SIMULATION OF LAMINATED STRUCTURES USING THE PROPER GENERALIZED DECOMPOSITION

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Abstract

Numerical simulations of composite materials are generally performed using laminated shell elements in the context of the finite elements method. This strategy has numerous advantages like a low computation time and the capability to reproduce the mechanical behavior of composites in most cases. However, shell simulations are not well adapted to simulate damaging and in particular delamination. The use of cohesive zone model with a full 3D simulation is interesting to deal with delamination but in practical the computational cost is to high and this solution is generally restricted to very simple structures. An alternative method is proposed between shell simulations and full 3D simulations. The idea is to solve the full 3D solid problem separating the in-plane and the out-of-plane spaces. This is possible with the Proper Generalized Decomposition. Only a shell mesh is required. Then, the computational cost is significantly reduced and cohesive elements may be used to treat delamination in multi-layer composites without having to manage a 3D mesh.

1. Introduction

Composite laminates are widely used due to their specific mechanical properties: a high stiffness and strength with regard to its weight. However, when these materials are subjected to loading, many failure mechanisms can occur. This damage leads to a local or global failure, like intralaminar failure (fiber fracture or matrix cracking), and interlaminar failure (delamination). Delamination is one of the most critical and frequent damage in laminated composites. The study of failure mechanisms and the fracture propagation is required in these materials to ensure good knowledge of their mechanical behavior.
Different approaches were developed to study delamination. The analysis of the onset of delamination generally uses a stress based criterion. The Linear Elastic Fracture Mechanics approaches (LEFM) have been developed to predict the propagation of delamination when the non linearities are negligible. In many cases of crack growth in laminated composites, a non linear zone exists in the crack tip and cannot be neglected. It is characterized by a softening behavior, and it is referred to as fracture process zone.

With a small process zone size, the LEFM has been proven to be reliable to predict propagation of a pre-existing crack using the Finite Element Method (FEM) [1]. However, the LEFM approaches presents many difficulties when implemented within finite element code. They require: i) a previous knowledge of the location of the crack and its direction of propagation, ii) remeshing at the crack tip during the growth of delamination. To overcome these limitations of the LEFM, other approaches have been proposed to study delamination using Damage Mechanics. One of these approaches is the Cohesive Zone Models (CZM) [2]. It is based on the use of interfacial finite elements between the layers of the laminated composite. These cohesive elements are delimited by two cohesive surfaces, linked together by cohesive forces. Compared to the fracture mechanic approaches, the CZM has the capability to predict both onset and propagation of the delamination, in conjunction with the FEM [3].

Nevertheless, the implementation of the CZM in finite element codes has several disadvantages. First of all, it can lead to convergence problems, numerical instabilities, mesh sensitivity and computing inefficiency in the presence of significant materiel non linearities. Then, a large number of finite element calculations is often required to evaluate the sensitivity of the model to the interface parameters. Moreover, a relatively refined mesh is needed for increased accuracy [4], but this would lead to excessive computation time when applied to industrial structures. Furthermore, those industrial structures are composed of a large number of layers, which require increased number of interface elements, leading to increased computational cost as well. These limitations lead to the necessity of efficient numerical tools.

We propose a new approach based on Reduced Order Modeling (ROM) in order to treat the delamination in composite laminates. One famous and efficient model reduction method is the Proper Generalized Decomposition (PGD), which has been used in this work in conjunction with the CZM. The use of the PGD discretization leads to major reductions of the computing time and storage cost [5, 6], especially when the resulting mesh involves a high number of degrees of freedom. The PGD is based on the use of separated representations of the solution. It enables to reduce the size of the multidimensional and parametric problems [7, 8]. This strategy was successfully employed by Ammar et al. for a kinetic theory description [9] of a complex fluid.The PGD has also been applied in other studies for thermal problems in composite materials [10] and to compute efficiently full 3D solutions using in-plane/out-of-plane separated representation of composite laminates [11, 12].

In the present paper, the model using the PGD along with CZM (PGD-CZM) is applied to mode I fracture problem. One aim of this work is to examine the efficiency of cohesive interface elements, using the PGD, for the prediction of delamination growth under static loading. Unidirectional 2-ply carbon/epoxy laminates were tested. Specifically, DCB (Double Cantilever Beam) mode I fracture test was implemented.

2. Constitutive cohesive law under single mode delamination

The formulation of the cohesive zone model used in this work is the Crisfield law [13] shown in Fig (1). It is used to describe the behaviour of the interface, which presents linear elastic and linear softening behaviour. The process of degradation begins when the stresses satisfy one imposed damage initiation criterion. A two-parameter cohesive law was defined for each pure mode. These two parameters are the maximum stress (T_{coh}) and the energy release rate (G_{ic}).

