INITIATION OF FINITE CRACKS IN NON-POSITIVE GEOMETRIES

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Abstract
Coupled stress and energy criteria in the framework of Finite Fracture Mechanics have been widely used to study crack initiation at stress concentrations. In many cases, the incremental energy release rates of the considered cracks of finite size increase monotonically with increasing crack sizes. But in certain structural situations, referred to as non-positive geometries, non-monotonic energy release rates are found. In this work the effect of this finding on the solution of coupled stress and energy is studied in detail. Using the example of an adhesively bonded joint the solution is discussed and it is found that the predicted crack lengths can be discontinuous with regard to a changed structural parameter. The non-monotonic energy release rate prevents the formation of cracks with certain range of crack lengths. The stability of the possible crack configurations is addressed.

1. Introduction

It has been shown that Finite Fracture Mechanics approaches can be used successfully to assess crack initiation at stress concentrations. Even crack initiation at weak stress singularities can be studied with coupled stress and energy criteria in the framework of Finite Fracture Mechanics [1–5]. Several studies show that coupled criteria can give a physical explanation for structural size effects [6–8]. A major advantage of these approaches is that they do not require a characteristic length that depends on the structural situation. The fundamental material parameters strength and fracture toughness suffice for an analysis. To analyze a structural situation with the coupled criterion the local stress distribution in the vicinity of the considered stress concentrations and the incremental energy release rate of possible crack configurations are required. The incremental energy release rate is the energy released per crack length in the process of formation of a crack of finite size. In many cases the incremental energy release rate shows a monotonic increase with larger crack lengths. Following Bažant's definition [9] such structures are referred to as positive geometries. In literature some structural situations with non-monotonic energy release rates have been reported. Free-edge delaminations in laminates where studied by Rybicki et al. [10] and by Wang and Crossman [11]. The debonding of composite stringers was studied by Krueger et al. [12] and it was also found to be a non-positive structure. Similar findings were obtained for debonding of patches by Krueger [13] and for crack initiation in adhesive joints by Hell et al. [14]. All given structural situations share the presence of thin layers with strong elastic mismatch. The latter example of an adhesive joint will be used in the following to study implications of the non-monotonic energy release rates on the evaluation of a coupled stress and energy criterion. The stability of the resulting crack configurations and its dependence on the energy release rate will be discussed.

2. Theoretical background

In Finite Fracture Mechanics the instantaneous formation of cracks of finite size is considered [15]. A coupled stress and energy criterion [1] within this framework allows for analysis of crack initiation at strong stress concentrations. The energy release rate of a crack of finite size ΔA is typically called incremental energy release rate $\bar{\mathcal{G}}$. It can be obtained by comparing the total potential of the uncracked state $^{(1)}$ and the equally deflected cracked state $^{(2)}$:

$$\bar{\mathcal{G}} = -\frac{\Delta\Pi^i}{\Delta A} = \frac{\Pi^{i(1)} - \Pi^{i(2)}}{\Delta A}. \tag{1}$$

or alternatively by using the Virtual Crack Closure Technique. For crack opening modes I and II, it reads:

$$\begin{aligned} \bar{\mathcal{G}}_I &= \frac{1}{2\Delta A}\int_0^{\Delta A} \sigma_y^{(1)}(x)\left(v^{(2)+}(x) - v^{(2)-}(x)\right)\mathrm{d}x, \\ \bar{\mathcal{G}}_{II} &= \frac{1}{2\Delta A}\int_0^{\Delta A} \tau_{xy}^{(1)}(x)\left(u^{(2)+}(x) - u^{(2)-}(x)\right)\mathrm{d}x \\ \bar{\mathcal{G}} &= \bar{\mathcal{G}}_I + \bar{\mathcal{G}}_{II} \end{aligned} \tag{2}$$

