechanics approach and b) approach where stress state between two cracks is analyzed (called micromechanics modeling). The approximate stress field is further used to determine certain thermo-elastic constant of the repeating element representing the damaged laminate. The simplest stress distributions are obtained based on shear lag assumption [2] or variational principles [1,3,4]. Interaction between damage in different layers can be approximately accounted for by replacing one of the damaged layers by non-damaged layer with effective elastic properties as it was done, for example, in [2] using modified shear lag model and so called “equivalent constraint model”.

In the present paper we use a similar approach to predict the bending stiffness change of a laminate with intralaminar cracks in the surface 90-layer. We suggest using Classical Laminate Theory (CLT) to calculate the bending stiffness of the damaged laminate. For the layer with intralaminar cracks the so called “effective stiffness” is used.

The key issue in this approach is the method of calculation of the effective stiffness of a layer with cracks. The most conservative solution called “ply-discount model” is well known: as soon as the layer fails transverse modulus and in-plane shear modulus is zero. This approach does not consider the fact that the crack density increases gradually and the effective stiffness change is progressive. The correct way of determining the effective layer stiffness would be by finding the stiffness of the undamaged laminate (experimentally or by calculations knowing layer properties) and also the stiffness of the damaged laminate (experimentally or FEM). Then the effective stiffness of the damaged layer as a function of crack density can be found by back-calculation using CLT. We suggest tensile test on damaged cross-ply laminate to obtain this dependency. The obtained effective stiffness will be used to predict the laminate bending stiffness change with crack density.

The laminate elastic properties degradation due to cracks in layers is uniquely related to the relative displacements of the corresponding points on both crack surfaces. As long as points on both surfaces coincide (relative displacements are zero) the thermo-mechanical properties of the laminate are not affected. The opening and sliding of crack surfaces reduce the average strain and stress in the damaged layer, thus reducing the part of the load carried by this layer. This part of the load is distributed to the rest of the laminate leading to additional deformation which means reduction of laminate thermo-elastic properties. Thus the crack opening displacement (COD) and crack face sliding displacement (CSD) together with the crack density are the micromechanical parameters governing the amount of macroscopic stiffness reduction. In linear elasticity COD is proportional to the applied load, ply thickness and, therefore, it has to be normalized to be used in stiffness modeling. A theoretical framework (called GLOB-LOC approach) expressing the damaged laminate thermo-elastic properties via density of cracks in layer and the microdamage parameters COD and CSD was presented in [5]. Only the average values of COD and CSD enter the stiffness expressions and the details of the relative displacement profile are not important. Simple relationships for COD and CSD at low crack density (non-interactive cracks) were proposed in [5]. The COD and CSD of interacting cracks were analyzed theoretically in [6].

In the present paper the bending stiffness of cross-ply laminates with one damaged surface layer will be obtained simulating the 4-point bending test by 3-D FEM. The results are compared with CLT simulations using effective stiffness of the damaged layer. The effective stiffness of the surface layer with a given crack density is back-calculated using CLT for in-plane stiffness of damaged and undamaged laminate in two ways: a) using FEM for stiffness of the symmetric damaged laminate with the same lay-up; b) using the GLOB-LOC model.

2. Bending resistance of damaged laminate

We will analyze initially symmetric [90n/0m]s cross-ply laminate subjected to bending, for example, in 4-point bending test where a region with constant applied moment M_x exists leading to constant curvature k_x. When intralaminar cracks are introduced in the surface 90-layer on the tensile side, the laminate becomes unsymmetrical. Due to the lack of symmetry the B-matrix is not zero and some midplane strains ε_{x0}, ε_{y0} are also present. When intralaminar cracking with or without local delaminations develop in some layers the laminate A, B and D-matrices change in a different way. We will analyze the change of the laminate bending resistance with damage development plotting the bending moment to create unit curvature versus the crack density in a layer, i.e., $(M_x / k_x) \sim \rho$. Other considered damage parameters in addition to crack density will be delamination length l_d normalized with respect to the crack size (damaged ply thickness t_{90}). Boundary conditions will be used with applied k_x leading to M_x and, $\gamma_{xy0} = k_{xy} = k_y = 0$. In this case the relevant CLT equations are

