MICROMECHANICAL ANALYSIS OF WOVEN COMPOSITES USING VARIATIONAL ASYMPTOTIC METHOD

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Keywords: Woven Composites, Variational Asymptotic Method, Homogenization, Voxel-based meshing

Abstract

The aim of this work is to develop an efficient micromechanical model to obtain the mechanical properties of woven textile composites. In this model, we assume the configuration of the woven composite to be represented by an equivalent mosaic structure. The equivalent mechanical properties of woven composites are obtained by the process of homogenization based on the micromechanics of the constituent material phases. Variational asymptotic method (VAM) is used as the mathematical framework for the homogenization procedure. The geometry of the repeating unit cell (RUC) belonging to 3D woven composite is discretized into uniform grids/mosaics using automated voxel technique. VAM is used to obtain the discrete governing equations valid on each mosaic substructure of the 3D woven composite. Subsequently, properties for each mosaic is determined based on the element of the geometry it holds within itself. The homogenization procedure is applied on a 3D woven composite geometry for which the experimentally tested properties are available in literature. The effects of grid refinement on the numerical results obtained are evaluated against the experimentally tested values. It was found that refining the geometry captures the fiber undulations and leads to the convergence of the numerical results towards the experimentally determined mechanical properties.

1. Introduction

Structural materials like unidirectional composites have been extensively used in engineering applications over the last decade owing to their high specific stiffness. More recently, they are being replaced by textile composites in varied industries as structural materials. This is due to the enhanced stability of the textile composites in their warp and fill directions, balanced in-plane properties, good impact resistance, etc. These textile composites come in woven, braided, knitted and stitched forms. Their complex material architecture combined with other factors that affect the thermomechanical properties of these materials make their characterization very difficult and expensive. Additionally, analysis of structures made of woven composites also becomes difficult because of the material intricacies at the microlevel. Addressing these challenges using traditional finite element method (FEM) based models that require careful and detailed meshing of the structure, including its microlevel architecture details, are very time consuming and computationally not viable. This has led to the development of micromechanics models that can give homogenized material properties, which can be used in structural level analysis. The global structural analysis results can then be feedback into the micomechanics model to determine the local fields of interest. This capability of micromechanics models is especially important for the prediction of onset and evolution of damages in structures.

Central to all micromechanics models is the identification of the representative volume element (RVE) or repeating unit cell (RUC), depending on the heterogeneous material of interest. Over the years, numerous approaches have been proposed for the homogenization of RVEs/RUCs. A comprehensive review of the micromechanics models developed before 1985 can be found in [1]. Recent developments in micromechanics modeling are elucidated elaborately in [2]. In general, the micromechanics modeling strategies can be categorized as: (a) self-consistent models; (b) variational approach based models ; (c) third-order bounded models; (d) method of cells model; (e) recursive cell method models; (f) mathematical homogenization models; and (g) FEM based models. A recent entrant into this category is the variational asymptotic method unit cell homogenization (VAMUCH) technique. VAM was first used in the homogenization of isotropic materials with cavities [3]. This was extended by Yu and Tang in [4] to develop the VAMUCH model to predict the homogenized effective properties of periodically heterogeneous materials. Here, the periodicity was considered as a small parameter and was used in expanding the energy functional asymptotically to obtain a variational statement for the unit cell. The minimization of this statement yielded the required governing equation and boundary conditions. The numerical implementation was done using FEM which was effectively used to solve sample problems. Later, VAMUCH was used in [5, 6, 7] to predict the effective thermoelastic, electromagnetoelastic and piezoelectric properties. These micromechanics models were developed by the constrained minimization technique, beginning with the Helmholtz free energy, total electromagnetic enthalpy and total electric enthalpy, respectively. VAMUCH has also been used to optimize the periodic microsructure of the material in order to achieve the prescribed effective properties [8].

