DISCRETE DAMAGE SIMULATION AND MEASUREMENT IN COMPOSITE LAMINATES UNDER FATIGUE LOADING

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Keywords: composite, fatigue, delamination, X-FEM

Abstract
Fatigue properties of CFRP were studied intensively in the past and satisfactory described for most isolated damage modes. What represents a challenge is the structural response to fatigue loading, which is characterized by evolution and interaction of various modes of damage. Recently proposed eXtended Finite Element Methodology (X-FEM) methods for mesh independent modeling of matrix cracking combined with cohesive zone based simulation of delamination have shown promising results in laminates with and without stress concentrations under static loading. In the present paper these methods are extended for simulation of fatigue loading and compared to experiment.

1. Introduction

Fatigue properties of CFRP were studied intensively in the past and satisfactory described for most isolated damage modes. The transverse strength properties have been shown to satisfy exponential S-N behavior and the fracture related damage modes such as delamination have been shown to follow the Paris law. The delamination onset and propagation investigation in composite laminates has been a critical research topic for several decades and is the subject of many reviews, e.g. [1-3]. Depending upon the layup and loading profile, the delamination propagation can be precipitated by matrix crack formation, which can drastically affect its propagation. Several scenarios directly influencing the damage tolerance assessment are possible: matrix cracking can temporarily arrest the delamination; it can divert the delamination to a different interface; it may cause an avalanche of multiple delaminations through the thickness of the part. Availability and rapid increase of computer power has enabled recent successes in the development of the discrete damage modeling (DDM) technique, which is based on the direct simulation of displacement discontinuities associated with individual instances of matrix cracking occurring inside the composite plies, and delaminations at the interfaces between the plies. These methods employ variants of eXtended Finite Element Methodology (X-FEM) [4] and its regularized implementation (Rx-FEM) [5-7] in particular. The Rx-FEM allows modeling the displacement discontinuity associated with individual matrix cracks in individual plies of a composite, without regard to mesh orientation, by inserting additional degrees of freedom in the process of the simulation. The propagation of the mesh independent crack is then performed by using the cohesive zone method. The goal of the present work is to extend these methods for simulation of fatigue
loading and verify the accuracy of the predictive techniques by using delamination migration specimens [8] under static and fatigue loading.

2. Computational Methodology

2.1 Fatigue Fracture Simulation

The DDM approach consists of mesh-independent crack (MIC) modeling of transverse cracks in each ply of the laminate, and in modeling the delamination between the plies by using a cohesive formulation at the ply interface. The matrix cracks are modeled by using the regularized formulation, termed Rx-FEM, developed and verified for static loading in [5-7]. Conceptually, the extension of the DDM framework to fatigue loading is relatively straightforward. Indeed, the kinematic aspects of the DDM framework in terms of MIC insertion and delamination modeling remain unchanged. What requires development is the constitutive modeling of MIC and delamination opening, as well as crack insertion criterion. In the present paper, we develop a ply level phenomenological framework for modeling fatigue response. Namely, we will use the S-N curves for ply level strength properties for modeling the damage initiation phase and Paris law for modeling the growth of damage. Two types of algorithms can be envisioned for modeling progressive failure in fatigue:

(i) **Cycle based algorithm (CBA)**. In this algorithm, one predefines a number of fatigue cycles on each solution step and simulates the damage which occurs during these cycles.

(ii) **Event based algorithm (EBA)**. In this algorithm, one defines an increment of damage or damage event, such as new crack insertion or delamination extension for one element and computes the number of cycles required to advance to this event.

In the present work we will be using mostly EBA, however, assumptions have to be made for the implementation of either algorithm, and depending on the type of problem, the combination of the two would be preferable. In static, we tracked one damage variable in each integration point, which was associated with the cohesive zone model. When this variable, often denoted as \(d\), becomes greater than zero, it starts affecting the structural response by softening the interface bonds and completely separating the interfaces when \(d=1\). In the static regime, the event when \(d > 0\) corresponds to stresses exceeding the static strength value \(Y\). The damage variable \(d\) will be used in the propagation stage of the fatigue cohesive law to advance the crack as well. However, an additional variable is needed during the fatigue loading in order to follow the material loading history when the stress is below static strength \(Y\). We shall denote this variable as \(d_I\) and use it in the virgin material to track the crack insertion event and in fatigue cohesive zone model to track the material state before the propagation damage variable \(d>0\). In the EBA algorithm, we will track the number of cycles on each loading step for each of the three events.