$$\begin{aligned} 0 &= A_{11}\varepsilon_{x0} + A_{12}\varepsilon_{y0} + B_{11}k_x \\ 0 &= A_{12}\varepsilon_{x0} + A_{22}\varepsilon_{y0} + B_{12}k_x \\ M_x &= B_{11}\varepsilon_{x0} + B_{12}\varepsilon_{y0} + D_{11}k_x \\ M_y &= B_{12}\varepsilon_{x0} + B_{22}\varepsilon_{y0} + D_{12}k_x \end{aligned} \tag{1}$$

$$A_{ij} = \sum_{k=1}^{N} \overline{Q}_{ij}^{k} t_k \,, \quad B_{ij} = \sum_{k=1}^{N} \overline{Q}_{ij}^{k} \frac{z_{k+1}^2 - z_k^2}{2} \,, \quad D_{ij} = \sum_{k=1}^{N} \overline{Q}_{ij}^{k} \frac{z_{k+1}^3 - z_k^3}{3} \tag{2}$$

In (2) t_k, k=1,2…N is the ply thickness; the overbar is used to denote the stiffness of the layer in the global system of coordinates. Solving (1) with respect to M_x we can express it in form

$$M_x = C_{11}(\rho) k_x \tag{3}$$

Parameter C_{11} dependence on crack density and delamination length will be investigated in this study. It has to be emphasized that C_{11} is not equal to bending stiffness element D_{11}.

3. Effective stiffness of the damaged layer

As discussed in introduction an extreme case of effective stiffness reduction of the damage layer is obtained using the ply-discount model. With increasing crack density the effective stiffness of the damaged layer has to approach to this asymptotic value. We will calculate the effective stiffness of the damaged 90-layer from the in-plane stiffness change of the [90n/0m]s laminate assuming that the same crack density ρ is in both 90-layers. This technique is described more in detail in [7]. The undamaged and the damaged laminate stiffness is denoted $[Q]_0^{LAM}$ and $[Q]^{LAM}$ respectively. In bending this laminate will behave as unsymmetrical because cracks in the compressed 90-layer will be closed. The advantage of the assumed symmetry in the damaged state is that for symmetric damaged laminate the [A]-matrix represents the stiffness matrix $[Q]^{LAM}$ of the laminate. When the two equal surface layers with index i=1 and N are damaged their effective stiffness is changing from $[\overline{Q}]_1$ to $[\overline{Q}]_1^{eff}$. According to CLT the damaged laminate extensional stiffness matrix can be written as

$$[Q]^{LAM} = \frac{t_1}{h}[\overline{Q}]_1^{eff} + \sum_{k=2}^{N-1} \frac{t_k}{h}[\overline{Q}]_k + \frac{t_1}{h}[\overline{Q}]_1^{eff} \tag{4}$$

The undamaged laminate stiffness matrix can be written in a similar form

$$[Q]_0^{LAM} = \frac{t_1}{h}[\overline{Q}]_1 + \sum_{k=2}^{N-1} \frac{t_k}{h}[\overline{Q}]_k + \frac{t_1}{h}[\overline{Q}]_1 \tag{5}$$

Subtracting (4) from (5) we obtain

$$[Q]_0^{LAM} - [Q]^{LAM} = 2\frac{t_1}{h}[\overline{Q}]_1 - 2\frac{t_1}{h}[\overline{Q}]_1^{eff} \tag{6}$$

From (6) the effective stiffness matrix of the damaged k-th layer in global axes is found

$$[\overline{Q}]_1^{eff} = [\overline{Q}]_1 - \frac{h}{2t_1}\left\{[Q]_0^{LAM} - [Q]^{LAM}\right\} \tag{7}$$

For the analyzed cross-ply laminate shown in Fig.1

$$h = 2t_{90} + 2t_0 \tag{8}$$

$$[\overline{Q}]_{90}^{eff} = [\overline{Q}]_{90} - \frac{h}{2t_{90}}\left\{[Q]_0^{LAM} - [Q]^{LAM}\right\} \tag{9}$$

Equation (7) can be transformed to the local coordinate system using expression

$$[Q]_1^{eff} = [T][\overline{Q}]_1^{eff}[T]^T \tag{10}$$

Then the effective compliance matrix of the damaged layer in local axes can be calculated

$$[S]_1^{eff} = \left([Q]_1^{eff}\right)^{-1} \tag{11}$$

From (11) the effective engineering constants of the damaged layer are

$$E_L^{eff} = 1/S_{11}^{eff}\ ,\ E_T^{eff} = 1/S_{22}^{eff}\ , \nu_{LT}^{eff} = -E_L^{eff} S_{12}^{eff}\ , G_{LT}^{eff} = 1/S_{66}^{eff} \tag{12}$$

Parametric analysis for large number of material systems and laminate lay-ups has shown that only the effective transverse modulus E_T^{eff} and the shear modulus G_{LT}^{eff} of the layer are reduced due to intralaminar cracks. The longitudinal modulus E_L^{eff} and ν_{LT}^{eff} are not changing. G_{LT}^{eff} does not affect the bending resistance of a cross-ply laminate.