The incremental energy release rate can also be obtained by integrating the differential energy release rate over the finite crack size:

$$\bar{\mathcal{G}} = \frac{1}{\Delta A}\int_A^{A+\Delta A} \mathcal{G}(\tilde{A})\mathrm{d}\tilde{A}. \tag{3}$$

The coupled stress and energy criterion with consideration of a crack length dependent crack resistance [6] (R-curve) reads:

$$f(\sigma_{ij}(\mathbf{x})) \geq \sigma_c \;\forall\; \mathbf{x} \in \Omega_c \;\wedge\; \bar{\mathcal{G}}(\Delta A) \geq \frac{1}{\Delta A}\int_0^{\Delta A} R(\tilde{A})\mathrm{d}\tilde{A}. \tag{4}$$

Here f is an appropriately chosen stress criterion and Ω_c is the surface of the considered crack. The quantities σ_c and $\bar{\mathcal{G}}_c$ are the strength and the toughness, respectively. In most applications it is sufficient to assume a constant crack resistance that is equal to the fracture toughness: $R(\Delta A) = \mathcal{G}_c = \text{const}$. Besides a point-wise formulation of the stress criterion in (4), consideration of the mean stresses was proposed by Cornetti et al. [16].

In the general case the identification of the critical crack initiation loading and the corresponding finite crack size requires solving the corresponding restrained optimization problem. The smallest loading must be found that satisfies both criteria of the coupled criterion for any kinematically admissible finite crack configuration:

$$F_f = \min_{F,\Delta A}\left\{ F \mid f(\sigma_{ij}(x_i)) \geq \sigma_c \;\forall x_i \in \Omega_c \;\wedge\; \bar{\mathcal{G}}(\Delta A) \geq \frac{1}{\Delta A}\int_0^{\Delta A} R(\tilde{A})\mathrm{d}\tilde{A} \right\}. \tag{5}$$

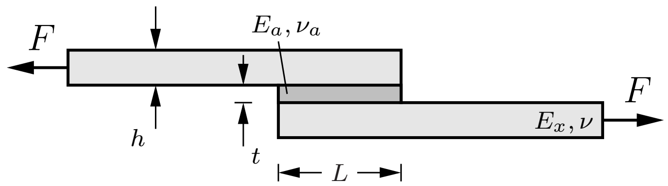

Figure 1. Single lap joint configuration under axial tensile loading.

The stability of cracks can be assessed by considering the derivative of the differential energy release rate. Cracks under constant load are stable and do not grow further when the derivative of the differential energy release rate is smaller than the derivative of the crack resistance function:

$$\frac{\partial \mathcal{G}}{\partial a} < \frac{\partial R}{\partial a}. \tag{6}$$

3. Structural analysis

To allow for a detailed analysis of arbitrary structural situations the Finite Element (FE) method is used. While the required stress field can be obtained by a single (possibly nonlinear) analysis of the uncracked configuration, the incremental energy release rate calculation needs an additional FE analysis for each crack configuration. Then the energy release rate can be calculated by relating the change of the total potential (1) or by evaluation with the Virtual Crack Closure Technique (2) in its discretized form using the nodal forces p and displacements u, v [10]:

$$\bar{\mathcal{G}}_I = \frac{1}{2\Delta A}\sum_{k=1}^{N} p_{yk}^{(1)}\left(v_k^{(2)+} - v_k^{(2)-}\right), \tag{7}$$

$$\bar{\mathcal{G}}_{II} = \frac{1}{2\Delta A}\sum_{k=1}^{N} p_{xk}^{(1)}\left(u_k^{(2)+} - u_k^{(2)-}\right). \tag{8}$$

It must be noted, that the separation of crack opening modes is only defined for cracks in an homogeneous body in its original sense [17]. Nevertheless, this terms will be used in the following as this allows for identification of the contributions of individual stresses and displacements to the overall energy release rate. The former method, that considers the change of the total potential, has the disadvantages that it does not allow for a separation of the individual crack mode contributions and numerical rounding errors can occur when the ratio of the released energy to the released energy to the total energy becomes too small. But using this method allows to compute the energy release rate in the case on nonlinear analyses. In the following the incremental energy release rate will be calculated with the change of the total potential whenever nonlinear computations are necessary. The method of Virtual Crack Closure Technique is used to allow for a separation of the mode I and II contributions.

