THE OUT-OF-PLANE BEHAVIOUR OF SPREAD-TOW FABRICS

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Abstract

In this paper the constitutive compressive behaviour of nearly parallel spread-tow textile reinforcement is studied. The striking result of our analysis is that the spread-tow type of reinforcement should obey linear relation between force and deformation. This is in contrast to standard textile reinforcements that obey a power-law type of behaviour. To support the theoretical investigation we have developed an test rig who's chief purpose is to achieve compression between nearly perfectly parallel surfaces. This is achieved using a mechanical arrangement consisting of a ball-joint.

1. Introduction

Presently, the aerospace industry is looking into spread-tow textile reinforcement for use in primary structural composites. This is mainly due to their relatively low price and improved mechanical performance, hand ability and manufacturability. As introduction of new structural composite materials into aero structures requires a timely and costly certification process, there is presently a large effort towards characterisation, understanding and modelling various aspects of the spread-tow based composite materials. Since forming of reinforcements plays a key role in terms of the subsequent composite manufacturing and the performance of the final product, we focus in this contribution on the elastic compression of such a preform.

Since, the mechanical properties of a fibre mass are important in many fields of engineering, including composite manufacturing, a significant effort has been spent to describe and model these properties. In particular, Van Wyk [1] pioneered the mechanistic analysis of the compressibility of 3D random fibre masses. Van Wyk regarded the fibre mass as a system of bending units consisting of fibre beam elements between adjacent fibre contacts and he ignored twisting, slip, and extension of the fibres. His key assumptions were that the mean contact spacing is proportional to the reciprocal of the fibre volume fraction and that the segments deform as in bending of straight slender beams. The result of his analysis is a simple power law for the compressive stress as

$$P = k\,E\left(\phi^3 - \phi_0^3\right), \tag{1}$$

where k is a structure dependent constant; E is the Young's modulus of the fibres; and ϕ_0 is the limiting fibre volume fraction below which P=0. Most of the work following van Wyk has accepted the form Eq. 1 and focused on finding an appropriate expression for k by extending the theories describing the structure development during deformation. It was not until 1998 that Toll [2] suggested a more general expression for the packing of fibre masses

$$P = k\,E\left(\phi^n - \phi_0^n\right), \tag{2}$$

where the exponent, n, was shown analytically to take the value of 3 for the 3D and 5 for the 2D random cases. The author also succeeded to fit experimental data for materials where the contacts between fibres are lines rather than points, e.g., fibre bundles, by adjusting the exponent in Eq. 2. Nevertheless, since the assumptions used in the analyses in [1, 2] proceeds from an assumption of point contact and bending of fibre segment, the fitting of Eq. 2 to data for highly parallel fibre architectures in [2] is to be considered as plainly phenomenological.

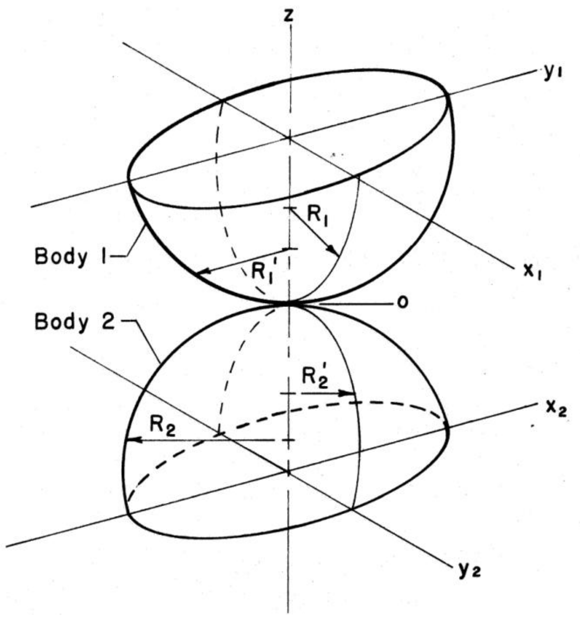

Figure 1. Geometry of the contact between elliptic paroboloids with principal axes of body 1 (x_1, y_1, z_1) and the principal axes of body 2 (x_2, y_2, z_2). The angle ω is the angle between the x_1, z and x_2 z planes, i.e. the principal radii of curvatures R_1 and R_2.

A striking quality of the spread tow technology is the possibility to obtain extremely parallel fibre assemblies. This is in contrast to a typical reinforcement where the fibre architecture are usually somehow irregular. Therefore in the context of mechanical behaviour, the compressive response of the highly oriented spread-tow fabrics is expected to be significantly different from the standard reinforcement. The main difference between the two types of fabrics is the constitutive assumption of load transfer mechanism between the fibres [2]. Namely, in standard fabric the load transfer is assumed via bending of fibre segments, while in spread tow fabric the main mechanism is assumed to be Hertzian contact between adjacent fibres. Therefore, in this paper we study the constitutive behaviour of highly parallel fibre network governed by Hertzian contact. The theoretical study is also supported by experimental characterisation of the transverse compressibility of the highly anisotropic and packed CF spread-tow fibre assemblies. The resulting force-deflection curve is analysed against the Hertzian contact modelling effort to give effective transverse modulus.