An overview of the micromechanics techniques used in the homogenization of textile composites is given in [9]. In general, homogenization of textile composites has been attempted using analytical [10] and numerical methods [11, 12]. However, analytical techniques become very cumbersome when the material architecture is complicated and are not capable of accurately predicting the local stress and strain fields. They are, however, simple and easy to implement. Numerical methods are predominantly based on FEM, see reference [13]. FEM based homogenization techniques have been successfully used to determine the effective properties and, to some extent, the local stress and strain fields in textile composites [14]. Most numerical techniques require refined and carefull meshing of the unit cells to accurately predict the local stress and strain fields. For complex material architectures this become very time consuming, both from the point of modeling and computation. The process of meshing can be overcome by adopting a voxel based technique to autmoate the grid generation process [12]. Computational efficiency can be improved by adopting VAMUCH framework.

This work is an attempt to develop a voxel based VAMUCH framework to homogenize textile composites. The paper discusses the VAMUCH model followed by the voxel based meshing approach. Finally, the application of the model to homogenize textile composites and model validation is presented.

2. VAM based unit cell homogenization of periodically heterogeneous materials

In this section a brief description of the VAMUCH model is given. The framework is based on the model described in [4, 15]. Within this framework, periodicity of the heterogeneous material is considered as a small parameter and a variational statement of the unit cell is formulated through an asymptotic expansion of the energy functional. The variational statement of the energy functional is solved using VAM to determine the relation between local and global fields. FEM is used as the numerical tool to solve the problem. The methodology allows the determination of the effective material properties without the use of multiple loadings. Further, due to the variational structure of the problem the periodic boundary conditions are naturally derived during the energy minimization process.

Following the steps described in [4, 15], the governing equations for the unit cell, which are mosaic cells (see Fig. 1) are derived as,

$$\frac{\partial}{\partial y_l} C_{ijkl}(\bar{\epsilon_{ij}} + \chi_{(i|j)}) = 0 \quad in \quad \Omega \tag{1}$$

where, Ω is the domain; x_1, x_2 and x_3 are used to describe coordinate system of the macroscopic structure and y_1, y_2 & y_3 are used to describe the unit cell. The dimensions of the unit cell along x_1, x_2 and x_3 directions are denoted by d_1, d_2 & d_3, respectively. The material constitutive tensor terms are denoted by C_{ijkl}; $\bar{\epsilon_{ij}}$ are the average strain terms in the unit cell and $\chi_{(i|j)}$ are the strain terms due to the fluctuation function.

The periodic boundary conditions for fluctuation functions are obtained as:

$$\chi_i(\mathtt{x}; d_1/2, y_2, y_3) = \chi_i(\mathtt{x}; -d_1/2, y_2, y_3) \tag{2}$$

$$\chi_i(\mathtt{x}; y_1, d_2/2, y_3) = \chi_i(\mathtt{x}; y_1, -d_2/2, y_3) \tag{3}$$

$$\chi_i(\mathtt{x}; y_1, y_2, d_3/2) = \chi_i(\mathtt{x}; y_1, y_2, -d_3/2) \tag{4}$$

The periodic boundary conditions for the local stress are obtained as:

$$C_{ijkl}(\bar{\epsilon_{ij}} + \chi_{(i|j)}) \mid_{y_1=d_1/2} = C_{ijkl}(\bar{\epsilon_{ij}} + \chi_{(i|j)}) \mid_{y_1=-d_1/2} \tag{5}$$

$$C_{ijkl}(\bar{\epsilon_{ij}} + \chi_{(i|j)}) \mid_{y_2=d_2/2} = C_{ijkl}(\bar{\epsilon_{ij}} + \chi_{(i|j)}) \mid_{y_2=-d_2/2} \tag{6}$$

$$C_{ijkl}(\bar{\epsilon_{ij}} + \chi_{(i|j)}) \mid_{y_3=d_3/2} = C_{ijkl}(\bar{\epsilon_{ij}} + \chi_{(i|j)}) \mid_{y_3=-d_3/2} \tag{7}$$

also,

$$< \chi_i >= 0 \tag{8}$$

It may be noted that each of the layer/mosaic has a set of three differential equation to be satisfied, which is stated below using Voigt's contracted notation.

