DYNAMIC RESPONSE OF SYNTACTIC FOAM CORE SANDWICH USING A MULTIPLE SCALES BASED ASYMPTOTIC METHOD

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Abstract

The free vibration response of a simply supported sandwich composite plate with syntactic foam core is analyzed using a multiple scales asymptotic method. No apriori assumption is made on the through-thickness variation of the field variables. The formulation transforms the basic 3-D equations of elastodynamics into recursive sets of equations for different orders of approximation. The lowest order approximation corresponds to the classical plate theory and higher order approximations include effects of rotary inertia and shear deformation. The influence of the size, thickness and volume fraction of inclusions on the natural frequency of the plate has been studied. The variation of the interlaminar stresses through the thickness is also presented.

1. Introduction

Sandwich composites with different material combinations for face sheet and core (Figure 1) have become popular in recent years due to their extremely high stiffness-to-weight and strength-to-weight ratios. The use of syntactic foams as core materials is relatively new and has several advantages over other materials [1, 2]. Syntactic foams are particulate composites obtained by embedding hollow inclusions, with diameters ranging from 1-500μm and thickness 0.5-5μm, in a polymeric resin (Figure 2). They are closed cell structured foams with an excellent combination of compressive strength, low density, excellent damping property, low radar detectability, low moisture absorption coefficient and damage tolerance compared to other core materials, which make them very attractive for structural applications. On a macro scale, syntactic foams behave nearly as isotropic material.

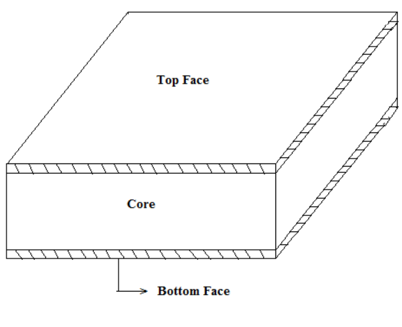


Figure 1. Schematic of a sandwich composite panel with laminated face sheets and foam core

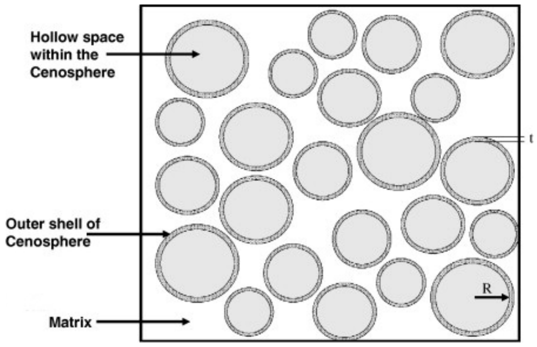


Figure 2. Schematic of a syntactic foam with differently sized inclusions embedded in a matrix [2]

In this work, a multiple-scales based asymptotic expansion formulation is developed to model the linear elastodynamic response of sandwich plates. The formulation has been applied to investigate the free vibration response of a sandwich composite with cross-ply laminate face sheets and syntactic foam core. The method can accurately model the effect of rotary inertia and transverse shear deformation on the mechanical response, which can be significant for sandwich plates.

2. Formulation of the asymptotic method

The method consists of the expansion of all field variables and introduced length scales in terms of a plate perturbation parameter characteristic to the problem [3]. The field variables are first appropriately non-dimensionalized before expanding in powers of a small plate parameter. The scaling of the coordinate and the field variables is done to ensure that the non-dimensionalized terms are of the same order so that the effect of each parameter can be examined easily. Importantly, for dynamic problems, multiple time-scales are introduced in the expansion to eliminate the secular terms and obtain uniformly valid expansions regardless of time span [4]. The formulation decomposes the basic three-dimensional elastodynamic equations into recursive sets of equations of various orders.

These equations can be integrated successively through the thickness to determine the three-dimensional solutions for the sandwich plate. The leading order equations yield the simplest plate equations corresponding to the classical laminated plate theory (CLT). The higher order solutions from the asymptotic theory incorporate various effects such as transverse shear deformation, transverse normal strain/stress and a nonlinear through-thickness variation of the displacements and yield corrections to the lower order solutions. Modifications to the leading-order approximation are obtained systematically by eliminating the secular terms by means of multiple scales and by considering the solvability conditions of the higher-order equations. There is no need to treat the system layer by layer and the interfacial continuity conditions are automatically incorporated in the procedure. The asymptotic solution converges rapidly and gives accurate results.

