INVARIANT-BASED FAILURE CRITERIA FOR COMPOSITE LAMINATES

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Abstract
A new three-dimensional failure criteria for fibre-reinforced composite materials based on structural tensors and invariant theory is proposed. Failure envelopes for several fibre-reinforced polymers under different stress states are generated and compared with the test data available in the literature. For more complex three-dimensional stress states, where the test data available shows large scatter or is not available at all, a computational micro-mechanics framework is used to validate the failure criteria. In general, the failure predictions were in good agreement with previous three-dimensional failure criteria and experimental data. The computational micro-mechanics framework is shown to be a very useful tool to validate failure criteria under complex three-dimensional stress states.

1. Introduction

The effective use of polymer composite materials reinforced by unidirectional fibres relies on the ability to obtain reliable predictions of the onset and propagation of the different failure mechanisms. Accordingly, the development of accurate failure criteria that predict the onset of ply damage mechanisms in fibre-reinforced polymers (FRPs) is extremely important. Indeed, this has been the subject of a great number of studies [1, 2, 3, 4, 5].

In the past, we have developed efforts to derive failure criteria for plane stress [4] that yielded good predictions of failure envelopes under such simple stress state. One major development was an improved criterion for fibre kinking, which is based on the mechanics of deformation of a composite ply with an initial fibre misalignment [4]. However, there are several applications of composite structures where the out-of-plane components of the stress tensor cannot be neglected (e.g. bolted joints under in-plane, out-of-plane loading and combinations of thereof). Therefore, the objective of this paper is to present a recently proposed failure criteria [6] that accounts for fully general stress states.

1.1. Summary of the failure criteria

The full details of the derivation of the failure criteria are presented in [6]. In the following, a summary of the equations that represent the failure criteria are presented.

Matrix failure

1. *Invariants*

$$I_1 = \frac{1}{4}\sigma_{22}^2 - \frac{1}{2}\sigma_{22}\sigma_{33} + \frac{1}{4}\sigma_{33}^2 + \sigma_{23}^2$$
$$I_2 = \sigma_{12}^2 + \sigma_{13}^2$$
$$I_3 = \sigma_{22} + \sigma_{33}$$

Matrix tension ($I_3 > 0$)	**Matrix compression** ($I_3 \leq 0$)
2. *Model parameters*	
$\alpha_1 = \frac{1}{S_T^2}$	$\alpha_1 = \frac{1}{S_T^2}$
$\alpha_2 = \frac{1}{S_L^2}$	$\alpha_2 = \frac{1}{S_L^2}$
$\alpha_{32}^t = \frac{1 - \frac{Y_T}{2Y_{BT}} - \alpha_1 \frac{Y_T^2}{4}}{Y_T^2 - 2Y_{BT}Y_T}$	$\alpha_{32}^c = \frac{1 - \frac{Y_C}{2Y_{BC}} - \alpha_1 \frac{Y_C^2}{4}}{Y_C^2 - 2Y_{BC}Y_C}$
$\alpha_3^t = \frac{1}{2Y_{BT}} - 2\alpha_{32}^t Y_{BT}$	$\alpha_3^c = \frac{1}{2Y_{BC}} - 2\alpha_{32}^c Y_{BC}$
3. *Failure criteria*	
$f_M = \alpha_1 I_1 + \alpha_2 I_2 + \alpha_3^t I_3 + \alpha_{32}^t I_3^2$	$f_M = \alpha_1 I_1 + \alpha_2 I_2 + \alpha_3^c I_3 + \alpha_{32}^c I_3^2$

Fibre failure

- **Fibre tension** ($\sigma_{11} \geq 0$)

1. *Failure criteria*

$$f_F = \frac{\varepsilon_{11}}{\varepsilon_1^T}$$

- **Fibre compression** ($\sigma_{11} < 0$)

1. *Model parameters*

$$\alpha_1 = \frac{1}{S_T^2}$$
$$\alpha_2 = \frac{1}{S_L^2}$$
$$\alpha_{32}^t = \frac{1 - \dfrac{Y_T}{2Y_{BT}} - \alpha_1 \dfrac{Y_T^2}{4}}{Y_T^2 - 2Y_{BT}Y_T}$$
$$\alpha_{32}^c = \frac{1 - \dfrac{Y_C}{2Y_{BC}} - \alpha_1 \dfrac{Y_C^2}{4}}{Y_C^2 - 2Y_{BC}Y_C}$$
$$\alpha_3^t = \frac{1}{2Y_{BT}} - 2\alpha_{32}^t Y_{BT}$$
$$\alpha_3^c = \frac{1}{2Y_{BC}} - 2\alpha_{32}^c Y_{BC}$$

2. *Angle of the kinking plane* (ψ)

If $\sigma_{12} = 0$ and $\sigma_{13} = 0$	Otherwise
$\psi = \frac{1}{2} \arctan \left(\frac{2\sigma_{23}}{\sigma_{22} - \sigma_{33}} \right)$	$\psi = \arctan \frac{\sigma_{13}}{\sigma_{12}}$