$$C_{11}(\frac{d^2\chi_1}{dy_1^2}) + C_{66}(\frac{d^2\chi_1}{dy_2^2}) + C_{55}(\frac{d^2\chi_1}{dy_3^2}) + (C_{21} + C_{66})(\frac{d^2\chi_2}{dy_1 dy_2}) + (C_{31} + C_{55})(\frac{d^2\chi_3}{dy_1 dy_3}) = 0 \tag{9}$$

$$C_{66}(\frac{d^2\chi_2}{dy_1^2}) + C_{22}(\frac{d^2\chi_2}{dy_2^2}) + C_{44}(\frac{d^2\chi_2}{dy_3^2}) + (C_{23} + C_{44})(\frac{d^2\chi_3}{dy_2 dy_3}) + (C_{12} + C_{66})(\frac{d^2\chi_1}{dy_1 dy_2}) = 0 \tag{10}$$

$$C_{55}(\frac{d^2\chi_3}{dy_1^2}) + C_{44}(\frac{d^2\chi_3}{dy_2^2}) + C_{33}(\frac{d^2\chi_3}{dy_3^2}) + (C_{13}+C_{55})(\frac{d^2\chi_1}{dy_1 dy_3}) + (C_{23}+C_{44})(\frac{d^2\chi_2}{dy_2 dy_3}) = 0 \quad (11)$$

For 3D mosaic models solving above set of equations becomes difficult owing to the orthotropic properties which make the coefficients vary from mosaic to mosaic in three dimensions. Hence, numerical method is adopted to solve the problem. In order to solve the coupled elliptic partial differential equations with variable coefficients, FEM is used. The layer equations within an element has constant coefficients and the elements can be assembled with periodic boundary conditions applied at the $\mathbf{y_1} - \mathbf{y_3}$ and $\mathbf{y_2} - \mathbf{y_3}$ plane of the model. It may also be noted that the mosaic model does not capture the undulations of the fiber yarn at the level of discretization used, which can result in larger values of homogenized properties along the $\mathbf{y_1} - \mathbf{y_2}$ plane and reduced values along the $\mathbf{y_3}$ direction properties. In order to avoid this problem the mosaic model should be refined sufficiently.

We write the fluctuation function χ in terms of the shape function, $[N]$, and discrete nodal variables, d, corresponding to χ as,

$$\{\chi\} = [N].\{d\} \quad (12)$$

The average strains are represented as,

$$\bar{\epsilon} = \begin{bmatrix} \bar{\epsilon}_{11} & 2\bar{\epsilon}_{22} & 2\bar{\epsilon}_{33} & 2\bar{\epsilon}_{23} & 2\bar{\epsilon}_{13} & 2\bar{\epsilon}_{12} \end{bmatrix}^T \quad (13)$$

The strain corresponding to the fluctuating functions are written as,

$$\begin{Bmatrix} \frac{\partial\chi_1}{\partial y_1} \\ \frac{\partial\chi_2}{\partial y_2} \\ \frac{\partial\chi_3}{\partial y_3} \\ \frac{\partial\chi_2}{\partial y_3} + \frac{\partial\chi_3}{\partial y_2} \\ \frac{\partial\chi_1}{\partial y_3} + \frac{\partial\chi_3}{\partial y_1} \\ \frac{\partial\chi_1}{\partial y_2} + \frac{\partial\chi_2}{\partial y_1} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial y_1} & 0 & 0 \\ 0 & \frac{\partial}{\partial y_2} & 0 \\ 0 & 0 & \frac{\partial}{\partial y_3} \\ 0 & \frac{\partial}{\partial y_3} & \frac{\partial}{\partial y_2} \\ \frac{\partial}{\partial y_3} & 0 & \frac{\partial}{\partial y_1} \\ \frac{\partial}{\partial y_2} & \frac{\partial}{\partial y_1} & 0 \end{bmatrix} \begin{Bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{Bmatrix} = \Gamma_h \chi \quad (14)$$