2.1 Modelling the elastodynamics of a sandwich composite plate with isotropic core

Consider a sandwich plate having an isotropic core of thickness t_c, specially orthotropic laminated face sheets of total thickness t_f. The in-plane dimensions are L x L. The coordinate plane $x_3 = 0$ is the plate mid-surface with the positive x_3 axis directed downwards as shown in Figure 3. The transverse load $q(x_1, x_2, t)$ is prescribed on $x_3 = -h$ and the lower surface is traction free. Appropriate edge boundary conditions are prescribed.

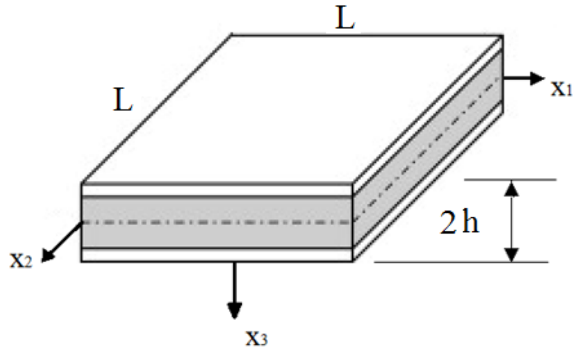


Figure 3. Schematic of the sandwich composite with the chosen coordinate system and dimensions

The governing equations for the linear elastodynamic behavior of the plate are:

$$\sigma_{ij,j} = \rho u_{,tt}, \; \epsilon_{kl} = \frac{1}{2}\left(u_{k,l} + u_{l,k}\right), \; \sigma_{ij} = c_{ijkl}\epsilon_{kl}$$
$$\sigma_i = c_{ij}\epsilon_k, \; i, j \text{ in contracted notation} \tag{1}$$

The stresses given in Equation (1) can combined and rearranged to express the derivative of each field variable with x_3 in terms of the other field variables and their derivatives with respect to x_1 and x_2. These equations are then non-dimensionalized in terms of a plate thickness parameter ε = h/L and a reference elastic constant Q. Suitably chosen quantities for each field variable are given in Equation (2).

$$x = \frac{x_1}{L}, \quad y = \frac{x_2}{L}, \quad z = \frac{x_3}{h}; \; u = \frac{u_1}{h}, \quad v = \frac{u_2}{h}, \quad w = \frac{u_3}{h}$$
$$\bar{\sigma}_{xx} = \frac{\sigma_{11}}{\varepsilon Q}, \quad \bar{\sigma}_{xy} = \frac{\sigma_{12}}{\varepsilon Q}, \quad \bar{\sigma}_{yy} = \frac{\sigma_{22}}{\varepsilon Q}\;; \; \bar{\sigma}_{xz} = \frac{\sigma_{13}}{\varepsilon^2 Q}, \quad \bar{\sigma}_{yz} = \frac{\sigma_{23}}{\varepsilon^2 Q}, \quad \bar{\sigma}_{zz} = \frac{\sigma_{33}}{\varepsilon^3 Q}; \tag{2}$$

ε = h/L is a plate thickness parameter and L a typical in-plane length dimension. Here $Q = c_{33}$. The field variables are expanded in terms of the plate thickness parameter as $f(x, y, z, \tau_k, \epsilon) = \sum_{k=0}^{\infty} f_{(k)}(x, y, z, t)\epsilon^k$. These expansions are substituted in the non-dimensionalized equations and suitable boundary conditions are derived. Comparing coefficients of the same power of the parameters, different orders of equations are obtained. The associated dimensionless traction and displacement boundary conditions are given by