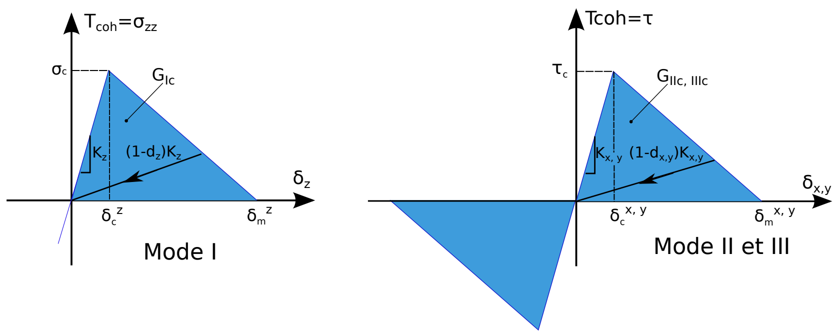

Figure 1. Cohesive law for the mode I.

K_I is the interface element stiffness. The critical separations (δ_c^I) is defined when the interfacial stress reaches maximum, and the maximum separations (δ_m^I) is defined when the stress becomes zero. The relation between local separation and the interlaminar stress (σ_{zz}), shown in Fig (1), can be expressed as:

$$T_{coh} = \begin{cases} K_i\delta_i & \delta_i < \delta_c^i \\ (1-d_i)K_i\delta_i & \delta_c^i \leqslant \delta_i < \delta_m^i \\ 0 & \delta_i \geqslant \delta_m^i \end{cases} \tag{1}$$

Where d_I is the damage variable :

$$d_i = \frac{\delta_m^i(\delta_i - \delta_c^i)}{\delta_i(\delta_m^i - \delta_c^i)}, \quad i = (x, y, z), \quad d_i \in [0, 1] \tag{2}$$

3. Problem formulation

A zero-thikness linear quadrilateral cohesive element shown in Fig (2) is used to simulate delamination problem in conjunction with the PGD model. The constitutive equations of these elements are mentioned in the previous sections in the case of mode I delamination. The 3D mesh is separated into a 2D and a 1D meshes as represented in Fig (2). In the case of the finite element approach, the number of cohesive elements is related to the number of nodes in the mid-plane surface and to the number of layers. In the PGD approach the number of cohesive elements in the thickness is only equal to the number of interfaces between layers. The displacement discontinuity δ across the interface can be expressed in terms of the displacement vector u computed on two sides of the discontinuity ($\mathbf{u}^+$ for the upper side and $\mathbf{u}^-$ for the lower side):

$$\delta = \mathbf{u}^+ - \mathbf{u}^- \Rightarrow \begin{pmatrix} \delta_x \\ \delta_y \\ \delta_z \end{pmatrix} = \begin{pmatrix} u^+ - u^- \\ v^+ - v^- \\ w^+ - w^- \end{pmatrix} \tag{3}$$

The weak form of the equilibrium equation for a linear elastic materials with a cohesive surface Γ_{coh} and a cohesive stress vector $\mathbf{T}_{coh}$, without body force gives:

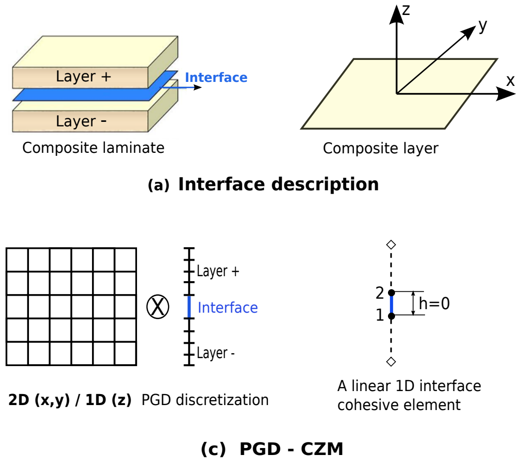

Figure 2. Definition of cohesive surface and mesh discretization.

$$\iint_{\Omega} \varepsilon(\mathbf{u}^*).(\mathbf{A}.\varepsilon(\mathbf{u}))d\Omega + \int_{\Gamma_{coh}} \mathbf{T}_{coh}\delta^* d\Gamma_{coh} = \int_{\Gamma} \mathbf{T}_{ext}\mathbf{u}^* d\Gamma \tag{4}$$

where $\mathbf{u}^*$ and δ^* are the virtual displacement and virtual separation, respectively. $\mathbf{T}_{ext}$ is the external force on the boundary Γ. ε is the strain tensor and $\mathbf{A}$ is a matrix related to the constitutive equation in each layer for an orthotropic material.
The displacement field $\mathbf{u}(x, y, z)$ is approximated using the following separated form of the PGD approach:

$$\mathbf{u} \approx \mathbf{u}^n(x, y, z) = \sum_{i=1}^{n} \mathbf{F}_i(x, y) \circ \mathbf{G}_i(z) \qquad \forall (x, y, z) \in \Omega \tag{5}$$

with $\mathbf{F}_i(x, y) = \begin{pmatrix} F_u^i(x, y) \\ F_v^i(x, y) \\ F_w^i(x, y) \end{pmatrix}$ are functions of the in-plane coordinate and $\mathbf{G}_i(z) = \begin{pmatrix} G_u^i(z) \\ G_v^i(z) \\ G_w^i(z) \end{pmatrix}$ are functions involving the thickness coordinate. $\circ$ denotes the Hadamard product. Equation (5) is then equivalent to:

$$\mathbf{u}^n(x, y, z) = \begin{pmatrix} u_n \\ v_n \\ w_n \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} F_u^i G_u^i \\ \sum_{i=1}^{n} F_v^i G_v^i \\ \sum_{i=1}^{n} F_w^i G_w^i \end{pmatrix}$$

It is assumed that the first n modes have been determined at previous iterations. In order to enrich the separated approximation, some new functions $R(x, y)$ and $S(z)$ have to be determined.

The new approximation can be written as:

$$\mathbf{u}^{n+1}(x,y,z) = \mathbf{u}^{n}(x,y,z) + \begin{pmatrix} R_u(x,y)S_u(z) \\ R_v(x,y)S_u(z) \\ R_w(x,y)S_w(z) \end{pmatrix} \tag{6}$$

The virtual separation δ defined by the equation (3) is approximated using the separated representation:

$$\begin{cases} u^+_{n+1} - u^-_{n+1} = R_u(x,y)\left(S_u(z^+) - S_u(z^-)\right) + \sum\limits_{i=1}^{n} F^i_u(x,y)\left(G^i_u(z^+) - G^i_u(z^-)\right) \\ v^+_{n+1} - v^-_{n+1} = R_v(x,y)\left(S_v(z^+) - S_v(z^-)\right) + \sum\limits_{i=1}^{n} F^i_v(x,y)\left(G^i_v(z^+) - G^i_v(z^-)\right) \\ w^+_{n+1} - w^-_{n+1} = R_w(x,y)\left(S_w(z^+) - S_w(z^-)\right) + \sum\limits_{i=1}^{n} F^i_w(x,y)\left(G_w(z^+) - G_w(z^-)\right) \end{cases} \tag{7}$$

The initial position of the two faces of the cohesive zone are defined by their coordinates on Ω_z denoted z^+ and z^- for all $x \in \Omega_x$. After discretization, z^+ and z^- define the coordinate of two nodes on Ω_z that may be initially at the same position. Finding the couple of functions (R, S) is a highly non linear problem. For that purpose, an alternating directions strategy is used. At each iteration a single function R or S is computed alternately assuming the other known. This procedure continues until convergence. So there are two steps:

1. finding **R** assuming **S**
2. finding **S** assuming **R**

For more details about the PGD resolution technique, the reader can refer to [5].

4. 3D simulation of a DCB test using the PGD approach

The specimen geometry of the DCB test, with boundary conditions and loadings are shown in Fig (3). This test considers a composite laminate with an initial delamination crack length denoted a.

The properties of the material (a unidirectional carbon/epoxy composite) and the ones of the cohesive interface are listed in Table (1).

A 3D DCB test case is realized to focus on the efficiency of PGD when increasing the number of nodes in the mesh. The main advantage of the PGD approach in comparison with the FEM approach is the reduction of the computational time. Another asset is the easy insertion of the cohesive elements. The functions $\mathbf{F}^i_w$ and $\mathbf{G}^i_w$ for $i = [1,2]$ of the separated representation are depicted in Fig (4).

The deformed shape and the longitudinal stress distribution (σ_{xx}) for an imposed displacement equal to 8mm are shown in Fig (5). This simulation was performed with 20000 nodes in the 2D mesh and with 30 nodes in the 1D mesh (thickness). In 3D, that represents a total of 1.810^6

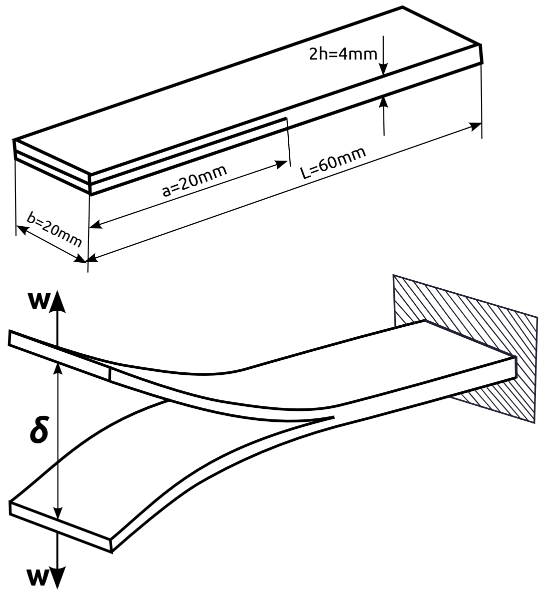

Figure 3. Specimen geometrical dimensions.