2. Theoretical background

The problem of contact between elastic bodies has long been of considerable interest. Assume that two elastic solids are brought into contact at a point **0**, as shown in Fig. 1. If collinear forces are applied so as to press the two solids together, deformation occurs, and we expect a small contact area to replace the point of the unloaded state. If we would able to determine the size and shape of this contact area and the distribution of normal pressure, then the interval stresses and deformation could be calculated.

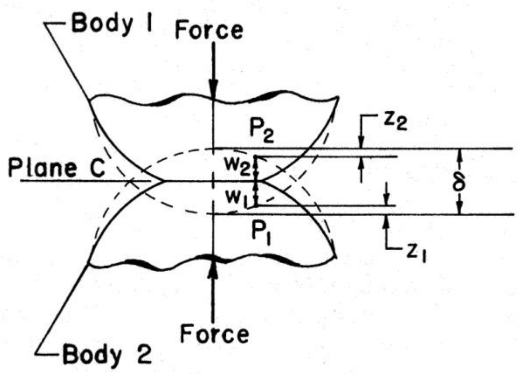

Figure 2. Geometry of deformed bodies. Dashed lines show the surface as they would be in the absence of deformation. Continuous lines show the surfaces of the deformed bodies.

In the analysis of the contact problem by Hertz, as extended by Timoshenko and Goodier [3], the following is assumed: (1) the contacting surfaces are perfectly smooth so that the actual shape can be described by a second degree equation of the form $z = Dx^2 + Ey^2 + Fxy$ where D, E and F are arbitrary constants.; the elastic limit of the materials are not exceeded during the contact; (3) only normal forces between the contacting surfaces are considered; and (4) the contacting surfaces are small in comparison to the entire surfaces. Based on the above assumptions and by applying potential theory it can be showed that: (1) the contact area is bounded by an ellipse whose semi-axes can be calculated from the geometric parameters of the contacting bodies; and (2) the normal pressure distribution over this area is $P = p_0\sqrt{1-(x/a)^2-(x/b)^2}$, where p_0 is maximum pressure at centre, a is the major axis of ellipse of contact and b is the minor axis of ellipse. If the two bodies are pressed together by applied normal forces (cf. Figure 3), then a deformation occurs near the original point of contact along the Z-axis. Here again, we consider only forces acting parallel to the z-axis where the distance from the z-axis is small. The displacements at a point are w_1 and w_2 where w_1 is the deformation of point P_1 of body 1 and w_2 is the deformation of point P_2 for body 2, plane C is the original plane of tangency; z_l is the distance from P_l to the undeformed state , and z_2 is the distance from P_2 to the undeformed state. For points inside the contact area, we have $\delta = (z_1 + w_1) + (z_2 + w_2)$, where $\delta(P)$ is the deformation function we seek. To solve the problem, consider the contact surfaces and define the following geometrical relations [4]

$$A + B = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_1^l} + \frac{1}{R_2} + \frac{1}{R_2^l}\right) \tag{3}$$

and

$$B - A = \frac{1}{2}\left(\left(\frac{1}{R_1} - \frac{1}{R_1^l}\right)^2 + \left(\frac{1}{R_2} - \frac{1}{R_2^l}\right)^2 + 2\left(\frac{1}{R_1} - \frac{1}{R_1^l}\right)\left(\frac{1}{R_2} - \frac{1}{R_2^l}\right)\cos(2\omega)\right). \tag{4}$$

The constants *A* and *B* are expressions for the combinations of the principal curvatures of the surfaces and the angle between the planes of curvature. These are used to calculate δ via the relation

$$\delta = Ax^2 + By^2 + w_1 + w_2. \tag{5}$$

The problem is now to find a pressures distribution and potential functions to satisfy the Eq. 5. The generic solution based on the use of elliptic integrals is to found in numerous textbooks onto contact mechanics, see e.g. [4]. Therefore we will here focus on the contact mechanics of fibres in contact with fibres and plane, i.e. the special case of line contact. To solve this problem we shall make use of the expressions already developed [3] to solve for the "pressure distribution" and size of the area of contact by allowing one axis of the ellipse of contact to become infinite. To determine the deformation, the contact area will be taken as being a finite rectangle with one side very much greater than the other. The derivation will be for the case of a pair of cylinders with their axes parallel and is based on the work presented in [3]. The solution for a cylinder to plane contact can easily be obtained by allowing the radius of one of the cylinders to become infinite.

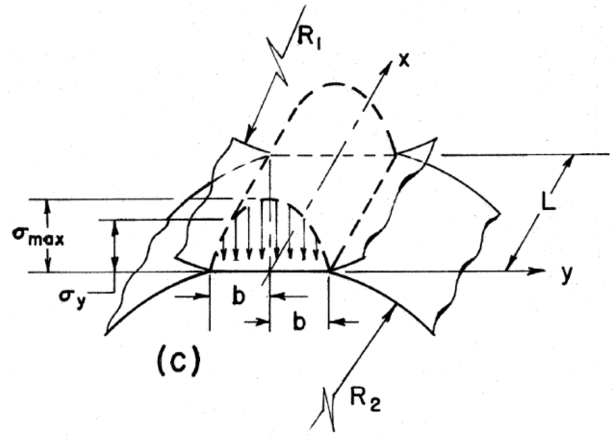

Figure 3. Contact geometry between two parallel cylinders.