$$\boldsymbol{\sigma}_{s(0)} = 0 \text{ on z } = \pm 1; \; \sigma_{z(0)} = \begin{cases} -\tilde{q} & \text{on } z = -1 \\ 0 & \text{on } z = 1 \end{cases}$$
$$\begin{bmatrix} n_1 & 0 & n_2 \\ 0 & n_1 & n_2 \end{bmatrix}\left[\mathbf{L}_3\right]\left\{\mathbf{u}_{(0)}\right\} = \begin{Bmatrix} \tilde{p}_1 \\ \tilde{p}_2 \end{Bmatrix}, \; \begin{bmatrix} n_1 & n_2 \end{bmatrix}\boldsymbol{\sigma}_{s(0)} = \tilde{p}_3 \text{ on edges } \Gamma_\sigma$$
$$\left\{\mathbf{u}_{(0)}\right\} = \{u_{(0)}, v_{(0)}\}^T \; ; u_{(0)} = u^0, \quad v_{(0)} = v^0, \quad w_{(0)} = w^{(0)}, \qquad \text{on edges } \Gamma_u \tag{3}$$
$$\tilde{p}_1 = \frac{p_1}{Q\varepsilon}, \quad \tilde{p}_2 = \frac{p_2}{Q\varepsilon}, \quad \tilde{p}_3 = \frac{p_3}{Q\varepsilon^2}, \quad \tilde{q} = \frac{q}{Q\varepsilon^3}, \; u^0 = \frac{u_1^0}{h}, \quad v^0 = \frac{u_2^0}{h}, \quad w^0 = \frac{u_3^0}{L}$$

$$
\begin{aligned}
&\boldsymbol{\sigma}_s = \left\{\sigma_{xz}, \sigma_{yz}\right\}^T ;\ \mathbf{L}_3 = \begin{bmatrix} l_{14} & l_{15} & l_{16} \\ l_{24} & l_{25} & l_{26} \end{bmatrix}^T \\
&l_{14} = \bar{Q}_{11}\frac{\partial}{\partial x_1},\ l_{15} = \bar{Q}_{12}\frac{\partial}{\partial x_1},\ l_{16} = \bar{Q}_{66}\frac{\partial}{\partial x_2} \\
&l_{24} = \bar{Q}_{12}\frac{\partial}{\partial x_2},\ l_{25} = \bar{Q}_{22}\frac{\partial}{\partial x_1},\ l_{26} = \bar{Q}_{66}\frac{\partial}{\partial x_1};\ \bar{Q}_{ij} = \frac{Q_{ij}}{Q} \\
&Q_{ij} = c_{ij} - c_{i3}c_{j3}/c_{33},\ (i,j = 1,2,6)
\end{aligned}
\tag{4}
$$

The equations corresponding to ϵ^0 are known as first order solutions. In a similar way the equations can be derived for higher order approximations corresponding to ε^{2k}. Here, the multiple dimensionless scales approach is used to avoid secular terms and ensure bounded solutions with time. The time scales are non-dimensionalized as

$$\tau_k = \frac{\varepsilon^{2k}}{L}\sqrt{\frac{c_{33}}{\rho_0}}t\ (\ k = 0,1,2,\ldots)\ \ ,\ \ \rho_0 \text{ is reference mass density} \tag{5}$$

The resulting sets of equations are integrated successively with respect to z and applying the boundary conditions on the free surface gives the governing differential equation for each order. As an illustration, the first order equations including the rotary inertia are:

$$
\begin{aligned}
&A_{11}u_{0,xx} + A_{66}u_{0,yy} + (A_{12}+A_{66})v_{0,xy} - B_{11}w_{0,xxx} - (B_{12}+2B_{66})w_{0,xyy} = I_{10}\frac{\partial^2 u_0}{\partial\tau_0^2} - I_{11}\frac{\partial^2 w_{0,x}}{\partial\tau_0^2} \\
&A_{22}v_{0,yy} + A_{66}v_{0,xx} + (A_{12}+A_{66})u_{0,xy} - B_{22}w_{0,yyy} - (B_{12}+2B_{66})w_{0,xxy} = I_{10}\frac{\partial^2 v_0}{\partial\tau_0^2} - I_{11}\frac{\partial^2 w_{0,y}}{\partial\tau_0^2} \\
&D_{11}w_{0,xxxx} + 2(D_{12}+2D_{66})w_{0,xxyy} + D_{22}w_{0,yyyy} - B_{11}u_{0,xxx} - (B_{12}+2B_{66})u_{0,xyy} - B_{22}v_{0,yyy} \\
&-(B_{12}+2B_{66})v_{0,xxy} = \tilde{q} - I_{20}\frac{\partial^2 w_0}{\partial\tau_0^2} + I_{12}\frac{\partial^2}{\partial\tau_0^2}(w_{0,xx}+w_{0,yy}) - I_{11}\frac{\partial^2}{\partial\tau_0^2}(u_{0,x}+v_{0,y}) \\
&I_{10} = \int_{-1}^{1}\rho_1 dz,\quad I_{11} = \int_{-1}^{1}\rho_1 z dz,\quad A_{ij} = \int_{-1}^{1}\bar{Q}_{ij}dz,\quad B_{ij} = \int_{-1}^{1}\bar{Q}_{ij}zdz,\ I_{12} = \int_{-1}^{1}\rho_1 z^2 dz,\ \rho_1 = \frac{\rho}{\rho_0}
\end{aligned}
\tag{6}
$$