Now an adhesively bonded single lap joint is considered. It is modeled as a two-dimensional plane strain continuum with width b. Linear elastic material behavior is assumed for the adherends and the adhesive. The geometry and material parameters are denoted in Fig. 1. Straight cracks of length Δa in the adhesive layer starting at the reentrant corner of the upper adherend and adhesive layer are considered. Fig. 2 shows the energy release rate of crack configurations

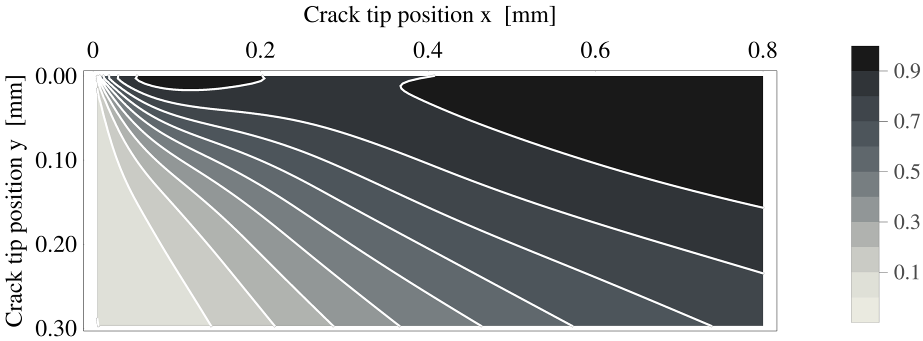

Figure 2. Incremental energy release rate for straight finite cracks at the reentrant corner of the upper adherend and the adhesive layer.

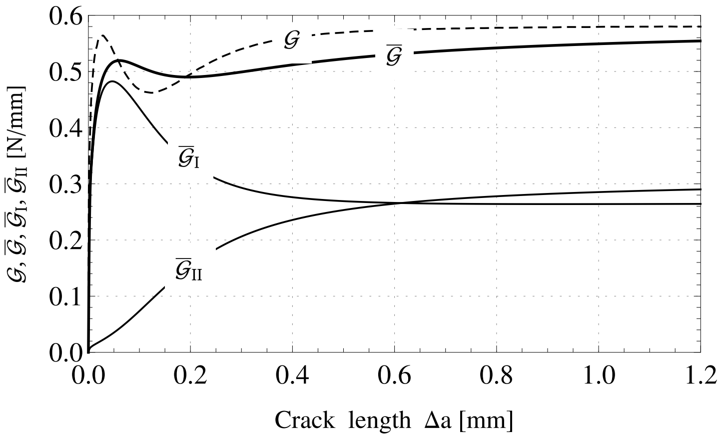

Figure 3. Energy release rates obtained by FE analyses as a function of the crack length for cracks in the adherend-adhesive interface. Both the differential and the incremental energy release rate are shown. For the latter the individual contributions by mode I and II are shown.

in a typical single lap joint configuration. The incremental energy release rate is shown in a contour plot as a function of the position of the crack tip. The point (0/0) is the reentrant corner of the adherend and adhesive where the weak stress singularity is located. The results show that cracks with small angles to the adhesive-adherend interface have higher energy release rates than cracks with large angles. It is interesting that the highest energy release rates do not occur for cracks on the interface but for small angles around 4°. It can also be seen that local maxima occur for cracks that are approximately 0.15 to 0.2mm long. Hence, in this structural situation a non-monotonic increase of the energy release rate is observed. To allow for a detailed examination of this effect Fig. 3 shows the energy release rate for cracks with a fixed angle (here 0°). Besides the total incremental energy release rate its mode I and II contributions and the corresponding differential energy release rate are shown. All energy release rates are zero for vanishing crack lengths which is a general feature of cracks at weak stress singularities [17]. Of course both the differential and the incremental energy release rate exhibit a non-monotonic dependence on the finite crack length with a local maximum. As the incremental energy release rate is an averaged differential energy release rate it has its maximum at higher crack lengths. Considering the individual contributions it can be seen that the mode II energy release rate