(a) \(\Delta N_c\) - insertion of a new crack

(b) \(\Delta N_I\) - onset of damage progression, which corresponds to initiation history variable reaching \(d_I=1\) at some integration point in the cohesive zone model

(c) \(\Delta N_p\) - delamination (MIC) advance for one interval, which corresponds to damage variable \(d=1\) at some integration point in the cohesive zone model

Each of these events will cause a discrete change either in the constitutive formulation or FE formulation through new MIC insertion and increase of degrees of freedom. The solution diagram is shown on Figure 1. Initially we start with total cycle count \(N_m=0\) and without any cracks and bypass to fatigue failure criterion (FFC), which provides \(\Delta N_c\) the number of cycles before crack insertion. Since \(N_m < \Delta N_c\), no cracks are inserted and we advance to a new step. However, this time we advance the cycle count by \(\Delta N_m=\Delta N_c\) since no other events were present and the total cycle count is \(N_m=\Delta N_m\). On the next step FCC signals that \(\Delta N_c=0\) and
we insert a crack and reset $\Delta N_c$ to the number of cycles to the next crack insertion. The inserted crack can now open and the fatigue cohesive law will provide us the respective number of cycles $\Delta N_i$, $\Delta N_{p}$ until the event of damage initiation and propagation. The latter will initially be infinite since $d=0$ everywhere. The number of cycles on this step will now be $\Delta N_m = \min(\Delta N_c, \Delta N_i)$. If $\Delta N_c < \Delta N_i$ then the next step inserts another crack,

otherwise we update the cohesive law as described in subsequent sections. Subsequent steps can involve propagation cycles $\Delta N_p$ and in general the cycle count increment on a given step is $\Delta N_m = \min(\Delta N_c, \Delta N_i, \Delta N_p)$. The fatigue failure criterion and fatigue cohesive law will be discussed below.

2.2 Fatigue Failure Criterion

The fatigue failure criterion is built upon 3D static failure criterion LaRC04 described in [9]. The idea of constructing a fatigue failure criterion based upon a static criterion is due to Hashin and Rotem [10]. For a given frequency $\omega$ and $R = \sigma_{\text{min}}/\sigma_{\text{max}}$ ratio the fatigue failure load $\sigma$ is defined as the failure load amplitude vs. number of cycles to failure and can be expressed as

$$\sigma = \sigma^s f(R, N, \omega)$$

where $\sigma^s$ is the static strength and $f(R, N, \omega)$ is the material fatigue function. In the present case, we only consider matrix failure modes and limit ourselves to the tension shear envelope. The idea expressed and experimentally verified in [10] consists of applying static failure criterion with degraded ply level strength properties, which are given by two S-N curves, namely normal tension $y(R, N, \omega)$ and shear $s(R, N, \omega)$, such that

$$Y(N)/Y_t = 1 - s_1 \log(N) \quad \text{and} \quad S(N)/S_\omega = 1 - s_2 \log(N)$$

where $Y(N)$ and $S(N)$ are reduced strength values as number of cycles for transverse normal and shear strength and $Y_t$ and $S_\omega$ their static values respectively; $s_1$ and $s_2$ are material parameters. According to [10] the fatigue strength can be predicted by applying static failure criterion with the respective ply level strength values provided by S-N curves (2). Two modifications are required to the methodology [10] for application in the DDM framework.
First, we need to employ a 3D failure criterion capable of predicting the transverse cracking angle relative to the normal direction to the ply interface. Second, due to constantly changing loading amplitude during progressive damage simulation, we need to generalize the approach to variable amplitude loading. To address the first requirement, we employ the LaRC04 criterion [9]. In the tension-shear quadrant, it represents a quadratic criterion similar to [10]; however, it is applied to the so-called failure which provides the maximum value of the failure index. In 2D case, this plane is perpendicular to a ply midsurface; however, in the presence of transverse shear stresses it can form an angle, which is essential for the problem at hand to assure correct matrix crack insertion direction. The second step required to complete the fatigue failure criterion development for DDM is generalization to variable amplitude loading. It is accomplished by applying the Palmgren-Miner linear damage accumulation hypothesis. For implementation purposes, we define a material point loading history parameter $d_i$ as