3. Geometry and material properties

The formulation outlined in the previous section was applied for the analysis of the dynamic behaviour of a symmetric cross-ply sandwich plate simply supported on all four edges with the arrangement given by [0°/90°/core/90°/0°]. The face sheets are graphite-epoxy laminates with elastic properties E_1 = 131 GPa, E_2 = E_3 =10.34 GPa, G_{12} = G_{13} = 6.895 GPa, G_{23} = 6.205 GPa, ν_{12} = ν_{13} = 0.22, ν_{23} = 0.49, ρ_f = 1627 kg/m^3. The syntactic foam core is made of hollow glass inclusions and epoxy resin with E_m = 3.3 GPa, ν_m = 0.37, ρ_m = 1160 kg/m^3, E_g = 85 GPa, ν_g = 0.3, ρ_g = 2600 kg/m^3. The inclusion radius R_p = 30μm, inclusion thickness t_p = 1μm, filler volume percentage V_f = 60%. A parametric study was conducted by varying the the thickness ratios of the face sheets to the core and the dimensions and volume fraction of the inclusion. All the inclusions are of the same size and shape, randomly distributed in the matrix and free of voids.

4. Solution Methodology

To accurately estimate the effective elastic properties of the foam core with the inclusions and matrix, a multi-phase self-consistent scheme proposed by Bardella and Genna [1] is used. The foam core can be assumed to be macroscopically homogeneous and isotropic. The effective shear modulus and the bulk modulus are estimated in terms of the constituent properties and subsequently the Young's modulus and Poisson's ratio are calculated. For the effective density of foam, it is sufficient to use the simple rule of mixtures to get an accurate estimate. The equations given in the earlier section get further simplified for symmetric cross-ply laminates. For free vibration analysis of a square plate simply supported on all edges, the solution for the first order approximation can be taken to be of the form:

$$\begin{aligned} u_0 &= U_0 cos\alpha x sin\beta y cos(\omega_{mn}\tau_0 - \delta_{mn}) \\ v_0 &= V_0 sin\alpha x cos\beta y cos(\omega_{mn}\tau_0 - \delta_{mn}) \\ w_0 &= W_0 sin\alpha x sin\beta y cos(\omega_{mn}\tau_0 - \delta_{mn}) \end{aligned} \tag{7}$$

Here, $\alpha = m\pi, \beta = n\pi$ and ω_{mn} are the circular frequencies of the motion. U_0, V_0, W_0 are the amplitudes. The phase angles δ_{mn} are independent of τ_0 and are as yet undetermined functions of time scales τ_1, τ_2, etc. Substitution into the governing equations gives the equations for the flexural and in-plane displacements. For the chosen configuration, the flexural displacement is uncoupled with the in-plane displacement giving the required eigenvalue problem. The frequency for the flexural motion for the first order approximation is given by

$$\omega_{mn} = \frac{\left[D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4 \right]^{\frac{1}{2}}}{[I_{20} + I_{12}(\alpha^2 + \beta^2)]^{\frac{1}{2}}} \tag{8}$$

The first order solutions for all the displacements and stresses can then be determined from Equation (6). The process can be repeated for higher order approximations to include thickness effects in the solution to correct the frequency and then the other field variables. As the thickness of the plate increases, higher order approximations are needed to get increased accuracy but in general one need not go beyond the 3rd order solution.