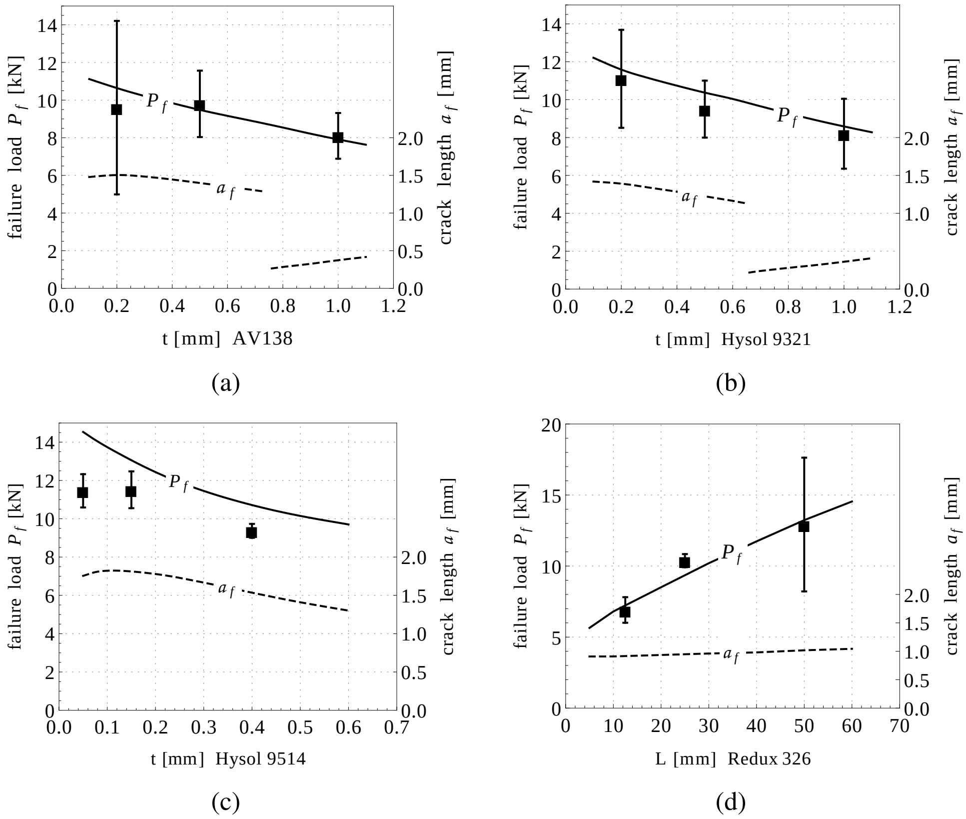

Figure 4. Comparison of the failure load predictions by the coupled criterion [14] (solid lines) to experimental results from da Silva et al. [18, 19] and Castagnetti et al. [20]. The corresponding finite crack lengths are shown as dashed lines.

contribution shows a monotonic behavior whereas the mode I contribution has a maximum for short cracks and then decays to an asymptotic value. For cracks longer than 0.5mm both crack mode contributions are about the same magnitude but for small cracks the mode I contribution dominates with its local maximum.