3. *Misalignment angle at failure when a pure longitudinal compression is applied* (φ_C)

$$\varphi_C = \frac{1}{2} \arccos \left\{ \left[4\sqrt{\alpha_1 - 4\alpha_2 + \alpha_2^2 X_C^2 + (\alpha_3^c)^2 + 2\alpha_2\alpha_3^c X_C + 4\alpha_{32}^c} + \right. \right. \\ \left. \left. + (\alpha_1 + 4\alpha_{32}^c) X_C + 4\alpha_3^c \right] \cdot \left[(\alpha_1 - 4\alpha_2 + 4\alpha_{32}^c) X_C \right]^{-1} \right\}$$

4. *Initial misalignment angle* (φ_0)

Linear shear and small angles	*Nonlinear shear*
$\varphi_0 = \varphi_C \left(1 + e\frac{\lvert X_C \rvert}{G_{12}}\right)^{-1}$	Defining the function $F(\varphi_0)$ as: $F(\varphi_0) = \varphi_C - \varphi_0 - \left\lvert \frac{X_C \sin 2\varphi_0}{2G_{12}} + \beta \frac{X_C^3 \sin^3 2\varphi_0}{8} \right\rvert$ and calculating its derivative with respect to φ_0: $\frac{\mathrm{d}F}{\mathrm{d}\varphi_0} = -1 - \left\lvert \frac{X_C \cos 2\varphi_0}{G_{12}} + \beta \frac{3}{4} X_C^3 \sin^2 2\varphi_0 \cos 2\varphi_0 \right\rvert$ the initial misalignment angle, φ_0, can be computed using the following recursive formula: $\varphi_0^{i+1} = \varphi_0^i - \frac{F(\varphi_0^i)}{\left.\frac{\mathrm{d}F}{\mathrm{d}\varphi_0}\right\rvert_{\varphi_0 = \varphi_0^i}}$

5. *Shear stress in the misalignment frame* ($\sigma_{12}^{(R)}(\varphi_0, \psi)$)

$$\sigma_{12}^{(R)}(\varphi_0, \psi) = \frac{1}{2}\left[-\sigma_{11} + \sigma_{22}\cos^2\psi + \sigma_{33}\sin^2\psi + \sigma_{23}\sin 2\psi\right]\sin 2\varphi_0 + \\ +(\sigma_{12}\cos\psi + \sigma_{13}\sin\psi)\cos 2\varphi_0$$

6. *Kinking-angle* (φ)

$$\varphi = \mathrm{sgn}\left\{\sigma_{12}^{(R)}(\varphi_0, \psi)\right\}\left\{\varphi_0 + \left\lvert \frac{\sigma_{12}^{(R)}(\varphi_0, \psi)}{G_{12}} + \beta\left[\sigma_{12}^{(R)}(\varphi_0, \psi)\right]^3 \right\rvert\right\}$$

7. *Preferred direction* ($\mathbf{a}$)

$$\mathbf{a} = \begin{bmatrix} \cos\varphi \\ \cos\psi\sin\varphi \\ \sin\psi\sin\varphi \end{bmatrix}$$

8. *Structural tensor* ($\mathbf{A}$)

$$\mathbf{A} = \mathbf{a} \otimes \mathbf{a}$$

9. *Reaction stress tensor* ($\boldsymbol{\sigma}^r$)

$$\boldsymbol{\sigma}^r = \frac{1}{2}(\operatorname{tr}\boldsymbol{\sigma} - \mathbf{a}\boldsymbol{\sigma}\mathbf{a})\mathbf{1} - \frac{1}{2}(\operatorname{tr}\boldsymbol{\sigma} - 3\mathbf{a}\boldsymbol{\sigma}\mathbf{a})\mathbf{A}$$

10. *Plasticity inducing stress tensor* ($\boldsymbol{\sigma}^p$)

$$\boldsymbol{\sigma}^p = \boldsymbol{\sigma} - \boldsymbol{\sigma}^r$$

11. *Invariants*

$$\begin{aligned} I_1 &= \frac{1}{2}\operatorname{tr}\left(\boldsymbol{\sigma}^p\right)^2 - \mathbf{a}\left(\boldsymbol{\sigma}^p\right)^2\mathbf{a} \\ I_2 &= \mathbf{a}\left(\boldsymbol{\sigma}^p\right)^2\mathbf{a} \\ I_3 &= \operatorname{tr}\boldsymbol{\sigma} - \mathbf{a}\boldsymbol{\sigma}\mathbf{a} \end{aligned}$$

Matrix tension ($I_3 > 0$)	**Matrix compression** ($I_3 \leq 0$)
12. *Failure criteria*	
$f_K = \alpha_1 I_1 + \alpha_2 I_2 + \alpha_3^t I_3 + \alpha_{32}^t I_3^2$	$f_K = \alpha_1 I_1 + \alpha_2 I_2 + \alpha_3^c I_3 + \alpha_{32}^c I_3^2$

2. Validation examples

2.1. *Effect of hydrostatic pressure on E-Glass/MY750/HY917/DY063*

Hine et al. [7] investigated the effect of hydrostatic pressure on the mechanical properties of E-Glass/MY750/HY917/DY063 fibreglass-epoxy. Figure 1 shows the experimental results obtained by [7] for the effect of hydrostatic pressure on the in-plane shear response, and a comparison with the predictions of the proposed failure criteria.