5. Results and discussion

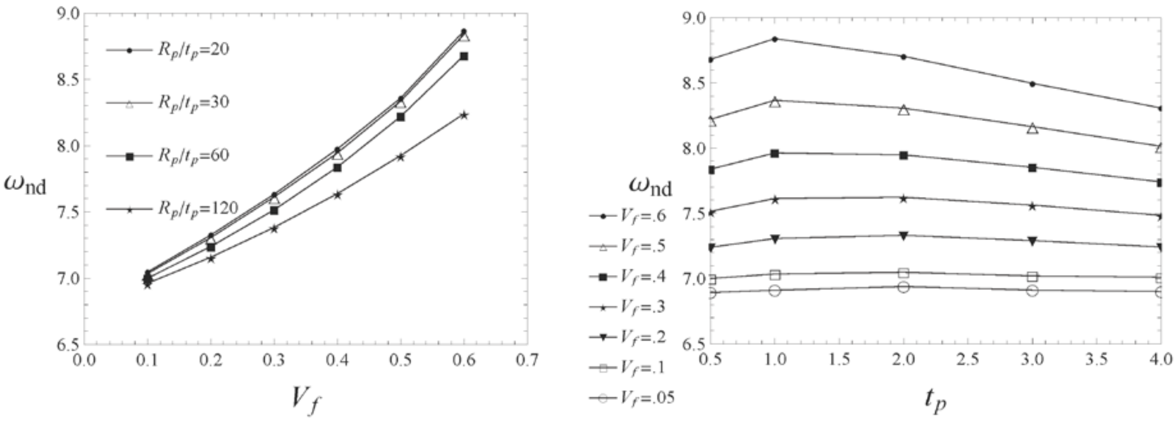
The fundamental frequency of the sandwich composite plate was obtained for a chosen volume fraction and inclusion radius using different orders of approximation. A parametric study was done to investigate the effect of different parameters including plate thickness and core properties like radius, thickness and volume fraction of the inclusion on the dynamic response. The values of the non-dimensionalized fundamental frequency for different plate thickness ratios are given in Table 1 for an inclusion volume fraction $\nu_f = 0.6$, inclusion radius $R_p = 30\mu m$ and inclusion thickness $t_p = 1\mu m$ and thickness of core to face (t_c/t_f) = 10. A detailed finite element modeling of the problem was also performed and the results were compared with the analytical results.

L/2h	1st order	2nd order	3rd order	4th order	5th order	FEA
100	9.683	9.674	9.674	9.674	9.674	9.868
50	9.680	9.644	9.644	9.644	9.644	9.804
25	9.668	9.527	9.530	9.530	9.530	9.625
10	9.587	8.742	8.853	8.840	8.840	8.959
5	9.315	7.397	7.641	7.858	7.914	7.945

Table 1. Non-dimensional fundamental frequency $\boldsymbol{\omega_{nd} = \omega \frac{L^2}{2h}\left(\frac{\rho_f}{E_f}\right)^{1/2}}$ for different L/2h

The results show the convergence natural frequency for the higher order approximations. For thin plates, the higher order corrections are small and the convergence occurs with the second order approximation itself. For thicker plates, the corrections from the higher order approximations become significant and these include the effect of transverse shear deformation and rotary inertia terms. For moderately thick plates, convergence is seen by the fourth or fifth order whereas for very thick plates still higher order approximations are necessary to achieve convergence. For the finite element model for thin plates upto a/2h=25 25, shell elements are used for meshing the entire plate but for thick plates i.e. for a/2h < 25, shell element are used for the face sheets and brick elements are used for meshing the core.

The effect of the core properties on the natural frequency of the sandwich plate was examined. For a thick plate (a/2h = 10) with $t_c/t_f = 10$, the variation of the natural frequency is plotted for different volume fractions of the inclusion and inclusion thickness as shown in Figure 3(a) and Figure 3(b). The natural frequency increases with an increase in filler volume fraction because of increase in effective stiffness of core and hence that of the plate. Further, configurations with a higher R_p/t_p have a lower frequency for the same volume fraction because of the relative reduction in the particulate stiffness. As R_p/t_p decreases, the effective stiffness and density of particle (core and plate) increases due to lower volume of void available. Thus the natural frequency increases as the increment in stiffness predominate. For volume fraction and inclusion radius the density and elastic moduli increases with an increase in inclusion thickness, the frequency initially increases due to increasing stiffness but then decreases due to the density effect. For lower volume fraction the inclusion thickness has negligible effect on natural frequency as the matrix control the stiffness of the foam.



