HYSTERETIC SHEAR BEHAVIOUR OF FIBRE-REINFORCED COMPOSITE LAMINATES

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Abstract
This work deals with the hysteretic shear behaviour modelling of woven carbon fibre reinforced composite laminates by means of fractional operators. The model includes both in-ply damage and irreversible strains. In-ply failure behaviour and damage is described by means of continuum damage mechanics and irreversible strain evolution equations are derived to finally obtain material parameters from experimental data. Hysteretic behaviour observed under in-plane shear loading-unloading cycles are modelled with the use of a fractional derivative model. The main advantage of fractional models is their ability to deal with history-dependent phenomena, such as hysteretic behaviour, with a very few number of parameters for the material behaviour model. The efficiency of this model has been analysed by comparison with experimental data, showing the model's ability to reproduce the damage and hysteretic behaviour of the material.

1. Introduction

Composite materials are essential in applications in which significant stiffness combined with lightweight are needed. Knowledge of their behaviour until rupture is necessary in order to optimise the structures made of these materials. Carbon fibre reinforced composite materials have been studied for many years. Their linear orthotropic elastic behaviour is well known and important work has been done on the understanding and modelling of damage by analysis at the mesoscale of the elementary ply [1, 2, 3, 4].

This work focuses on the hysteretic shear behaviour of these materials together with its coupling with damage mechanics. In this work the hysteretic shear behaviour will be dealt with by the use a non-modified fractional model, based on the experience on previous works [4, 5]. The main advantage of fractional models is the need of a very few number of parameters for the material characterisation and their ability to deal with phenomena which are history-dependent, such as hysteretic phenomena.

Hysteresis modelisation enables, for instance, to improve composite thermomechanical behaviour modelling in fatigue, where, in general, hysteresis is not included within the damage models [6, 7]. Another area of interest could be heating modelling during fatigue tests [8], where thermomechanical behaviour could be improved by hysteresis-loops modelling with fractional operators.

The aim of this analysis is to develop a model which includes both damage phenomena and irreversible strain, together with hysteretic behaviour under shear. The main advantage of fractional models is the need of a very few number of parameters for the material characterisation and their ability to deal with phenomena which are history-dependent, such as hysteretic phenomena.

2. Experimental

In order to define the shear behaviour, mainly the damage and irreversible strain evolution laws and model parameters, loading-unloading shear tests have been performed.

2.1. Materials and samples

The carbon fibre reinforced composite plates are composed of Toray GV 170U carbon fibre and SiComin SR8100 resin, manufactured at Mondragon Unibertsitatea. The samples have been obtained by water jet cutting.

2.2. Shear tests

The tests have been fulfilled in an Instron universal test machine (figure 1) on samples with ±45° fibre orientation at 50 mm/min (v50).

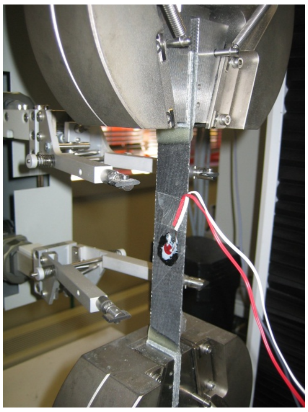

Figure 1. Universal test machine.

3. Material model

The model is written at the mesoscale of the layer which establishes a good compromise between the scales of the constitutive materials and the structure. A plane-stress state is assumed and only small strains are taken into account. In what follows, subscripts 1 and 2 stand for the fibre direction and the transverse direction, respectively.

3.1. Constitutive law

The model is applied to a fibre reinforced composite ply, which is modelled as a homogeneous orthotropic elastic or viscoelastic damaging material whose properties are degraded after loading mainly by matrix microcracking. Continuum damage mechanics theory is used to describe ply degradation by the use of internal damage parameters [1]. In fiber-reinforced materials, matrix cracking is usually accompanied by interfacial debonding followed by sliding, giving rise to irreversible strains and hysteresis loops. To model hysteresis loops, different models have been proposed [4, 9, 10].

In order to study the behaviour of the material, shear tests with discharge were performed on woven carbon fibre epoxy-matrix laminates with a $[\pm 45]_{2s}$ stack sequence following the same procedure as in [4]. The irreversible strains and loading-unloading hysteretic behaviour may be observed in fig. 2.

