Fatigue Analysis of Wind Turbine Composites using Multi-Continuum Theory and the Kinetic Theory of Fracture

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Abstract

Multi-continuum theory and the kinetic theory of fracture have been used to predict the fatigue life of wind turbine laminate coupon tests from the OPTIMAT project. In order to apply this approach in a manner that would be effective for turbine blade analysis, several alterations to the method described in the literature were made. These include the treatment of the stress time history, the method used to integrate the bond-rupture equation, the treatment of low stresses and the method used to estimate temperature increase due to hysteresis. In this paper, a comparison is made between the results that are obtained with and without these enhancements and the results of a benchmarking exercise are presented. Good agreement was found with coupon tests under circumstances where the loading was not completely compressive.

1. Introduction

The fatigue behaviour of fibre reinforced composites is difficult to predict because of their anisotropic strength and stiffness characteristics. This can lead to several different failure modes depending on the magnitude, direction and frequency of the applied loading.

Fertig and Kenik [1] aimed to address this problem by using the multi-continuum theory to access the matrix stress of the composite, reasoning that the majority of the fatigue life of composites is spent developing matrix cracks, so the matrix stress rather than the composite stress is in fact the relevant quantity. The multi-continuum theory was developed by Garnich and Hansen [2] and is an efficient method of accessing the volume averaged stresses of the constituents of a composite, namely the fibres and the matrix in the case of fibre reinforced polymers. The method is described in detail in [2].

The matrix stress tensor is then converted to a scalar 'effective stress' value using the failure criterion described in [1]. There are different failure criteria for on-axis (longitudinal) and off-axis (transverse) cracks.

The fracture of the matrix is then predicted using the kinetic theory of fracture first developed by Zhurkov [3], combined with a damage accumulation model originally developed by

Hansen and Baker-Jarvis [4] which was used to link the microscopic process of bond rupture to macroscopic crack growth.

The method requires knowledge of the 'in situ' fibre and matrix stiffness properties. These are found by using measured composite stiffness data and a micromechanical model in an optimization routine in a preprocessing pass. Composite strength data is also required to calibrate the failure criterion for transverse cracks.

Finally, some on axis and off-axis fatigue data is required to calibrate the constants in the kinetic theory equation. These are found by using a Nelder-Mead optimization routine to minimise the difference between the calculated life and the life of physical test specimens.

2. Method

In the present work, no changes were made to the multi-continuum theory or the method used to calculate the effective stress. The focus was instead on enhancing the kinetic theory part of the process, with the goal of improving results to the point where the method can be used in the fatigue analysis of wind turbine blades. These enhancements include:

- Treatment of the stress time history. Fertig and Kenik [1] reduced the time history by looking for turning points in each of the components of the matrix stress tensor. In the present work, two alternatives were tried:
- Retain the complete time history. This is more accurate, but will come at the cost of increased computational burden.
- Retain zero crossing points as well as turning points. This will increase the accuracy, as the effective stress is always positive.
- Use of an alternative method used to integrate the bond rupture rate equation through the time history. Fertig and Kenik used the trapezium rule between two points in the time history [1]. In the present work, the numerical integral of the bond rupture equation for a linearly changing stress was also tried for comparison purposes.
- Using a different stress variable to calculate the temperature increase. Fertig and Kenik recognized that temperature rises due to hysteretic heating were responsible for the different behaviour at a range of R-Values. They used the effective stress, which can never be negative. In addition to this method, the signed Von Mises matrix stress was used, and a temperature calculation which allows for the fact that the temperature will eventually stabilize was used. The sign for the Von Mises stress is taken from the principal component with the largest absolute value.

These enhancements were compared by looking at the sum of the residuals in the objective equation when calibrating the constants used in the kinetic theory equation. If the model matches up to the physical test better then this number will be lower.

Finally, the original method and the enhanced method were compared by analyzing the failure of multidirectional laminates under a range of loading conditions.