Line contact occurs when two cylinders rest on each other with their axes parallel, cf. Fig. 3, and when a cylinder rests on a plane. As the two cylinders are pressed together along their axes , the resulting pressure area is a narrow "rectangle " of width 2*b* and length *L* (assuming no taper in the cylinders). In other words , the area of contact is an elongated ellipse with the major axis of the ellipse equal to *L* and the eccentricity approaching unity. Without going into details, due to limited space, the total deformation of a pair of cylinders with their axes parallel (after some lengthy algebra) can be obtained as

$$\delta = 2(\lambda_1 + \lambda_2)\frac{P}{L}\left(\ln L - \ln b + \frac{1 + \ln 4}{2}\right), \tag{6}$$

where *b* is given by

$$b^2 = \frac{4(\lambda_1+\lambda_2)P\,R_1R_2}{L(R_1+R_2)} \tag{7}$$

and where

$$\lambda_i = \frac{1-\nu_i^2}{\pi\,E_i}\,. \tag{8}$$

The Eq. 6 (together with Eq. 7) is simplified to the following expression for a cylinder on a flat surface

$$\delta = 2(\lambda_1+\lambda_2)\frac{P}{L}\left(\ln\left(\frac{L^3}{4(\lambda_1+\lambda_2)PR}\right)+(1+\ln 4)\right). \tag{9}$$

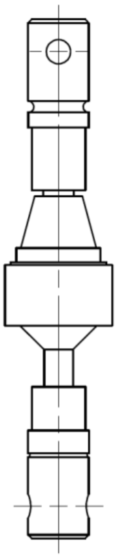

Figure 4. Self-adjusting compaction rig.

The main result of the above derivations is the observation that the relation between the deformation and the applied force is nearly linear for moderate loadings such as the expected for typical manufacturing processes of composite material. This is evident when representative material data for carbon fibres and steel plate are used in Eqs. 6 and 9: CFs transverse stiffness of $E_T \approx 15\cdot 10^9$ Pa and Poisons ratio of $\nu_T \approx 0.35$, CFs diameter of $R_{fib} \approx 3\cdot 10^{-6}$ m, together with data for steel. This leads to a relation between deformation and force in terms of

$$\delta = P\left(3.22\cdot 10^{-7} - 1.17\cdot 10^{-8}\ln P\right) \text{ or } P \approx 3.1\cdot 10^6\,\delta \tag{10a}$$

for contact between two fibres, and as

$$\delta = P\left(1.73\cdot 10^{-7} - 6.29\cdot 10^{-9}\ln P\right) \text{ or } P \approx 5.8\cdot 10^{6}\delta \tag{10b}$$

for contact between a fibre and steel surface. Of course, the above derivations are for a single fibre only and needs to be further revised for multiple fibres and fibre assemblies. However, one can still conclude that compressibility of parallel fibre assemblies is (nearly) linear with respect to the applied force (following from a simple assumption of P in Eq. 10a,b being a sum of individual fibre forces). This is in contrast to the commonly observed behaviour in standard reinforcement where the behaviour is expected to be power law, cf. Eqs. 1 and 2.

2. Experimental method and results

Since compressive testing of extremely thin fabric (in the case of the studied TeXtreme® UD spread-tow band approximately 0.05 mm) requires an terrific parallelism of the testing equipment, we decided to develop an self-adjusting test rig where the adjustment is performed using a ball-joint, cf. Fig. 4. The test rig was machined from a single piece of steel and consist of machine attachment, testing area and a ball-joint used for the parallelism adjustment. Machine compliance was subtracted from the experimental force-deflection data by assuming a spring model. The tested material was TeXtreme® UD spread-tow bands from Oxeon consisting of TR50S 15k CF fibres. A typical force-deflection data for the TeXtreme® UD spread-tow, after correcting for machine compliance, results in an almost linear relation between Force and Deflection given as $P \approx 186\cdot 10^{3}\delta$.

3. Concluding remarks

In this contribution we have studied the constitutive behaviour of nearly parallel fibre assemblies in the form of TeXtreme® UD spread-tow bands from Oxeon. The results indicate that the compressive behaviour of the nearly parallel fibres should obey a linear relation, see Eqs. 10a and b, which is in contrast to the standard textile reinforcements that obey power law type of behaviour. This theoretical investigation is further supported by our initial experimental data which shows almost linear behaviour. Compared to the theoretical investigation, the experimental data shows however somehow higher compliance. This could be due to various phenomena such as, amongst other; limited fibre parallelism resulting in partial or elliptical-type of contact, non-homogeneous fibre distribution and theoretical development performed for a single fibre only. This needs to be further investigated into in the near future.

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