4. Discussion

The incremental energy release rate results obtained for the adhesive joint configurations can be used to solve the optimization problem (5) associated with the coupled criterion (4). This has been done in the comprehensive analysis by Hell et al [14]. A good agreement with experimental findings was obtained showing the soundness of the employed approach. The results of this comparison are shown in Fig. 4. It can be seen that in general a good agreement of the predicted failure loads with the experiments is obtained. The effects of the overlap length and the adhesive layer thickness on the failure load are rendered correctly [21, 22]. No fitting of the failure parameters was performed but only results from standard tests were used. The predicted crack lengths cannot be compared to experimental findings as they are typically not monitored during the possibly unstable crack initiation and crack growth process. The crack lengths shall be object of the further discussion as they show the effect of the non-monotonic energy release rate on the solution of the optimization problem of the coupled criterion.

In case of a monotonic decrease of the stress function and a monotonic increase of the energy

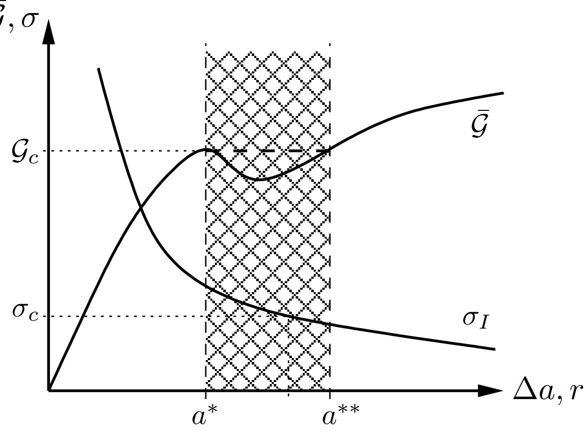

Figure 5. Schematic representation of the energy release rate and stress function in the discussed structural situations with non-positive geometries.

release rate the inequalities in the coupled criterion (4) revert to equalities. Then the energy condition can be understood as a lower bound for the crack length and the stress condition as an upper bound. The minimum load fulfilling both conditions is found when both bounds coincide. In the present case of non-monotonic energy release rates the general formulation must be considered. As the stress function shows a monotonic decrease with increasing distance from the stress concentration the stress condition still represents an upper bound for the crack length. Figure 5 shows a schematic representation of the stress function and the energy release rate function for non-positive geometries. The crack length associated to the local maximum is denoted as a^* and the corresponding crack length that leads to the same value of the energy release rate is denoted as a^{**}. It is clear that no crack configurations can initiate that are between these two crack lengths (marked as shaded in Fig. 5). Because then there is always a lower load that fulfills the energy condition with $\Delta a = a^*$. The stress condition may remain overfulfilled in such cases. By incident such a situation can be found in the previously addressed comparison to experimental results. Of course, the strength and the fracture toughness do not change throughout the study but the adhesive layer thickness has a marked effect on the energy release rate. With increasing adhesive layer thickness the magnitude of the energy release rate rises lowering the lower bound of possible crack lengths. Simultaneously, the crack length of the maximum a^* rises as well. It is found that a^* is proportional to this geometric parameter. In case of the considered steel-epoxy joints $a^* \approx 0.3t$ holds. The stresses are reduced by increasing adhesive layer thickness but the effect to increase the energy release rate dominates and hence the failure load decreases with increasing adhesive layer thickness. In the two comparisons shown in Fig. 4(a,b) the predicted crack length is not continuous. For thin adhesive layer the predicted crack length is larger than a^{**}. With increasing adhesive layer thickness the crack length reduces and eventually reaches a^{**}. Then, as discussed before, the crack length must jump to a^*. With further increase of the adhesive layer thickness the corresponding crack length increases as a^* is proportional to the adhesive layer thickness t.

The two possible crack configurations $\Delta a = a^*$ and $\Delta a \geq a^{**}$ have different stability characters. As only one minimum in the energy release rate function exists, the crack length a^* of the first maximum is associated with a differential energy release rate that has a negative derivative. Hence, this crack will be stable after loading, given that the loading is constant. The finite cracks that are predicted to initiate with a crack length $\Delta a \geq a^{**}$ are unstable as the derivative of the differential energy release rate is greater than zero. Although not yet observed in experiments it should be possible to identify such a change in the stability behavior in experiments.

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