2.1 Background to the Kinetic Theory of Fracture

The kinetic theory of failure of solids was first proposed by Zhurkov [3]. The basis of the concept is that all atoms and molecules at a temperature above absolute zero vibrate at a frequency proportional to υ_0 given by equation (1).

$$\upsilon_0 = \frac{kT}{h} \tag{1}$$

where kT is the thermal energy (the product of the Boltzmann constant and the absolute temperature) and h is the Planck constant. The thermal energy is in fact a distribution, not a single number, so there is always a chance that any given oscillation will have enough energy to overcome the energy barrier U and move from one equilibrium state to another. The likelihood of this occurring is given by equation (2).

$$\exp\left(\frac{-U}{kT}\right) \tag{2}$$

In the case of fracture, the first equilibrium state is unbroken and the second is broken. Clearly, there is also an equal likelihood that a bond will move from a broken state back to the unbroken state.

An applied stress has the effect of reducing the likelihood of 'bond healing' and increasing the likelihood of 'bond breaking'. The bond rupture rate K_B can therefore be calculated as the product of the number of attempts to break bonds (the vibration frequency) multiplied by the likelihood of an attempt having sufficient energy to break the bonds.

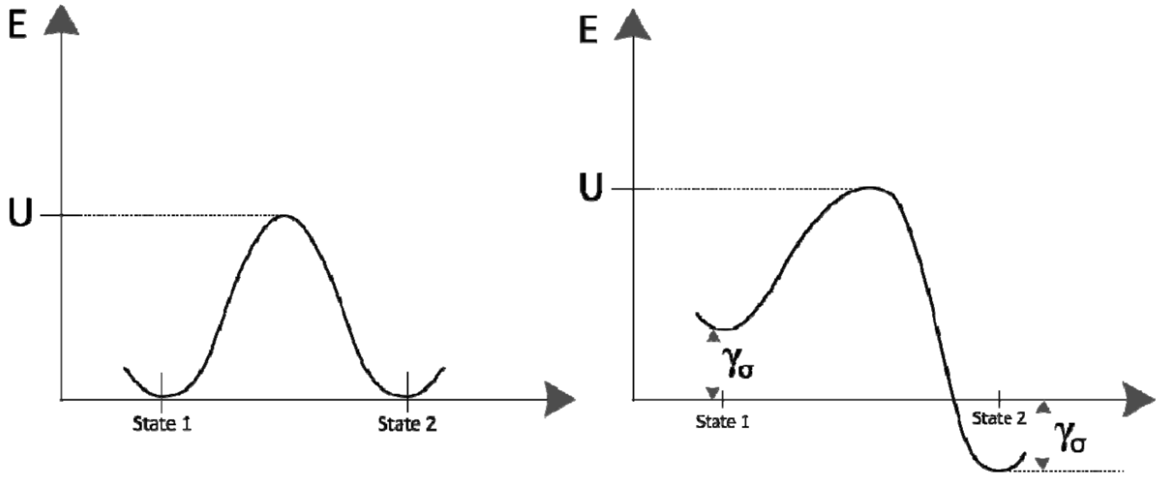

Figure 1. Energy barrier [19]

$$K_B = \frac{kT}{h} exp\left(-\frac{U - \gamma\sigma}{kT}\right) \tag{3}$$

where γ is called an activation volume, and is related to the volume of material which is involved in the process. Equation (3) is bond rupture rate and is the basis of the fatigue algorithm used by Fertig and Kenik [1]. The possibility of bonds moving from a broken state back to an unbroken state is not considered, because even at low stresses it is very unlikely. There is an issue with this method when it is used for the analysis of wind turbine blades; because they must be designed to survive a 50 year gust of wind without clipping the tower, the stresses in everyday operation are very low. In equation (3) K_B can never be zero, even if the applied stress σ is 0, so the matrix will always accumulate damage even without an applied stress. For this reason, the possibility of bond healing has been considered in the new implementation presented here, so K_B is calculated using equation (4), which takes account of bond healing.

$$K_B = \frac{kT}{h}\left[exp\left(-\frac{U-\gamma\sigma}{kT}\right) - exp\left(-\frac{U+\gamma\sigma}{kT}\right)\right] \tag{4}$$