Material properties		Interfacial properties	
$E_x(GPa)$	151.4	$G_{Ic}(N/mm)$	0.3
$E_z = E_y(GPa)$	12	$G_{IIc}(N/mm)$	1.6
$G_{xz} = G_{xy}(GPa)$	5.11	$\sigma_c(MPa)$	60
$G_{yz}(GPa)$	4.3	$\tau_c(MPa)$	139
$\nu_{xz} = \nu_{xy}$	0.31	$K_z(N/mm^3)$	1.10^4
ν_{yz}	0.39	$K_{x,y}(N/mm^3)$	5.10^4

Table 1. Material properties for carbon/epoxy.

degrees of freedom. The PGD algorithm enabled running the simulation on a simple laptop in less than 15 minutes. This represent an enormous gain of time when compared to classical 3D FEM simulations with comparable mesh refinement.

The cohesive surface is shown in Fig (6). In this figure, the blue color indicates the undamaged zone, the red color indicates the damaged zone and the process zone is the small part between them.

5. Conclusion

In this paper, an approach based on the PGD have been proposed to simulate mode I delamination in composite laminates in conjunction with CZM. It is shows that Proper Generalized Decomposition can be used as an alternative to overcome the computational drawbacks of the Finite Element Method such as the rapid increase in the number of degrees of freedom, the large computational time and the storage limitation. The reduction of the number of interface elements was achieved due to the PGD-CZM new discretization strategy, which minimize modeling complexity.

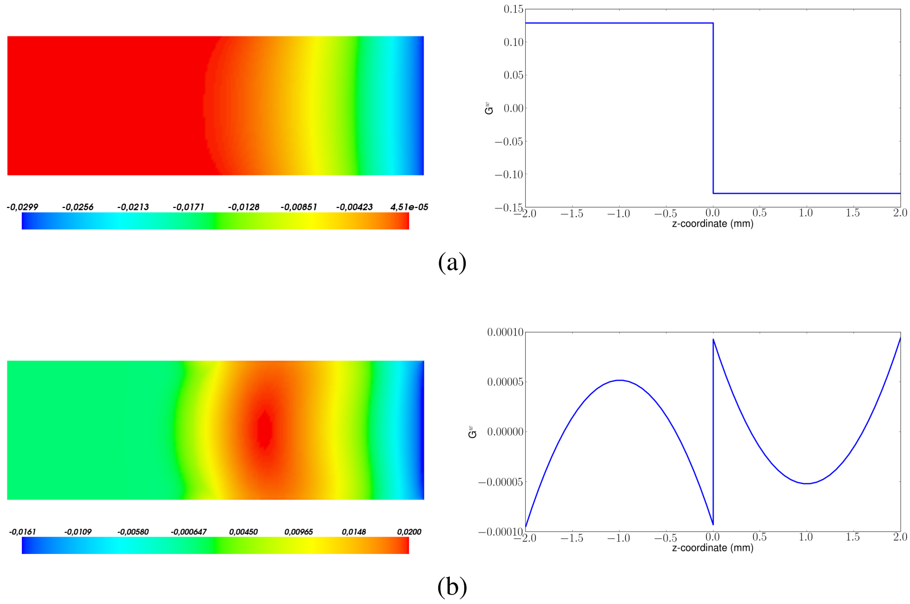

Figure 4. Functions F_w^i and G_w^i in the separated representation of the displacement field: (a) i=1, (b) i=2.

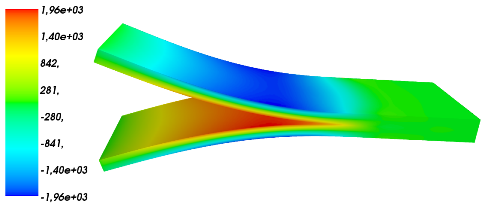

Figure 5. The σ_{xx} stress distribution for the 3D DCB specimen.

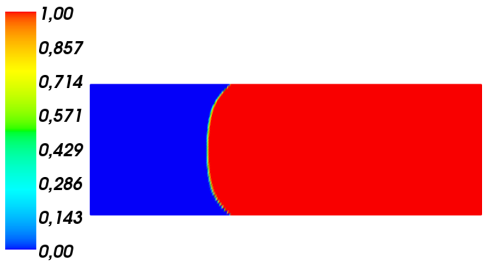

Figure 6. Crack surfaces of 3D DCB test.

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