A MULTISCALE METHOD FOR THE ANALYSIS OF FIBER REINFORCED COMPOSITE STRUCTURES USING THE CW APPROACH

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A hierarchical multiscale approach for the analysis of fiber-reinforced composite structures is presented in this work. Two scales are taken into account by carrying out uncoupled analysis, and subsequently, results are bridged by introducing a proper information-passing scheme. The macro-scale is modeled through 3D Finite Elements within the ABAQUS environment while the CW approach is used to model the microscale level. Results for a notched laminate structure are presented where the finer scale has been used to provide the homogenized composite stiffness for the macroscale.

1. Introduction

This work proposes a novel multiscale approach for the analysis of fiber-reinforced composite structures. Despite the great benefits that composite materials provide, there are issues to be addressed to enhance the design of composite structures. Their behavior is affected by phenomena that happen at different length scales, the development of analysis capabilities involving many length scales is necessary in order to properly understand these phenomena. Nevertheless, the computational time and cost required to perform multiscale analyses can still be prohibitive for extensive applications. Reducing the computational cost is then one of the most important issues to overcome in the context of multiscale modeling. Different multiscale methods have been developed in recent years, and a comprehensive overview about the available techniques has been provided by Lu and Kaxiras [1]. An extensive review of micromechanics theories has been recently given in Aboudi et Al. [2] An exhaustive description of a new bottom-up multiscale modeling strategy for fiber-reinforced structural

composites can be found in [3], Llorca et al. Referring to fiber reinforced composite laminates, the micro- (fiber/matrix), meso- (lamina) and macro-scale (whole laminate) are usually three scales at which analysis is carried out. Molecular dynamics (MD) analysis can also be performed to investigate the nanoscale level [4]. Many computational models are developed in the framework of finite element approximations. 2D and 3D finite elements can be employed to discretize single components (fibers and matrices) or to directly model layers. For this purpose, 1D higher order elements were used by Carrera at al. [5,6] in the Component-Wise approach (CW). Within the CW, different scale components (fiber, matrix, laminae and laminates) can be simultaneously considered in the same structural model, the stress and strain fields are provided with accuracy comparable to solid element modeling, and very low computational cost. Predicting reliable stress fields is of primary importance to evaluate the structural integrity of a composite structure. Focusing on two scale analysis, the Method of Cells [7,8] is being extensively used to homogenize composite stiffness first and then, investigate failure mechanisms. Recently, the Integrated Finite Element Method (IFEM) has been used to capture progressive failure within the constituents.
In this paper, the CW approach is used to model cells at the microscale level while the macroscale is modeled using the commercial software, Abaqus. In section 2, a brief description of the multiscale technique herein adopted and of the CW approach within the CUF are furnished. Results and discussion are provided for a notched laminate in section 3 where the main conclusions are then outlined.

2. The multiscale approach and boundary conditions

The multiscale approach proposed in this paper focuses on a two length scales. The macroscale, characterized by the global dimension of the model, is described as a homogenized continuum through a model of a mathematically homogenized material. The microscale, characterized by the fiber diameter dimensions, and which includes the heterogeneity of materials is directly modeled as constituents, with fiber and matrix phases. This approach is based on the micromechanics assumption that in the whole structure a repeating cell containing fiber and matrix phases can be identified. The present method is developed in the framework of the Finite Element Analysis (FEA) where, for the first scale, solid elements are used to discretize the structure while for the latter (micro-scale), 1D CUF elements are employed. The analyses are decoupled at the two different scales, information obtained by results at the fine scale are passed back to the macroscale assuming that each point of the structure can be represented by a unit cell. The scheme of this approach is shown in Figure 1 where the quantity passed back to the coarse scale is the average stress computed on the Representative Unit Cell (RUC), and where $<\cdot dV>$ indicates the volume integral $\int_V dV$ of the stress components on the RUC. The procedure is applied for each integration point of the macroscale. Since the periodicity of the microscale is assumed, the strain at each integration point is applied to the RUC by means of the Periodic Boundary Conditions (PBC).
The macroscale is modeled in in ABAQUS, the microscale is introduced through a UMAT subroutine where the Jacobian matrix of the constitutive model, $[\partial\Delta\sigma/\partial\Delta\varepsilon]$, has to be provided. The tangent stiffness, which relates, $\partial\Delta\sigma$ (the stress increments) to $\partial\Delta\varepsilon$ (the strain increments) at each step, is computed in the microscale analysis. The microscale is modeled by means of the Component-wise approach (CW), where "Component-wise" means that each typical component of a composite structures can be separately modeled by means of a unique formulation. In previous work, this approach has been used in a concurrent multiscale scheme where homogenized laminates or laminae were directly interfaced with fiber and matrix phases. Each component can be modeled with its own mechanical properties using 1D elements.