2.2 Temperature dependency

Equation (4) shows that low amplitude oscillations at a high mean stress will result in a higher average bond rupture rate and therefore a shorter life than high amplitude oscillations with a lower mean stress. In reality the reverse is true, and Fertig and Kenik [1] suggest that this is because higher amplitude oscillations increase the temperature more due to hysteresis losses. The energy dissipated per cycle is calculated using equation (5) [1], and the temperature increase is assumed to be proportional to the energy dissipated and so it is calculated using equation (6), where ψ is a constant found as part of the calibration process [1].

$$\dot{E} = \pi f J'' \sigma_a^2 \tag{5}$$

$$T^* = T + \psi \sum_{i=1}^{i=n} \frac{\Delta\sigma_{Eff}{}^2}{\Delta t^2} \tag{6}$$

This calculation of the temperature rise presents a problem for wind turbine blade analysis, as the design standards [5] [6] require that time histories at least 10 minutes long are used, so that the turbulence can be statistically representative of long term operation at a particular average wind speed. In this situation equation (6) will incorrectly increase the temperature indefinitely, and so the temperature is calculated in the new implementation by combining Newton's law of cooling with equation (6). Instead of the effective stress (which may not change much from point to point in the cycle), the signed Von Mises stress σ_{SVM} is used.

$$T_{i+1}^* = \left(T_i^* - T - \psi\frac{\Delta\sigma_{SVM\,i}{}^2}{\Delta t_i^2}\right)e^{-k\Delta t_i} + T + \psi\frac{\Delta\sigma_{SVM\,i}{}^2}{\Delta t_i^2 k} \tag{7}$$

The value of the cooling constant, k, is fairly arbitrary; as ψ is chosen by calibrating against physical test data. The value of T^* is found by stepping through the entire stress time history.

2.3 Damage Evolution

In the work performed by Fertig and Kenik [1] the microscopic bond breaking is connected to macroscopic cracks by evolving a damage parameter that is set to zero initially and to a value of one when a macro crack is formed. The method used is based on that developed by Hansen and Baker-Jarvis [4].

$$\frac{dn}{dt} = (n_0 - n)K_B \tag{8}$$

n_0 is a parameter that is found by enforcing the condition

$$\int_0^1 \frac{dn}{(n_0 - n)} = 1 \tag{9}$$

Equation (8) can then be integrated to obtain $n(t)$. Fertig and Kenik [1] solved the equation for a single load block. In the present work the solution has been extended to allow for block loading and progressive failure analyses. The value of n after the ith block can be calculated using equation (10).

$$n(t_i) = (n(t_{i-1}) - n_0)\exp\left(-N\int_{t_{i-1}}^{t_i} K_b(t)\,dt\right) + n_0 \tag{10}$$

This equation can be rearranged for N, the number of cycles to failure to implement progressive failure as described in [7]. The integral term can be calculated numerically or mathematically, as shown in equation (11). We might expect to see an improvement using this piecewise linear integral, because the bond rupture rate is highly nonlinear as shown in Figure 2. In this typical instance, the numerical method overestimates the area under the true ('Haversine') curve by a factor of 5 whilst the mathematical method ('Linear change') underestimates the area by a still poor but better factor of 2.