$$d_i^q = \sum_{k=1}^{q} \frac{\Delta N_m^{q_k}}{N_f^{k}}$$  \hspace{1cm} (3)$$

where summation is carried out over all previous load steps 1…q and $N_f^{k}$ is the limit number of cycles the specimen can survive within a given block of loading alone. The failure, according to the Palmgren-Miner hypothesis, occurs when $d_i$ attains a value between 0.7 and 2.2 depending on the material, $R$, and loading frequency. Without restricting generality, we will assume the fatigue failure corresponds to $d_i = 1$. Based on this hypothesis and Eqn. (3), we can compute the number of cycles until the failure event ($d_i = 1$), which in our context corresponds to the insertion of a MIC as

$$\Delta N_c = \min \left\{ \left[ 1 - d_i^q \right] / N_f^{q+1} \right\},$$  \hspace{1cm} (4)$$

2.3 Fatigue Cohesive Law

As seen in the previous section, the extension of static failure criterion in the tension-shear quadrant for matrix failure mode to fatigue is conceptually transparent. A significantly more complicated situation arises with cohesive zone models. Based on previous work [11-13]; two problem areas can be identified. One area is simulation of the propagation of existing delamination according to Paris law. The difficulty here is that Paris law is defined within the classical fracture mechanics framework and uses the Energy Release Rate (ERR) or stress intensity factor magnitude associated with ideal crack tip singularity. The cohesive zone model on the other hand introduces a process zone concept instead of the ideal crack tip. Two different approaches have been proposed to address this issue. One is based on explicitly bringing in the process zone length into the propagation mechanism as described in [12]. The other approach proposed in Ref. [11,12] extracts the ERR value of the classical crack tip from the process zone information and uses it to propagate a fatigue crack similar to VCCT. In the present work, we will use the second approach for the delamination propagation phase. The second problem area of cohesive zone model extension to fatigue analysis is the damage initiation phase and its transition into the propagation phase. It is this feature of the cohesive zone method, which has earned its popularity in static analysis. Figure 2 displays bi-linear static cohesive law (solid line) and LOG of the number of cycles required to achieve each event DI and/or DP by dash-dot lines. In the process of static deformation, material points move along the traction displacement jump curve in a continuous fashion from left to right assuring the transition from undamaged to damaged and eventually to the fully propagated
state under load or stored energy. In fatigue regime, the situation is completely different in both the initiation and propagation regime in that the material points do not follow continuously the cohesive traction law thorough the initiation and propagation phases, and in some situations are not even located on the static cohesive law curve at all. The latter situation will not be considered due to space limitations. The number of cycles until the DI event $\Delta N_I$ is calculated by using Eqn. (4), where we implemented the load history variable $d_I$ similar to that in the previous section. Assuming LOG type S-N relationship (2); we obtain linear relationship for LOG($\Delta N_I$) in the

$$ \Delta N_I = \frac{1}{C} \left( \frac{G_{\text{max}}}{G_c} \right)^{-m} $$

Figure 2. Schematics of a cohesive zone model and LOG cycles to Delamination Initiation (DI) event and to Delamination Propagation (DP) event.

In the propagation regime, we use Paris law

$$ \frac{da}{dN} = C \left( \frac{G_{\text{max}} (1 - R^2)}{G_c} \right)^m $$

(5)

to compute the number of cycles until DP, whereas $G_c$ is the static critical value of the ERR, $G_{\text{max}}$ is the maximum value of ERR during the cycle, and $C$ and $m$ are material fatigue constants. To calculate the number of cycles until DP we use a length value for crack extension $l_e$, normally equal to one element size, and rewrite (5) as

$$ \Delta N_p = \frac{l_e}{C \left( \frac{G_{\text{max}} (1 - R^2)}{G_c} \right)^m} $$

(6)