The total strain energy is then re-written in terms of the average strains and the fluction functions as,

$$\pi_\Omega = \frac{1}{2\Omega} \int_\Omega C_{ijkl}[\bar{\epsilon_{ij}} + \chi_{(i|j)}][\bar{\epsilon_{kl}} + \chi_{(k|l)}]d\Omega \quad (15)$$

The strain energy form in Eq. 16 is then written in terms of the shape functions and the nodal variables by substituting Eqs. 12, 13 and 14 in Eq.15 to obtain,

$$\pi_\Omega = \frac{1}{2\Omega}\left(\chi^T E \chi + 2\chi^T D_{he}\bar{\epsilon} + \bar{\epsilon}^T D_{ee}\bar{\epsilon}\right) \quad (16)$$

where, E, D_{he} and D_{ee} are given by

$$E = \int_\Omega (\Gamma_h N)^T D (\Gamma_h N) d\Omega \quad (17)$$

$$D_{he} = \int_\Omega (\Gamma_h N)^T D d\Omega \quad (18)$$

	Geometry Parameters			Material Properties				
	a	b	h	E_L	E_T	G_{LT}	ν_{LT}	ν_{TT}
Units	mm	mm	mm	GPa	GPa	GPa	-	-
Values	2.0	2.0	0.196	137.3	10.79	5.394	0.26	0.46

Table 1. Woven Composite : Geometric Parameters & Material Properties

$$D_{ee} = \int_{\Omega} D d\Omega \tag{19}$$

The strain energy form given in Eq. 16 is minimized to get,

$$E\chi = -D_{he}\bar{\epsilon} \tag{20}$$

and

$$\chi = \chi_0 \bar{\epsilon} \tag{21}$$

Based on the above mathematical statements the final energy stored in the unit cell is given as,

$$\pi_{\Omega} = \frac{1}{2\Omega}\bar{\epsilon}^T \left(\chi_0^T D_{he} + D_{ee}\right)\bar{\epsilon} \equiv \frac{1}{2}\bar{\epsilon}^T \left(\bar{D}\right)\bar{\epsilon} \tag{22}$$

where, $\bar{D}$ is the homogenized effective property.

3. Results and discussion

3.1. Geometry

As a verification problem, the experimentally tested woven composite from [16] made of carbon fibre reinforced plastic(CFRP) is considered. The geometric parameters and the material properties from [16] is reported in Table 1. The RUC for the woven composites and its idealization using a mosaic model is shown in Fig. 1. The geometeric parameters which govern the effective properties are indicated as a, b and h. The cells have been numbered as 1, 2, 3, 4 for top layer in anticlockwise direction when viewed from +ve $\mathbf{y_3}$ axis. The bottom layer is similarly numbered as 5, 6, 7 and 8. The mosaic model in exploded view (see Fig. 1) shows the cell-1 separated from the rest of the model. It can be seen that each cell is a transversely isotropic fibre bundle element; where, the direction **L** is along the axial/longitudinal direction and the transverse directions are denoted by **T**. It can be noted that the $\mathbf{y_3}$ direction properties for all the cells are the transverse direction (T) properties. However, the fluctuation functions along $\mathbf{y_1}$ and $\mathbf{y_2}$ directions does vary with $\mathbf{y_3}$ coordinates (note the $\mathbf{x_1}$ direction material variation along $\mathbf{x_3}$ direction for cells 1 & 5, 2 & 6 etc).

3.2. Voxelisation of Geometry

In order to keep the FE discretization independent of the geometry of the textile RUC, voxelisation is preferred over regular meshing in general purpose FEM pre-processors. This procedure of voxelisation has following benefits.

- The procedure of discretization is simplified even if the geometry is complex.