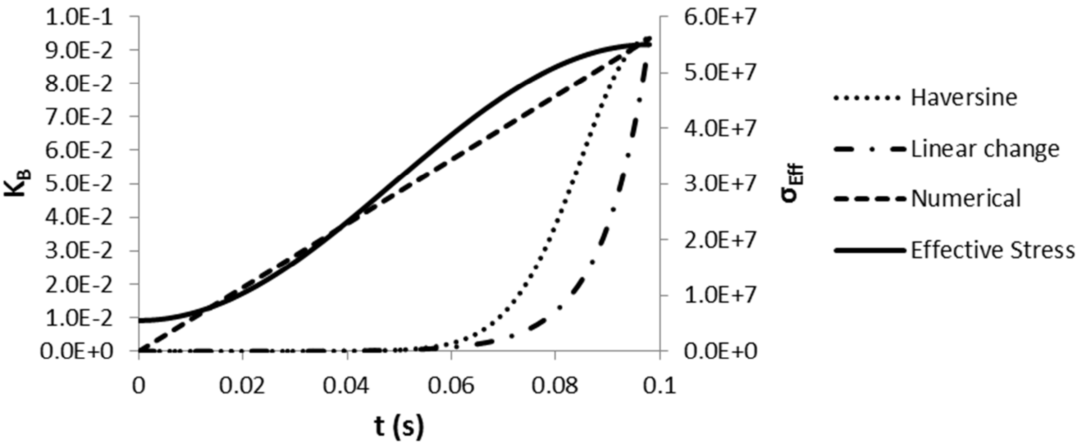

Figure 2. Comparison of bond rupture rate integration methods

$$\int_0^{t_1} K_B dt = \frac{(kT)^2\left[e^{\left(\frac{2\gamma c}{kT}\right)}e^{\left(\frac{\gamma m t_1}{kT}\right)} - 1\right]\left[e^{\left(\frac{\gamma m t_1}{kT}\right)} - 1\right]}{\gamma h m e^{\left(\frac{U}{kT}\right)}e^{\left(\frac{\gamma c}{kT}\right)}e^{\left(\frac{\gamma m t_1}{kT}\right)}} \tag{11}$$

where

$$m = \frac{\sigma_{eff}(t_{i+1}) - \sigma_{eff}(t_i)}{t_{i+1} - t_i},\ c = \sigma_{eff}(t_i) \text{ and } t_1 = t_{i+1} - t_i$$

The values of U and γ in equation (4) and ψ in equation (7) are found using a Nelder-Mead optimisation routine. Separate values are used for transverse and longitudinal failures of the ply. The objective function is given by equation (12).

$$\min_{N_c \in \mathbb{R}} \sum_{i=1}^{i=n} \left(\log_{10} N_{C_i} - \log_{10} N_{T_i}\right)^2 \tag{12}$$

N_{C_i} is the computed cycles to failure for a specimen i calculated using the same material properties, maximum stress, R-Value, load angle and test frequency as the physical test specimen i which failed after N_{T_i}cycles. The computed cycles are calculated using the composite fatigue algorithm described in [7].

3. Results

3.1 Quantification of best method of calculating kinetic theory constants

The approach used was validated using data from the Optidat project [8]. There are several composite material databases available for wind turbine laminates, but the Optidat database is particularly useful because all of the data required to characterise a ply is available, with consistent volume fractions and fibre/ matrix systems used throughout. The laminates referred to as 'UD2' and 'UD3' were used in this analysis (the two laminates are composed of different numbers of layers but are otherwise similar), and the material properties extracted from the database are shown in Table 1. Direction 1 is in line with the fibres, and direction 2 is perpendicular to them.

Strength	Value	Units	Stiffness	Value	Units
$\sigma_{1\,UT}$	800	MPa	E_1	39.105	GPa
$\sigma_{1\,UC}$	530	MPa	E_2=E_3	14.165	GPa
$\sigma_{2\,UT}$	53	MPa	$G_{12} = G_{13}$	4.659	GPa
$\sigma_{2\,UC}$	165	MPa	G_{23}	5.238	GPa
$\sigma_{12\,US}$	75	MPa	$\upsilon_{12} = \upsilon_{13}$	0.279	
			υ_{23}	0.352	

Table 1. UD2/UD3 Stiffness and strength data

All of the available applicable fatigue data for UD2 and UD3 in the database was used. This amounted to 47 tests (R=-1 and R=0.1) in the 90° off-axis direction and 245 tests (R=-1 and R=0.1) in the 0° direction. The results of the optimisation for the transverse constants are given in Table 2 and the results for the longitudinal optimisation are given in Table 3. The scores for the longitudinal direction have been normalised by the number of test specimens so that they are equivalent to those for the transverse direction.