The LOG($\Delta N_p$) is displayed with dash/point line on Figure 3 for a fictitious material. Note that all numerical values on Figure 2 were selected for scaling and display purposes only. Under fatigue loading, the material point is not anymore continuously passing along the cohesive zone traction curve. Consider a point A (Figure 2), after less than 10 cycles this point becomes damaged in the sense that according to S-N curve we have reached failure. However, what does it mean in terms of cohesive law? In [12] it is proposed to assign to this element the propagation damage variable of $d=1$ moving it all the way to the DP condition. The rationale behind it is that we will form a highly loaded crack tip zone, which will be in the propagation regime and can be treated by the VCCT like approach. This damage initiation
approach is however similar to assuming a certain size of initial flaw and this size is also tied

to the mesh size. We propose instead to modify the cohesive law in each point depending

upon the history of how the DI condition was achieved. Namely, we propose to reduce the

initiation strength of the cohesive law to that at which the DI condition was reached under

 cyclic loading while maintaining the critical ERR value for static propagation G_c. It is

 illustrated on Figure 2 where a new cohesive law for point A is shown by a dashed line after

the initiation variable \( d_i = 1 \). We change the slope of cohesive zone model to maintain the

static propagation characteristics. According to this hypothesis, each material point may have

its own cohesive law. The proposed approach eliminates any ambiguity or need for initial

damage size or presence of any cracks or delamination’s in the structure. Space limitations do

not allow describing additional details of variable mode implementation of the initiation and

propagation phase of the proposed formulation.

3. Experimental Methodology

Delamination Migration Specimen (DMS) was introduced in Ref. [8] to study delamination

migration phenomenon under static loading. The specimen dimensions are shown in Figure 3.

The specimens are 12.7mm wide and the initial crack length \( a_0 \) is 48mm. Specimens were

loaded on the top surface using a piano hinge, while the two lateral edges were clamped.

IM7/8552 material was used with a stacking sequence of

\[ [0/90/0_2/90_6/0_2/90/0/Teflon/90_4/0_10/90_12/0_4], \]

building from the bottom of the specimen to the

top. This lay-up was chosen because this stacking sequence allows for stable crack growth in

this testing configuration.

![Figure 3: Migration Specimen Configuration and Dimensions (all dimensions in mm).](image)

4. Results and Discussion

DMS specimens were simulated using static and fatigue loading. IM7/8552 stiffness, strength,

fracture toughness, and fatigue properties were the same as in Ref [11]. The fatigue specimens

were simulated using constant displacement control with amplitude corresponding to 60% and

40% of the critical ERR for static propagation with \( R=0.1 \) and frequency 10Hz. Under

displacement controlled fatigue loading, the energy release of the delamination decreases with

its length, and the ERR amplitudes in fatigue loading corresponds to the initial delamination

length. Load location of \( L=1.2a_0 \) was used for these three simulations. The results of the

progressive damage simulation are shown on Figure 4. Under static loading, the delamination

migration occurs at the distance of 10.5mm from the load application point compared to

experimental value of 12.5mm. As the applied displacement amplitude decreases, the crack

jump moves further away from the load application point. In the 40% max displacement
fatigue case, the crack does not migrate from interface to interface within the fine mesh region at all. A migration does occur at a distance of 38 mm in the area of coarse mesh and therefore requires further study. Fatigue loading experiments are in progress.

**Figure 4.** Damage in Migration Specimens at Different Loadings.

### 5. Conclusions

DDM framework for fatigue modeling of matrix cracking and delamination in laminated composites is proposed. Fatigue failure criterion based on LaRC04 static criterion is proposed. A consistent cohesive law for fatigue crack initiation and extension is proposed. It allows for damage initiation and propagation governed by S-N curves and Paris law respectively. The proposed model eliminates any explicit or implicit assumptions regarding initial crack size in fatigue loading much alike the cohesive zone model in static loading.

Static and fatigue behavior of DMS under displacement loading was modeled. The delamination migration location under static loading was predicted within 2mm from that observed experimentally. Fatigue loading under 60% and 40% initial ERR amplitude compared to critical static value was performed. The observed delamination migration location moved further out for 60% and did not occur within the refined mesh region for 40% amplitude.
Acknowledgement

Authors thank Dr. Kevin Obrien, NASA Langley Research Center Hampton, VA for support and Dr. J. Ratcliffe, National Institute of Aerospace, Hampton, VA for collaboration and experimental data. The work was co-funded under AFRL contract FA8650-10-D-5011.

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