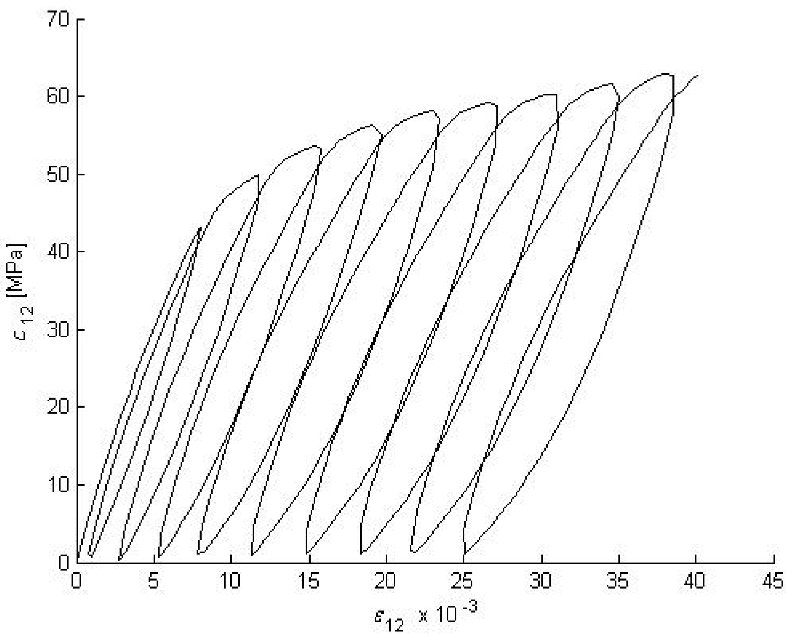

Figure 2. Experimental shear stress-strain relation at v50.

The following split of shear strain is assumed to deal with the reversible and irreversible effects:

$$\epsilon_{12} = \epsilon^{e}_{12} + \epsilon^{i}_{12} + \epsilon^{v}_{12} \tag{1}$$

where ϵ^{e}_{12} stands for the elastic shear strain, ϵ^{i}_{12} stands for the irreversible shear strain and ϵ^{v}_{12} stands for the viscous shear strain. Nevertheless, viscous shear strains have been neglected after considering the experimental results, which have revealed small values for them. Thus, the strain split yields:

$$\epsilon_{12} = \epsilon^{e}_{12} + \epsilon^{i}_{12} \tag{2}$$

Taking into account Helmholtz's free energy [2], and only considering shear loading conditions, the following constitutive law for in-plane stress is considered:

$$\sigma_{12}(t) = 2G^{0}_{12}\,(1 - d_{12})\,\epsilon^{e}_{12} \tag{3}$$

where G_{12}^0 and d_{12} stand for the undamaged elastic shear modulus and damage variable associated with stiffness loss in shear due to matrix microcracking, respectively. In the general damage mechanics formulation, the thermodynamic force Y_{12} , associated to damage in shear, monitors damage development and is defined by:

$$Y_{12} = 2G_{12}^0(\epsilon_{12}^{\mathrm{e}})^2 \tag{4}$$

In order to model the hysteretic behaviour observed in experimental tests (fig. 2), a fractional model is chosen [4]. To take into account the existence of clockwise hysteresis loops observed during the tests, a fractional fractional derivative constitutive law is used [4, 5]. In the presence of interfacial debondings and slipping is taken into account with the internal damage variable $d_{12}^{\mathrm{v}_k}$ [9]. The modified fractional model yields:

$$\sigma_{12}(t) = 2G_{12}^0\,(1 - d_{12})\,\epsilon_{12}^{\mathrm{e}}(t) + \sum_{k=1}^{N} 2G_k \left(1 - d_{12}^{\mathrm{v}_k}\right) \mathrm{D}^{\alpha_k} \epsilon_{12}^{\mathrm{e}}(t) \tag{5}$$

where G_k are material parameters and D^{α_k} is the fractional derivative of order α_k , $0 \leq \alpha_k \leq 1$ and $k \in [1, \ldots, N]$. Finally, α_k is assumed to be function of damage d_{12}. Furthermore, within the loops, the elastic strain is defined in the following way:

$$\epsilon_{12}^{\mathrm{e}} = \frac{\sigma_{12}}{2G_{12}^0(1 - d_{12})} \tag{6}$$

Fractional calculus allows defining the derivative and integral of generalised order. The fractional derivative of a function $f(t)$ can be defined as [11, 12]:

$$\mathrm{D}^{\alpha} f(t) = \frac{\mathrm{d}^{\alpha} f(t)}{\mathrm{d}t^{\alpha}} = \frac{1}{\Gamma(1-\alpha)} \frac{\mathrm{d}}{\mathrm{d}t} \int_0^t (t-y)^{-\alpha} f(y)\,\mathrm{d}y \tag{7}$$

where Γ is the gamma function [12].