(a) (b)

Figure 3. (a) Effect of volume fraction on natural frequency of sandwich plate (b) Effect of inclusion thickness on natural frequency of sandwich plate (L/2h = 10, t_c/t_f=10)

The through-thickness variation of the transverse displacement is captured accurately by this method. The nonlinear variation of the transverse displacement for a thick plate (L/2h = 10) can be seen in Figure 4. The inter-laminar shear and normal stresses are shown for a thick plate. For thin plates the first order approximation corresponding to the CLT method gives good results but for a thick plate (L/2h=10), as seen in Figure 5(a) and Figure 5(b), there is significant difference between the results from different order approximations and a higher order approximation is needed to get an accurate distribution through the thickness. The interlaminar normal stress shows a faster convergence relative to the interlaminar shear stress.

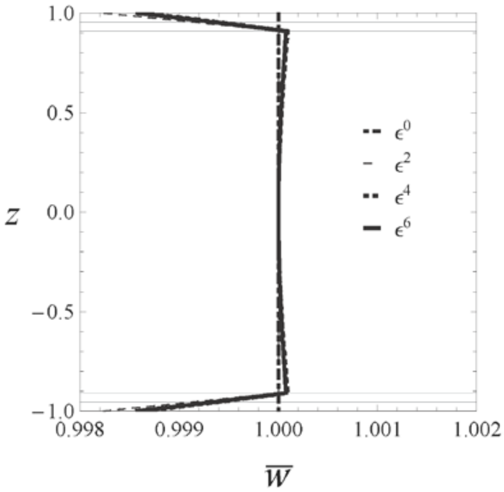
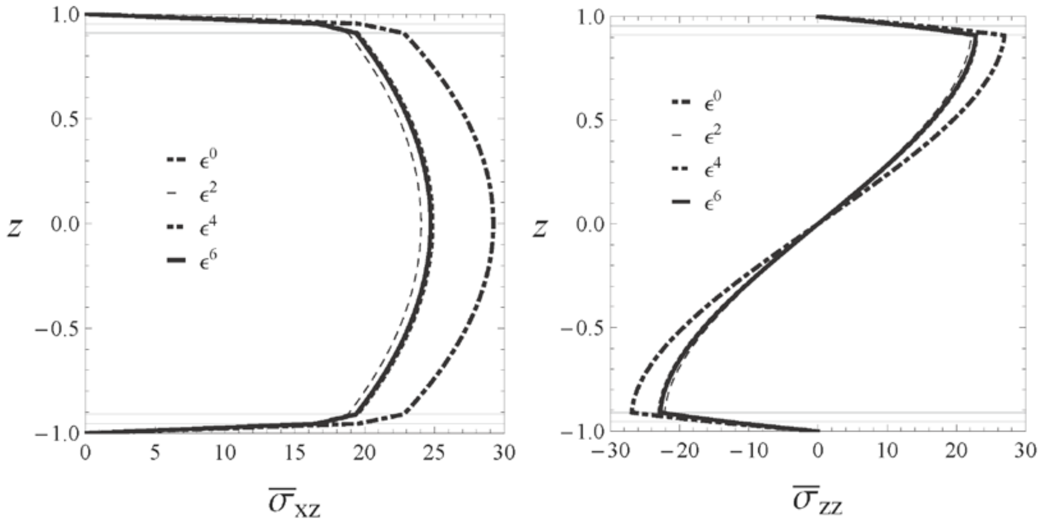


Figure 4. Variation of non-dimensional transverse displacement through the plate thickness for a thick plate (L/2h =10, $t_c/t_f = 10$)



(a) (b)

Figure 5. (a) Non-dimensional interlaminar shear stress variation through the plate thickness and (b) non-dimensional interlaminar normal stress for a thick plate (L/2h =10, $t_c/t_f = 10$)

6. Summary

The multiple scales based asymptotic expansions method was formulated as an accurate and effective analytical method for dynamic analysis of sandwich plates with syntactic foam core and laminated face sheets. The method gives rapidly converging bounded solutions by eliminating secular terms. The effective properties of the foam core were estimated using the self-consistent method of homogenization. The effect of plate thickness and core properties on the fundamental frequency and interlaminar shear and normal stresses were studied using higher order approximations.

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