This approach has been derived through the Carrera Unified Formulation (CUF). CUF is a hierarchical formulation which considers the order of the theory as an input of the analysis. The use of higher order elements permits us to compute the stress and strain fiels with a solid-like accuracy and to reduce the computational cost of the analysis. Within the CUF, the displacement field can be approximated through the expansion of generic functions, $F\tau$, as shown in Equation (1), where $F\tau$ vary over the element cross-section. u_τ is the displacement vector and M stands for the number of terms of the expansion. According to the Einstein notation, the repeated subscript, τ, indicates summation. The choice of $F\tau$ determines the adopted class of 1D CUF model. For developing this multiscale approaches Lagrange Expansion class (LE) has been exploited to build 1D refined models that have displacement variables only. In this work, a single fiber/matrix RUC is modelled by using L9 polynomials. The FE approach is adopted to discretize the structure along the y-axis, this process was conducted via a classical finite element methodology based on the Principle of Virtual Displacements. The shape functions, N_i, and the nodal displacement vector, q_i, are used and the displacement vector becomes as shown in Equation (2) and Equation (3) where K is the number of the nodes on the element. Elements with 3 nodes, here denoted as B3, are used in this paper, that is, a quadratic approximation along the y axis is adopted. The stiffness matrix is obtained via the Principle of Virtual Displacements is written in terms of Fundamental Nuclei. No assumptions on the approximation order have been done to obtain the fundamental nucleus. It is therefore possible to obtain refined 1D models without changing the formal expression of the nucleus components. This is the key-point of CUF which permits, with only nine FORTRAN statements, to implement any-order one dimensional theories. The displacement, as well as the strain and stress, fields can be computed with solid-like accuracy with a significant reduction of DOFs involved. Since CUF resolves for the 3D state of stress the Jacobian matrix of the constitutive model is a 6×6 matrix. Once the local strains are evaluated at the microscale, the RUC constitutive laws yield the local stresses, and the global stresses and homogenized stiffness of the RUC can be obtained. PBC Equations are shown in (4) respectively for sides 1, 2 and 3. Sides refer to the cell faces as shown in Figure (2) where l_1, l_2, l_3 are the characteristic dimensions of the cell respectively in x-, y- and z-directions.

2. Results and conclusions

A one layer notched laminate subjected to uniaxial tension is herein taken into account. The laminate geometry is shown in Figure 3, where the related dimensions are reported in Table 1. A solid mesh has been provided for the macroscale as depicted in Figure 4. A single fiber/matrix cell has been used to represent the microscale. The cross-section has been modeled using 20 L9 elements, 1 B3 has been used along the y-axis as in Figure 5.The cell is square with $l_1 = l_3 = 0.1$ mm, $l_2/l_1 = 10$. The fiber ν_f and matrix ν_m volume fractions are ν_f = 50.3%, ν_m = 49.7%. Material properties arerespectively E_f = 250634 MPa, ν_f = 0.2456 for the fibers and E_m = 3252 MPa, ν_m =0.355 for the matrix portions. The laminate is clamped at one end while a displacement u = 0.01 mm is applied to the opposite side of the structure. At each integration point of the solid elements at the macroscale, the 1D CUF RUC has been used to solve the boundary value problem and to compute the average value of the stress, which is passed back to the macroscale. Using the single fiber/matrix cell, the values for the macroscale elastic properties are shown in Table 2 .

Results in terms of the maximum principal stress distribution on the laminate are shown in Figure 6 and compared with the homogenized case study, Figure 7. The stress distribution close to the notch has also been provided for both cases, respectively, in Figure 8, 9. The present approach is of particular relevance when failure analysis wants to be performed.

Using this technique the failure can be evaluated directly on the components introducing appropriate criteria. If a criterion is satisfied, the cell stiffness can be reduced. The reduced stiffness can be used to update the global stress on the cell and therefore recording the loss in the stiffness of the composite at the macroscale. The use of a low-cost and accurate method to compute the stress and strain fields at the microscale is of prime importance for an extended use of multiscale approaches in the structural analysis of composite structures. In this framework the 1D CUF is an extremely powerful tool to reach a compromise between accuracy of results and computational cost. Furthermore, the orthotropic behavior of fibers would be easily introduced using the 1D CUF elements.