4. Stiffness of the damaged laminate

4.1. FEM modeling

The stiffness matrix of the damaged laminate $[Q]^{LAM}$ is needed in order to calculate the effective stiffness using (7)-(12). The used FEM model of the RVE is schematically shown in Fig.1. A 3-D FEM model was generated using FEM software ANSYS [8]. Taking advantage of the symmetry conditions, the FEM model in Fig.1 consists of a 90 degree layer of thickness t_{90} and a 0 degree layer of thickness t_0. The length of the model is l, which is the half distance between two transverse cracks in the 90 layer. Thus, parameter l defines the crack density ($\rho = 1/2l$) and can be parametrically varied to calculate $[Q]^{LAM}$ for different crack densities. The width of the FEM model of RVE was in all calculations equal to $w = 0.2h$ where h is the total thickness of the laminate. On the top surface of the FEM model ($z = t_0 + t_{90}$) symmetry boundary conditions were used. At $x = 0$, symmetry boundary conditions were applied on the 0 degree layer while the surface of the 90 degree layer is free representing the transverse crack surface. At $x = l$ a uniform displacement in the x axis direction was applied. Separately on each side edge of the model (y=0 and y=-w) displacement coupling in the y axis direction was used. Elastic modulus E_x^{LAM} and Poisson's ratio ν_{xy}^{LAM} of the damaged laminate were calculated from post-processing using reaction forces and resulting displacements. The transverse modulus E_y^{LAM} of the damaged laminate was found using the FEM model with the same geometry as in Fig.1 but with different boundary conditions: uniform displacement was applied in the positive y axis direction, symmetry boundary conditions were applied on nodes at y=-w, displacement coupling in x axis direction was applied on the nodes with coordinates x=l.

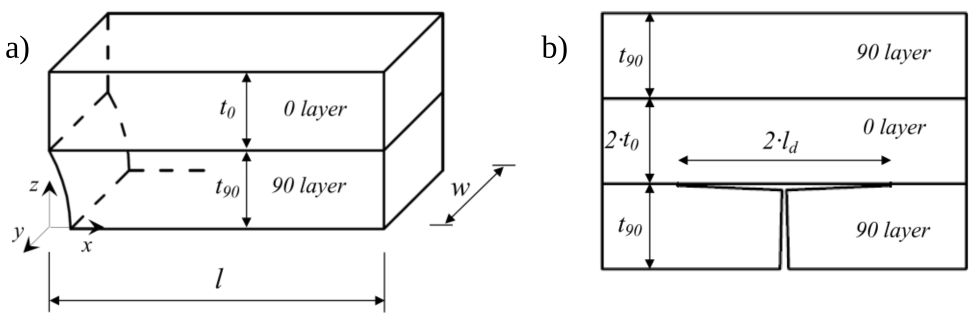

Figure 1. Schematic representation of a) FEM model of representative volume element (RVE); b) model of delamination cracks between 90 and 0 layers.

4.2. GLOB-LOC model for cross-ply laminates with cracks in surface 90-layers

The GLOB-LOC model [5] in case of cross-ply laminates leads to very simple but still exact expressions [6]

$$\frac{E_x^{LAM}}{E_{x0}^{LAM}} = \frac{1}{1+2\rho t_{90}(t_{90}/h_0)u_{2an}c_2}, \quad \frac{E_y^{LAM}}{E_{y0}^{LAM}} = \frac{1}{1+2\rho t_{90}(t_{90}/h_0)u_{2an}c_4} \tag{13}$$

$$\frac{\nu_{xy}^{LAM}}{\nu_{xy0}^{LAM}} = \frac{1+2\rho t_{90}(t_{90}/h_0)u_{2an}c_1\left(1-\dfrac{\nu_{LT}}{\nu_{yx0}^{LAM}}\right)}{1+2\rho t_{90}(t_{90}/h_0)u_{2an}c_2}, \quad c_1 = \frac{E_T}{E_{x0}^{LAM}}\frac{1-\nu_{LT}\nu_{xy0}^{LAM}}{\left(1-\nu_{LT}\nu_{TL}\right)^2} \tag{14}$$

$$c_2 = c_1\left(1-\nu_{LT}\nu_{xy0}^{LAM}\right), \quad c_4 = \frac{E_T}{E_{y0}^{LAM}}\frac{\left(\nu_{LT}-\nu_{yx0}^{LAM}\right)^2}{\left(1-\nu_{LT}\nu_{TL}\right)^2} \tag{15}$$