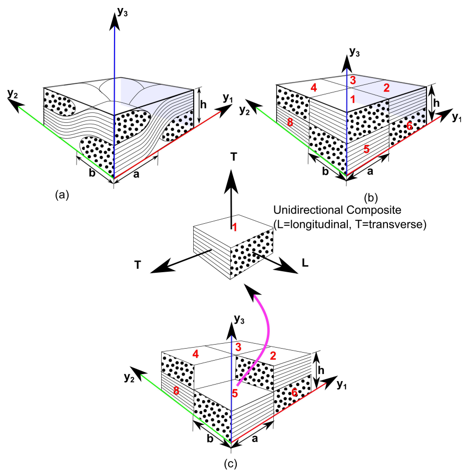

Figure 1. Woven Composite: 3D RUC

- The discretization procedure can be automated.
- Mesh refinement is easier.
- Multi level analysis becomes easier with sub voxelisation.

In the macro voxelisation, the RUC is divided into voxels. The macro-voxelised RUC is homogenized using VAM, hence this method provides recovery relation at the level of individual voxels. In order to get the homogenized properties for each voxel, individual voxels are further voxelised; this is referred to as micro voxelisation. At the micro level the homogenization procedure can be simple stiffness averaging method or based on VAM.

3.3. Homogenization

The voxel model used in the analysis is shown in Fig. 2. A convergence study was done on the homogenized properties and the converged results are compared with [16]. Comparative results are tabulated in Table 2. The results compare closely with the tested and predicted properties from presented in [16]. It may be noted that in [16] the undulations were modelled using analytical functions using a different homogenization procedure from the one adopted in this study. The results suggest that VAM in conjunction with mosaic based voxel model is an effective yet simplified approach for the homogenization of complex 3D textile RUC.

A study on the variation of Young's modulus, shear modulus and the Poisson's ratio with the geometric parameters, h/a, was also carried out. It is noted from the investigation that as the geometric parameter h/a varies from 0.1 to 1.0 (fibre bundle cross-section area is also increasing), the Young's modulus in the thickness direction shows only 6% drop where as the in-plane Young's modulus shows a significant drop of 37%. It may also be noted that as the geometric parameter h/a varies from 0.1 to 1.0, the out-of-plane shear modulus increases from a value of

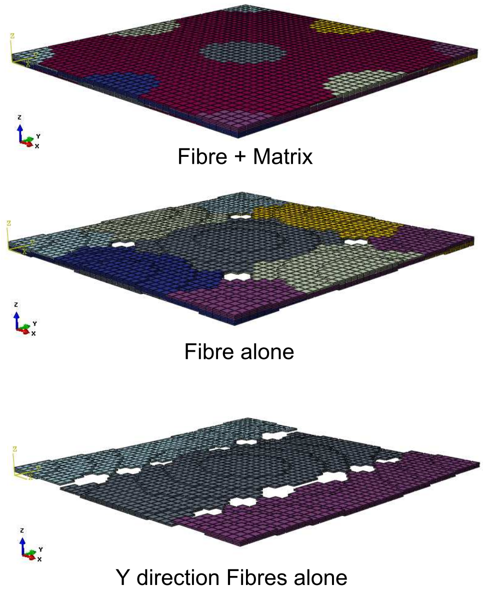

Figure 2. Woven Composite:Voxel Model of 3D Weave

values	E_x(GPa)	E_y(GPa)	G_{xy}(GPa)	ν_{xy}
Test	48.3	48.3	5.41	0.062
Prediction [16]	46.35	46.35	3.83	0.0538
Present	46.76	46.76	3.89	0.0558

Table 2. Woven Composite: Tested Properties & Analytical Results vs Voxel Based VAM

3.3 GPa to 3.9 GPa and comes closer to the in-plane shear modulus (G_{xy}). Similar trends were observed from the variation of Poisson's ratio with respect to the volume fraction.

4. Conclusion

A voxel based variational asymptotic method unit cell homogenization procedure is developed. The framework is capable of handling complex material architectures by avoiding the manual mesh generation process. Voxel based method is automatic and is capable of accurately describing the local heterogenity in the material without the need for fine grids; thus imrpoving the computational efficiency of the model. The framework was used to determine the effective mechanical properties of 3D woven composites. Model predictions matched well with the experimental results and confirmed the veracity of the framework.

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