2. Tables and figures

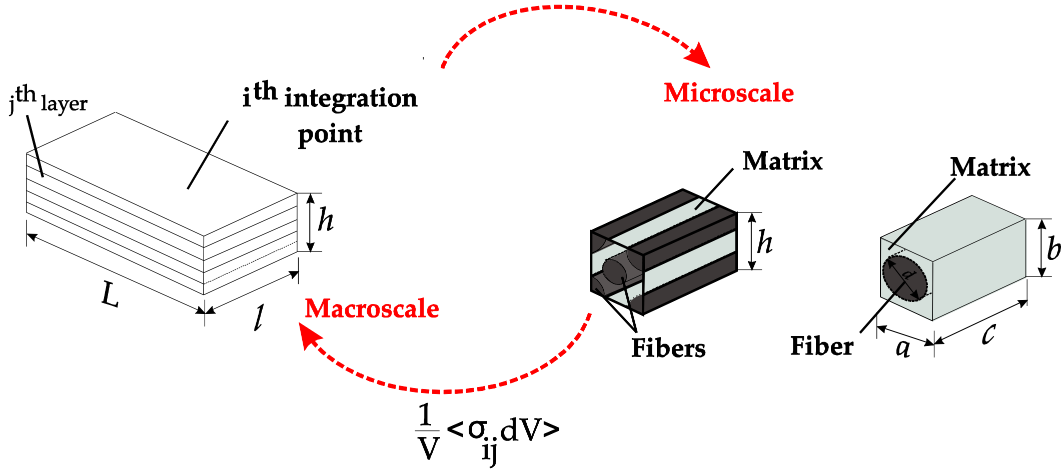

Figure 1. The information-passing scheme for the proposed multiscale approach.

$$u = F_\tau u_\tau , \tau = 1,2,....,M \qquad \text{Eq. (1)}$$

$$\mathbf{u}(x,y,z) = N_i(y)F_\tau (x,z,) \mathbf{q}_{\tau i}, i=1,2,\dots K \qquad \text{Eq. (2)}$$

$$q_{\tau i} = \left\{q_{u_{x_{\tau i}}}, q_{u_{y_{\tau i}}}, q_{u_{z_{\tau i}}}\right\}^T \qquad \text{Eq. (3)}$$

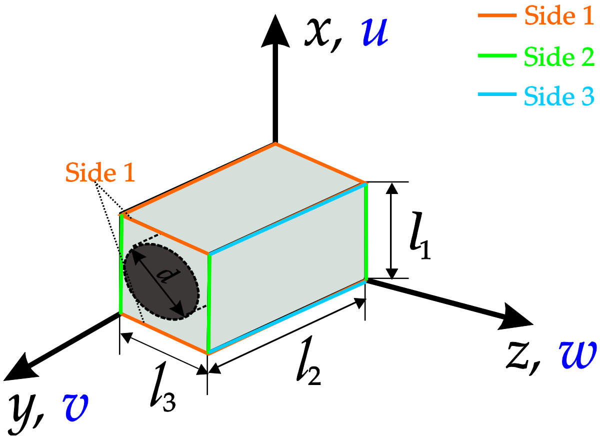

Figure 2. Periodic Boundary Conditions for the analysis of the microscale.

$$\text{Side 1:} \begin{cases} \boldsymbol{u(l_1, y, z) - u(0, y, z) = \varepsilon_{11} l_1} \\ \boldsymbol{v(l_1, y, z) - v(0, y, z) = 2\,\varepsilon_{12} l_1} \\ \boldsymbol{w(l_1, y, z) - w(0, y, z) = 2\,\varepsilon_{13} l_1} \end{cases}$$

$$\text{Side 2:} \begin{cases} \boldsymbol{u(x, l_2, z) - u(x, 0, z) = 2\varepsilon_{21} l_2} \\ \boldsymbol{v(x, l_2, z) - v(x, 0, z) = \varepsilon_{22} l_2} \\ \boldsymbol{w(l_1, l_2, z) - w(x, 0, z) = 2\,\varepsilon_{23} l_2} \end{cases} \qquad \text{Eq. (4)}$$

$$\text{Side 3:} \begin{cases} \boldsymbol{u(x, y, l_3) - u(x, y, 0) = 2\varepsilon_{31} l_3} \\ \boldsymbol{v(x, y, l_3) - v(x, y, 0) = \varepsilon_{32} l_3} \\ \boldsymbol{w(x, y, l_3) - w(x, y, 0) = 2\,\varepsilon_{33} l_3} \end{cases}$$