Depending on the definition used and on the order of fractional operators, fractional derivatives and integrals at any time t_n can be calculated numerically by using several numerical algorithms [11, 13]. Using the L1-algorithm, the expression obtained is:

$$\mathrm{D}^{\alpha} f(t_n) = \frac{(\Delta t)^{-\alpha}}{\Gamma(2-\alpha)} \left[\left(\frac{1-\alpha}{n^{\alpha}} f_0 + \sum_{j=0}^{n-1} \left((j+1)^{1-\alpha} - j^{1-\alpha} \right) \left(f_{n-j} - f_{n-j-1} \right) \right] \tag{8}$$

3.2. Evolution laws

The damage parameter d_{12} is defined by the damage evolution function f_{12}. After experimental results on unidirectional fibre reinforced composites [1], assuming that damage during unloading remains constant until further positive loading is applied and causes further damage accumulation, the parameter $\bar{Y}_{12}$, based on the maximum value reached by the thermodynamic damage force along the previous loading history, is defined to describe the damage development. Thus:

$$\bar{Y}_{12} = \max_{\tau \leq t} \left(\sqrt{Y_{12}(\tau)} \right) \tag{9}$$

A logarithmic form was found to be a good approach for shear behaviour, thus the damage evolution law is defined as:

$$d_{12} = a_{12} \left\langle \ln \bar{Y}_{12} - \ln \sqrt{Y_{12}^{0}} \right\rangle_{+} \text{if } d_{12} < 1 \text{ and } \bar{Y}_{12} < Y_{12}^{c} \tag{10}$$

where $\langle\ \rangle_+$ is the Macaulay bracket, a_{12} is a coeficient, Y_{12}^0 is the damage initiation thermodynamic force and Y_{12}^c is the critical thermodynamic force.

4. Parameters identification

The identification procedure consists in determining by experimental data the values of the parameters appearing in the constitutive relations of the model [1, 2, 4, 6].

4.1. Evolution master curves

After determining G_{12}^0, damage parameters d_{12}^M and $\epsilon_{12}^{i\,M}$ in loop M have been obtained. Then, once the damage parameters have been identified, the shear damage and irreversible strain evolution master curves can be determined. From experimental data, a quadratic evolution form has been obtained making it possible to estimate the irreversible strain threshold Y_{12}^i.

4.2. Fractional model parameters

In order to obtain the values for the parameters of the fractional model, an optimisation problem has been formulated aimed at minimising the error function e, which has been calculated adding all the individual data point errors in the interval considered. During the optimisation process and based on equation 5, the following model has been used:

$$\sigma_{12}(t) = 2G_{12}^{0}\,(1 - d_{12})\,\epsilon_{12}^{e}(t) + \sum_{k=1}^{N} G_k{}^{v} \mathrm{D}^{\alpha_k} \epsilon_{12}^{e}(t) \tag{11}$$

The error function e , which is calculated from the error e^m at time t_m, has been defined as:

$$e = \sum_{m=0}^{n} e^m = \sum_{m=0}^{n} (\bar{\sigma}_{12}^{m} - \sigma_{12}^{m})^2 \tag{12}$$

where $\bar{\sigma}_{12}^m$ and σ_{12}^m are the shear stress value obtained from experimental tests and from the mathematical model, respectively, both evaluated at time t_m. The unknown parameters of the model, namely G_{12}^0, $G_k{}^v$, α_k and $d_{12}^{v_k}$ can be then be obtained by minimising the error.