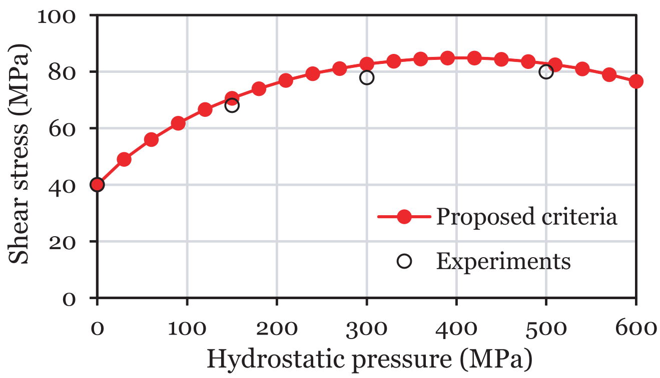
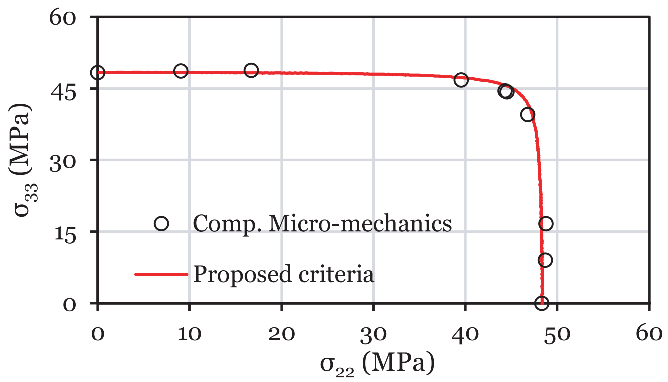


Figure 1: Shear response of E-Glass/MY750/HY917/DY063 fibreglass-epoxy subjected to hydrostatic pressure.

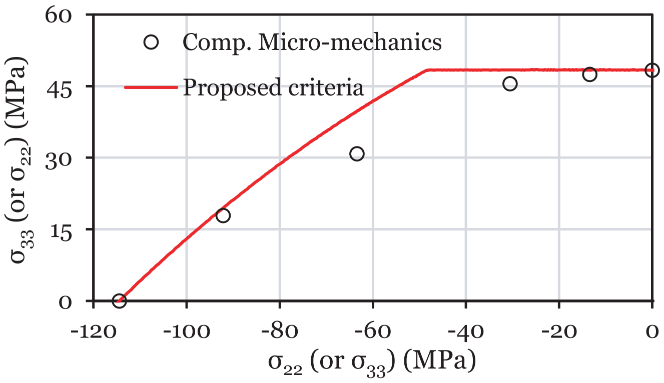
2.2. Validation based on computational micro-mechanics

The range of stress states that can be imposed by means of experimental tests is limited by the complexity of the load introduction systems. Therefore, a recently proposed approach based on computational micro-mechanics [8] is used here for generation of failure envelopes corresponding to stress states that cannot be experimentally imposed to the material.

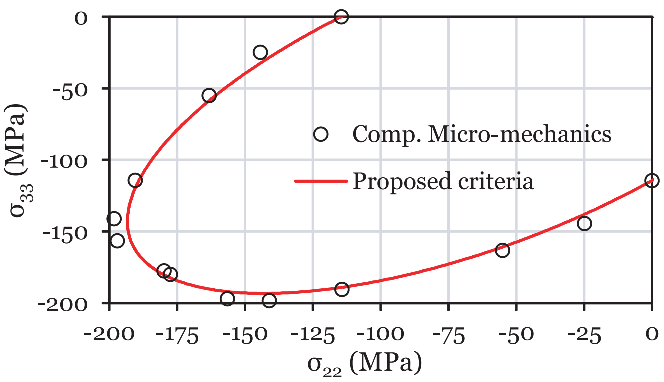
Figure 2 shows the comparison between the failure envelopes predicted by computational micro-mechanics and the failure criteria proposed in this paper. In general, a good agreement between the two modelling strategies is observed.



(a) Transverse biaxial tension.



(b) Transverse tension-compression.



(c) Transverse biaxial compression.

Figure 2: $\sigma_{22} - \sigma_{33}$ failure envelope — micro-mechanics versus analytical failure criteria.

3. Conclusions

The new failure criteria proposed is able to predict failure of composite laminates under complex stress states with good accuracy. A good agreement was observed when the predictions of the failure criteria are compared with experimental data and with results of computational micro-mechanical models. The computational micro-mechanics framework is shown to be a very useful tool to validate failure criteria under complex three-dimensional stress states.

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