$\int K_B\,dt$	Numerical						Mathematical					
Time history	Peak-Valley		Peak-Valley + Zero Crossings		Unchanged		Peak-Valley		Peak-Valley + Zero Crossings		Unchanged	
$\sigma\Delta T$	σ_{Eff}	σ_{SVM}	σ_{Eff}	σ_{SVM}	σ_{Eff}	σ_{SVM}	σ_{Eff}	σ_{SVM}	σ_{Eff}	σ_{SVM}	σ_{Eff}	σ_{SVM}
U	104046	112646	104046	112646	104310	111342	102012	110228	102012	110228	102614	109661
γ	3.97E-4	5.59E-4	3.97E-4	5.59E-4	4.41E-4	5.80E-4	4.66E-4	6.37E-4	4.66E-4	6.37E-4	4.41E-4	5.80E-4
ψ	2.1E-31	3.1E-17	2.1E-31	3.1E-17	4.8E-31	2.2E-17	1.6E-31	3.3E-17	1.6E-31	3.3E-17	6.0E-30	2.3E-17
Score	4.92	3.31	4.92	3.31	3.61	2.57	5.42	3.78	5.42	3.78	3.61	2.57

Table 2. UD2/UD3 Transverse fatigue data kinetic theory coefficients

The longitudinal scores are much higher than the transverse scores, reflecting the fact that longitudinal failure is dominated by fibre breakage so there is generally a lot more scatter for the longitudinal coupons. The most significant improvement in the variable fitting scores for both cases results from using the signed Von Mises stress as opposed to the kinetic theory effective stress when calculating the temperature rise due to hysteretic heating. This is because the effective stress can only be positive, so it does not reflect the real changes in stress state that are driving the temperature increase.

Surprisingly, using a mathematical integral between time history points does not prove to be as effective as the numerical integral, despite the highly nonlinear nature of the bond rupture rate which can be seen in Figure 2. The best results were obtained by retaining the full time history (in this case a simple sinusoid of one test cycle) and using the signed Von Mises stress for the temperature calculation.

$\int K_B\, dt$	Numerical						Mathematical					
Time history	Peak-Valley		Peak-Valley + Zero Crossings		Unchanged		Peak-Valley		Peak-Valley + Zero Crossings		Unchanged	
$\sigma\Delta T$	σ_{Eff}	σ_{SVM}	σ_{Eff}	σ_{SVM}	σ_{Eff}	σ_{SVM}	σ_{Eff}	σ_{SVM}	σ_{Eff}	σ_{SVM}	σ_{Eff}	σ_{SVM}
U	104842	109012	104461	106831	105293	107428	104319	108363	104012	106289	105292	107428
γ	2.69E-4	3.11E-4	2.53E-4	2.83E-4	2.93E-4	3.22E-4	3.11E-4	3.56E-4	2.93E-4	3.27E-4	2.93E-4	3.22E-4
ψ	9.2E-20	3.3E-18	3.2E-18	3.5E-18	2.7E-18	2.9E-18	2.1E-20	3.3E-18	3.1E-18	3.5E-18	2.7E-18	2.9E-18
Score	52.99	20.88	49.04	31.02	42.50	26.86	56.58	22.75	52.55	33.69	42.51	26.86

Table 3. UD2/UD3 Transverse fatigue data kinetic theory coefficients

3.2 Comparison of baseline results with enhanced method

The baseline (time history filtered to peaks and valleys, bond rupture integration performed numerically and temperature calculated using kinetic theory effective stress) and enhanced (time history unchanged, bond rupture integration performed numerically and temperature calculated using matrix signed Von Mises stress) methods have been compared by using the method to analyse the failure of Optidat laminate 'MD2' which is composed of layers of UD2/ UD3 as follows: [[±45°, 0°]$_4$;±45°]. The analysis was performed using the progressive failure fatigue algorithm described in detail in [7].