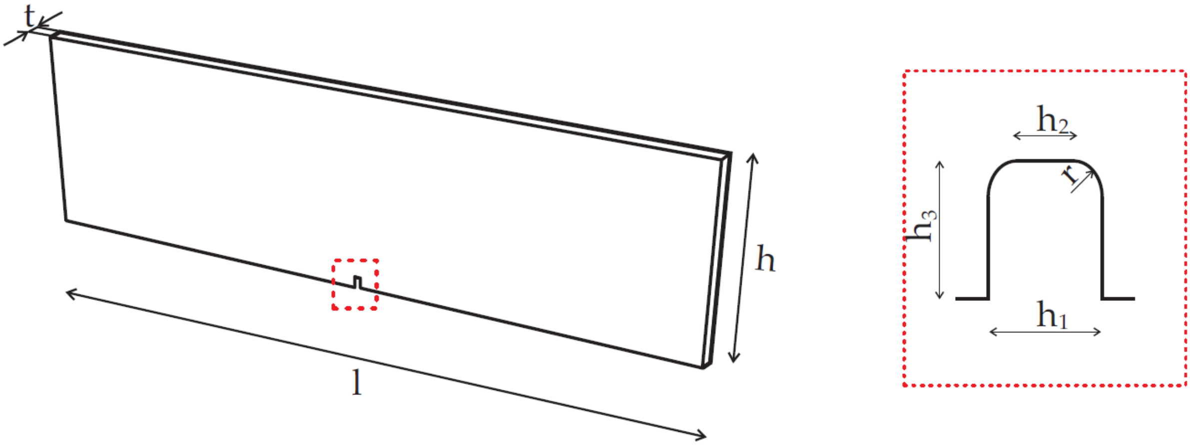

Figure 3. Geometry of the macroscale model.

Dimensions	[mm]
L	21.65
h	6.35
t	0.13
h_1	0.22
h_2	0.12
h_3	0.38
h_4	0.05

Table 1. Notched laminate model dimensions.

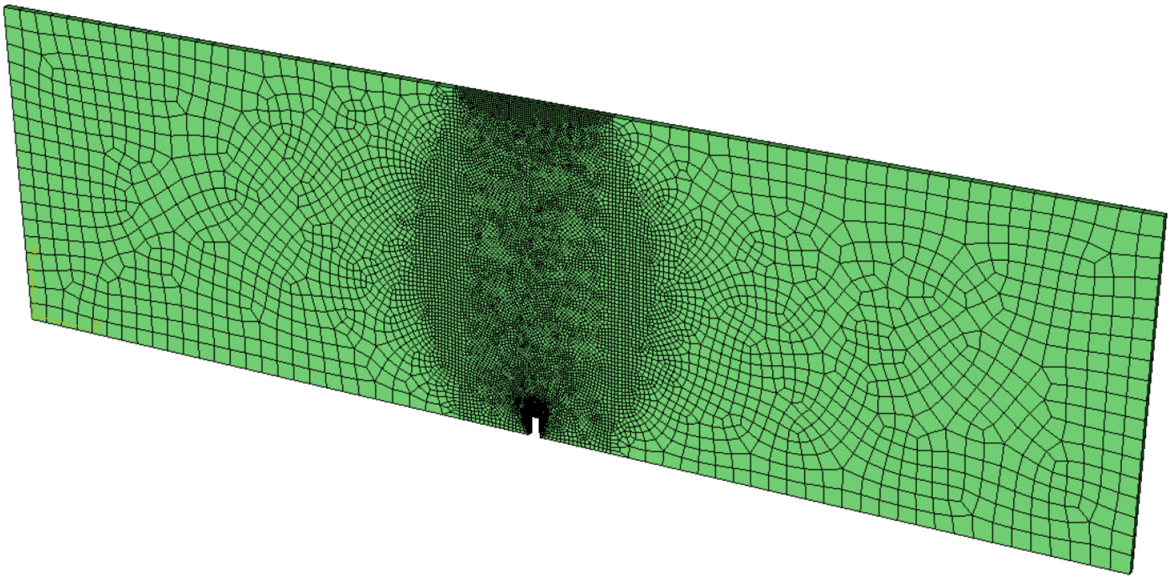

Figure 4. Solid model for the analysis of the macroscale.

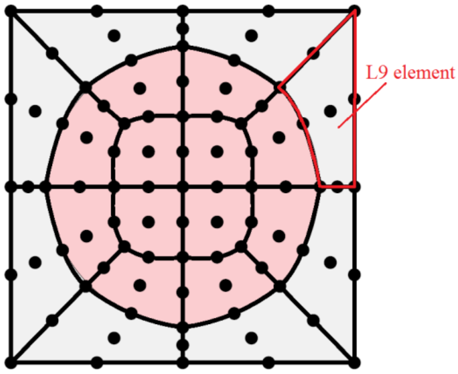

Figure 5. Element distribution on the RUC cross-section: 20 nine-point (L9) elements.