Notation h_0 is used for the half thickness of the laminate, $h_0 = t_{90} + t_0$. Quantities with index "LAM" are laminate constants, additional upper index 0 denotes undamaged laminate constants. The only remaining unknown entity in (13)-(15) is the average normalized crack opening displacement u_{2an}. At low crack density intralaminar cracks in layer are considered as non-interactive and $u_{2an} = u_{2an}^0$ does not depend on crack density ρ. The u_{2an}^0 for cracks in surface and inside layers are different. Fitting expressions for u_{2an}^0 of surface layers were presented in [5]

$$u_{2an}^0 = A + B\left(\frac{E_T}{E_x^S}\right)^n \tag{16}$$

In (16) E_x^S is the Young's modulus of the support layer measured in the x-direction which is the transverse direction for the cracked layer. For cross-ply laminate $E_x^S = E_L$, $A = 1.2$, $B = 0.5942 + 0.1901\left(2(t_{90}/t_S)-1\right)$,

$$n = -0.5229\left(\frac{t_{90}}{t_S}\right)^2 + 0.8874\frac{t_{90}}{t_S} + 0.2576 \tag{17}$$

In (17) $t_S = 2t_0$ is thickness of the adjacent support layer and t_{90} is thickness of the cracked layer. The normalized average COD is larger for less stiff support layers and approach to certain asymptotic value with increasing support layer and cracked layer stiffness ratio. For thicker support layers the COD is smaller. When the crack density is high the u_{2an} depends on crack density [6] and COD of non-interactive cracks, u_{2an}^0 by relationship

$$u_{2an} = \lambda(\rho)u_{2an}^0 \tag{18}$$

The interaction function λ is a function of the crack density in the layer. For non-interactive cracks $\lambda = 1$. Detailed analysis of the effect of different parameters on interaction function was performed in [6] and the following empirical relationship was obtained

$$\lambda_k = \tanh\left(\frac{k}{\rho_{kn}}\right) \tag{19}$$

$$k^2 = 0.085\frac{G_{23}(t_S/2)+t_{90}G_{LT}}{G_{23}((t_S/2)+t_{90})}\left(1+\frac{t_{90}E_T}{E_S(t_S/2)}\right) \tag{20}$$

5. Calculation of resultant moment-curvature relationship in 4-point bending test

3-D FEM model schematically shown in Fig.2 was generated using finite element software ANSYS version 13.0 [8]. The model consists of 3 layers representing the given cross-ply lay-ups [90m/0n]s . The total length of the laminate L was 150 mm with the equal distance between the loading points. The width of the laminate beam was $w = 0.8$ mm. The laminate was subjected to 4-point bending loading scheme by applying constant displacement $u_z = -3$ mm on the top surface points. Simple supports were added: the nodes corresponding to coordinates $x = 0$, $z = 0$ were constrained against displacement in x and z axis directions; the node at coordinate $x = 0$, $z = 0$, $y = -0.5w$ was additionally constrained against displacement in the y axis direction; the nodes corresponding to coordinates $x = L$, $z = 0$ were constrained against displacement in the z axis direction and the node corresponding to coordinate $x = L$, $z = 0$, $y = -0.5w$ was additionally constrained against displacement in y axis direction. Displacement coupling in the y axis direction was applied on the nodes of each edge.

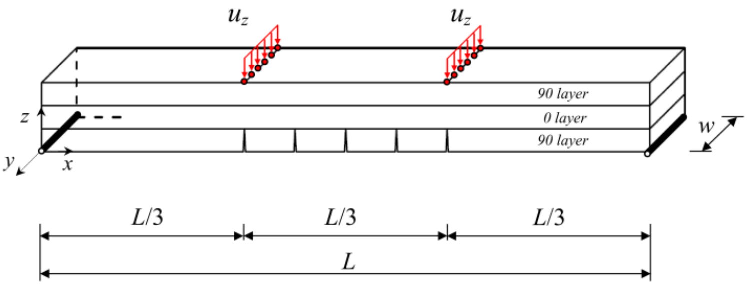

Figure 2. Schematic representation of a 4-point bending FEM model.