5. Results

After having obtained the values for the initial (undamaged) shear modulus from the shear stress-strain curve, damage initiation and irreversible strain threshold thermodynamic forces have been determined from the evolution master curves. In order to perform the optimisation problem, only one derivative order has been retained, that is, $N = 1$. Therefore, after the minimisation procedure only G_{12}^0, G_1 and α_1 have been taken into account. Nevertheless, in order to analyse their evolution, their respective values for the M-th loop, $G_{12}^{0\,M}$,G_1^M and α_1^M have been calculated. The results can be seen in table 1.

The model's ability to reproduce the clockwise hysteresis loops is shown (fig.3, 4 and 5). Furthermore, the values for the damage parameters are consistent with the ones obtained experimentally.

Loop	G^0_{12} [MPa]	G_1[MPa.s$^{-\alpha_1}$]	α_1
1	2236	1058	0
2	2823	543	0.0008
3	3126	288	0.0043

Table 1. Parameters of the fractional model for v50

Concerning the fractional parameter, its values are very small and suggest a quasi-linear behaviour with respect to the elastic strain. This may be due to the definition of the elastic strain within the loops. If one considers the order of the derivatives to be zero, the sum of the values obtained for G^0_{12} and G_1 turns out to be the same as the one obtained from the parameters identification process (3487 MPa).

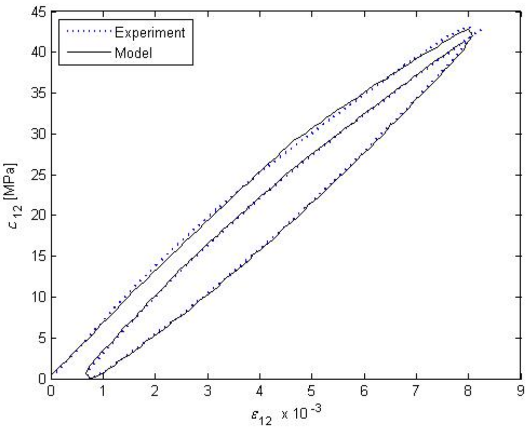

Figure 3. Stress-strain relation for the first loop v50

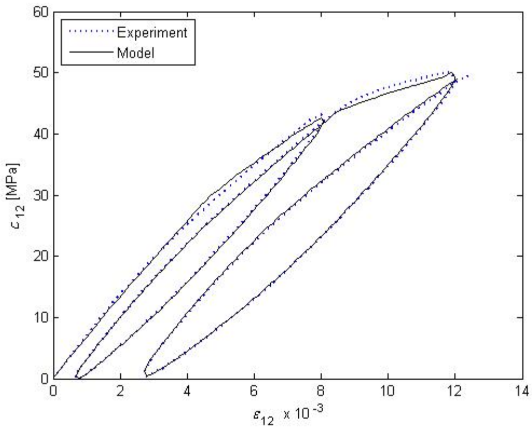

Figure 4. Stress-strain relation for the second loop v50

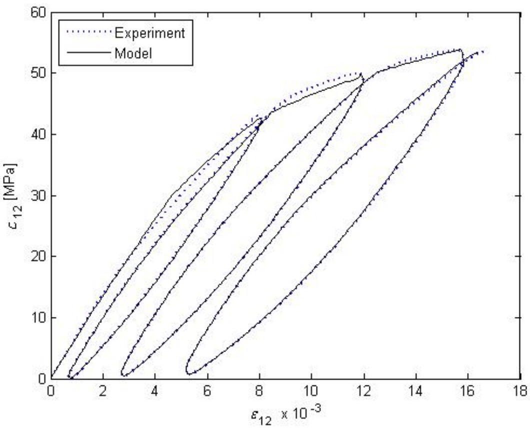

Figure 5. Stress-strain relation for the third loop v50

6. Conclusion

A fractional model has been developed for a carbon fibre reinforced composite, which includes both intra-ply damage and hysteresis under shear loadings with only a very few number of parameters. The construction of the model is based on the thermodynamics framework of irreversible processes, continuum damage mechanics and the use of fractional calculus in order to reproduce the hysteretic behaviour. The model's ability to reproduce the clockwise hysteresis loops has been shown. The values obtained for the fractional derivative are very low, which suggest a quasi-linear behaviour with respect to the elastic strain.

It has been shown that the model is able to reproduce the damage and hysteretic behaviour of the material for low strain rates without modifying the fractional model construction developed for quasi-static conditions. Further work is in progress concerning the explicit implementation of the strain rate effect in fractional models and its implementation in finite element codes.

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