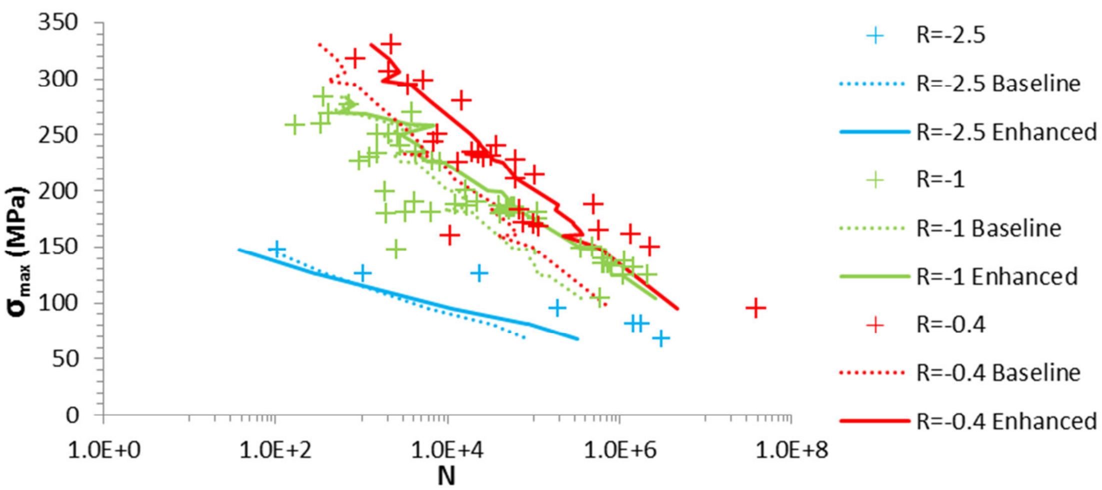

Figure 3. Comparison of physical tests and baseline and enhanced models (reversing loads)

Figure 3 shows a comparison between the predictions of the baseline MCT/KTF model described by Fertig and Kenik [1] and the enhanced method described in the present work for reversing loading, and Figure 4 shows the results for tensile loading. Qualitatively, the enhanced method does improve the results in both cases, and this is reflected when equation (12) is applied. For the baseline method, the error sum is 178.1 compared to 151.9 for the enhanced method (average life prediction out by a factor of 6.13 and 4.7 respectively). This slightly disappointing improvement reflects the large amount of scatter in the R=-1 and R=0.1

data (these datasets include samples from a huge number of laboratories). With these datasets removed the sums are 56.7 and 36.5 respectively (average life prediction out by a factor of 13.2 and 5.2 respectively). Overall, it is clear that the enhanced method does improve the results.

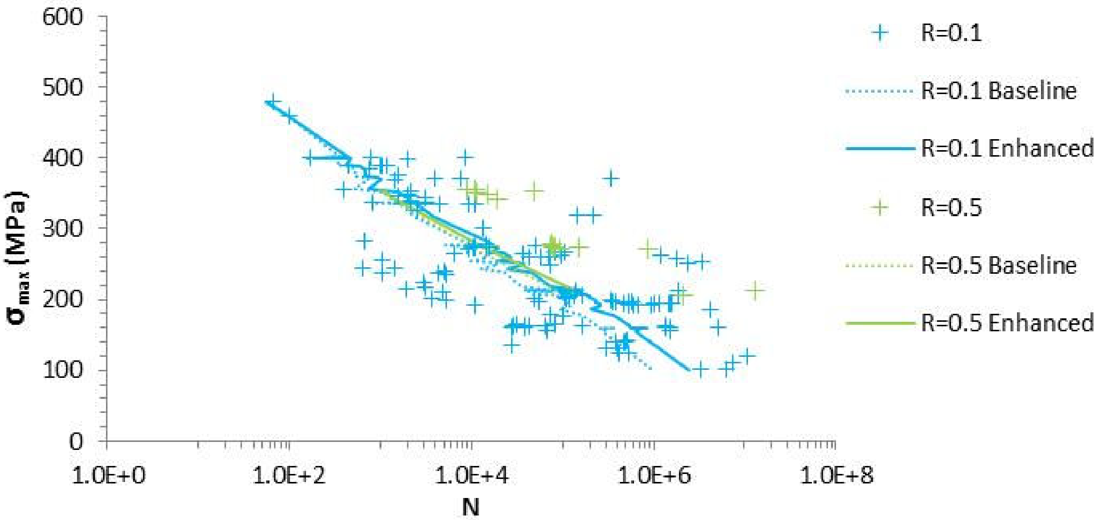

Figure 4. Comparison of physical tests and baseline and enhanced models (tensile loads) for material MD2 from the Optidat database

4. Conclusions

The changes to the methods used to calculate the temperature rise and the use of the raw time history rather than just the peaks and valleys result in a very significant improvement to the achieved lifetime predictions. Future work should focus on improving the composite stress model to allow delamination and fibre failure to be modelled to improve on-axis and compressive predictions.

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