Material Properties	[MPa]
E11	123935
E22	11603
E33	11626
ν12= ν13	0.294
ν32	0.343
G12	6904
G23	5075
G13	6916

Table 2. Composite material properties computed by means of the present multiscale approach.

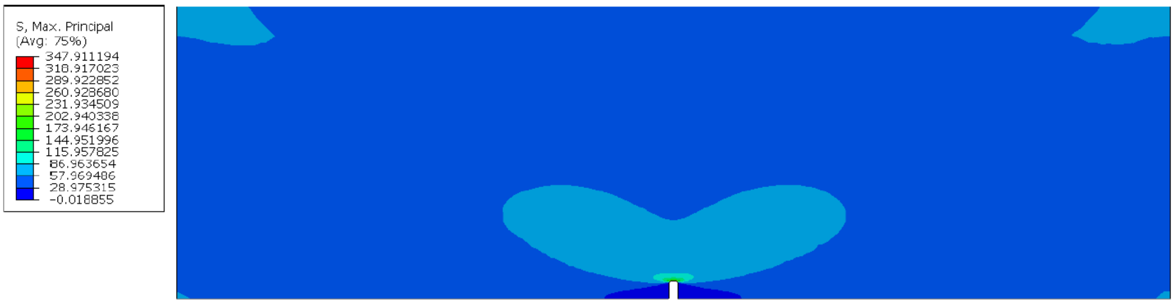

Figure 6. Stress distribution on the notched laminate computed through the present multiscale approach.

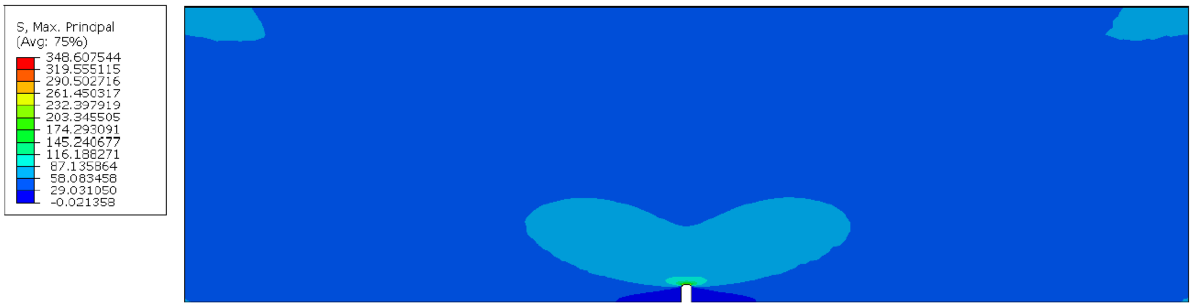

Figure 7. Stress distribution on the notched laminate computed through the homogenized solid model.

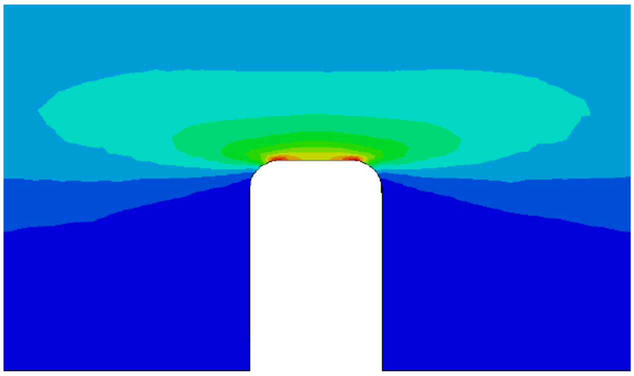

Figure 8. Stress distribution close to the notch computed through the present multiscale approach.

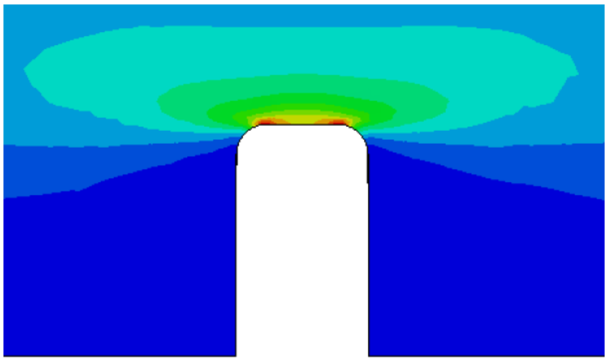

Figure 9. Stress distribution close to the notch computed through the homogenized solid model.

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