The top 90-layer and the 0-layer remained undamaged. In the bottom 90-layer transverse cracks where introduced only in the maximum bending moment zone, i.e., in the region between the displacement application points as shown in Fig.2. The number of transverse cracks was parametrically varied from 0 to 33 uniformly spaced cracks. Correspondingly, the bending stiffness of the laminate was reduced.
Apart from transverse cracks the effect of delaminations at transverse crack tip on bending stiffness was also investigated. Delamination cracks were introduced between the damaged 90 degree layer and 0 degree layer as shown schematically in Figure 1b. Delaminations were assumed symmetric with respect to transverse crack plane. The delamination length l_d was parametrically varied from 0 to $0.5t_{90}$. Bending stiffness was calculated using (3) where the bending moment M_x was calculated from the reaction forces and the curvature k_x was calculated from displacements in the constant bending moment zone.

6. Calculation of resultant moment-curvature relationship in 4-point bending test

Carbon fiber/epoxy and glass fiber/epoxy $[90/0_2]$s , [90/0]s and $[90_2/0_2]$s laminates with UD elastic properties presented in Table 1 were studied. The respective 90-layer thickness was 0.6 and 1.2mm.

Property	E_1	E_2	ν_{12}	G_{12}	ν_{23}
Unit	[GPa]	[GPa]	[-]	[GPa]	[GPa]
CF/EP	104.00	6.14	0.40	5.00	0.45
GF/EP	45.00	15.00	0.30	5.00	0.40

Table 1. Elastic constants of UD composites.

The effective transverse modulus of the damaged 90-layer at different crack densities calculating using the FEM RVE model and GLOB-LOC approach for CF/EP $[90/0_2]$ is shown in Fig.3a. The values at all crack densities are almost coinciding. As expected only the transverse effective modulus E_T^{eff} changed with the crack density, while the longitudinal effective modulus E_L^{eff} and the Poisson's ratio ν_{LT}^{eff} remained unchanged.

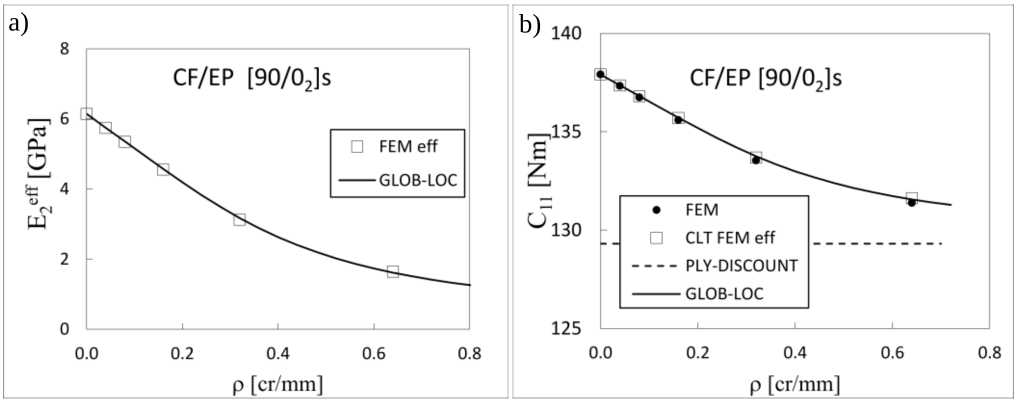

Figure 3. Simulations for CF/EP [90/0$_2$]s laminate: a) effective 90-layer modulus; b) bending stiffness.

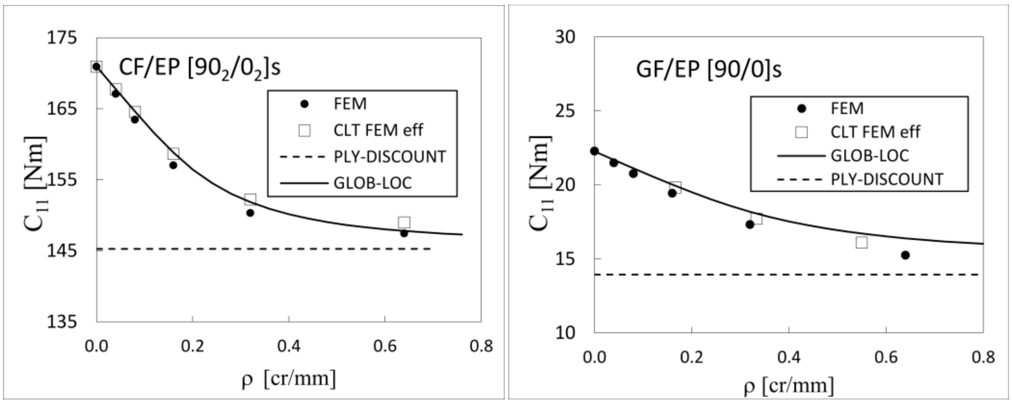

Figure 4. Bending stiffness of CF/EP and GF/EP cross-ply laminates according to different models.

In Fig. 3b and 4 bending stiffness parameter C_{11} is plotted for lay-ups [90/0$_2$]s , [90/0]s and [90$_2$/0$_2$]s respectively. FEM calculations for 4-point bending can be compared with two CLT calculations using effective transverse modulus of damaged layer from FEM RVE (denoted as "CLT FEM eff") and GLOB-LOC models (denoted as "GLOB-LOC"). Ply discount model results (assuming near zero properties of the damaged layer) are also presented for comparison. It can be seen that all models are in very good agreement in the whole range of studied crack densities ρ. Agreement is excellent for CF/EP laminate with relatively thin damaged surface layer.

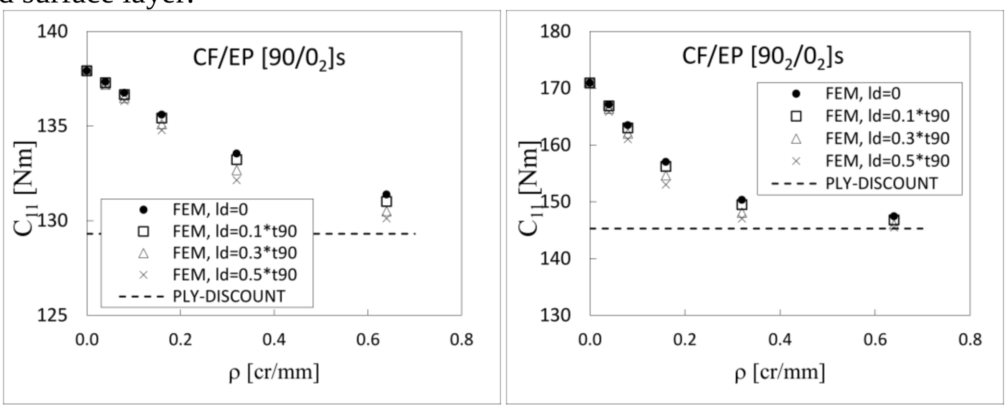

Figure 5. Delamination effect on bending stiffness of CF/EP laminates with cracks in 90-layer.

For laminates with relatively thicker 90-layer shown in Fig.4 the models based on CLT and effective stiffness slightly overestimate the residual bending stiffness. Better agreement for laminates with thinner surface layer may be because in this case the axial stress distribution across the ply thickness is more uniform and the COD in bending better corresponds to COD in the tensile calculation case used to determine effective constants of the damaged layer.
In Fig.5 the influence of delaminations on bending stiffness reduction for $[90/0_2]s$ and $[90_2/0_2]s$ lay-up is shown. Results are obtained simulating the 4-point bending test using FEM. The bending resistance of the laminate with cracks is additionally reduced by the presence of delaminations initiating from cracks. As expected, the reduction is larger due to longer delaminations. The effect of the delamination seems to be relatively smaller (comparing to the cracking effect) for laminates with a thick cracked layer.

7. Conclusions

Classical laminate theory (CLT) based approach to calculate the bending stiffness of laminate with intralaminar cracks in surface 90-layers is suggested. In this approach the damaged layer is replaced with undamaged layer with effective stiffness as for the damaged layer. The effective stiffness is back-calculated from the laminate in-plane stiffness difference in damaged and undamaged state. The damaged laminate stiffness was obtained by two methods: a) performing FEM calculations on representative volume element of the laminate between two cracks (this is considered the most accurate approach); b) using the earlier developed analytical GLOB-LOC model which is more convenient but not exact. The high accuracy of the effective stiffness approach was validated comparing with FEM simulation of 4-point bending test for laminate with intralaminar cracks in tensile surface layer. It is shown that the GLOB-LOC based analytical model gives sufficient accuracy for damaged layer effective stiffness. Significant effect of local delamination on bending stiffness was found simulating 4-point bending test.

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