ANALYSIS OF THE DYNAMIC FLEXURAL BEHAVIOUR OF SANDWICH BEAMS

I. Ivañez[a*], E. Barbero[a], S. Sanchez-Saez[a]

[a] *Department of Continuum Mechanics and Structural Analysis, University Carlos III of Madrid,*
Avda. De la Universidad 30, 28911, Leganes, Madrid, Spain
**idel@ing.uc3m.es*

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Abstract

In this work, the low-velocity impact response of sandwich beams, with carbon fibre/epoxy skins and aluminium honeycomb core, was studied by developing a non-linear two-degree-of-freedom mass-spring model. In order to determine the parameters which control the global impact response of the structure, the model has been formulated in terms of dimensionless groups. The groups with more influence on the analysis are the dimensionless global stiffness, the dimensionless non-linear relationship between the indentation force and the local displacement, and the dimensionless impact velocity. It was revealed that the influence of the dimensionless effective mass of the sandwich beam and the dimensionless effective mass of the upper skin is not negligible on the dynamic response.

1. Introduction

Composite sandwich structures achieve the same structural performance as traditional materials with less weight, and are used in a wide range of engineering applications. A structurally efficient design must withstand both normal and accidental load conditions, thus it is important to study the response of composite sandwich structures subjected to such loads. One area of potential concern relates to their relatively poor resistance to low-velocity impact loading (i.e. drop of tools during manufacturing and/or maintenance operations), as the resulting damage can be barely visible, but it can reduce significantly their strength and stiffness [1,2].

One of the main approaches to gain knowledge about the behaviour of composite sandwich structures subjected to low-velocity impact is the experimental testing; however, a broad testing programme has to be undertaken to set up a reliable experimental response of these structures, whereas modelling is more flexible and less costly. The finite-element method is often used to study the dynamic response of composite sandwich beams [3,4]. However, setting up a finite-element model is complicated and usually calculation times are high. In this context, analytical models can be developed to quickly predict the dynamic global response of composite sandwich structures [5].

In this work, the low-velocity impact response of sandwich beams, with carbon fibre reinforced epoxy skins and aluminium honeycomb core, was studied by developing an analytical model. The proposed model is based on Hoo Fatt and Park work [6], although it considers the effect of the non-linear relationship between the indentation force and the local displacement of the upper composite skin. The effects of the inertial masses corresponding to the composite upper skin and the whole sandwich structure are also included.

The model permits to evaluate the behaviour of sandwich beams subjected to dynamic three-point bending if the damaged area on the upper skin does not affect significantly to the global stiffness of the beam. In addition, the model has been formulated in terms of dimensionless parameters, in order to determine the key dimensionless groups which control the dynamic response of the sandwich beams. The predicted results were validated by comparing with experimental tests carried out in a drop-weight tower.

2. Model description

The low-velocity impact of the composite sandwich beams has been modelled as a discrete system of two-degrees-of-freedom (Figure 1).

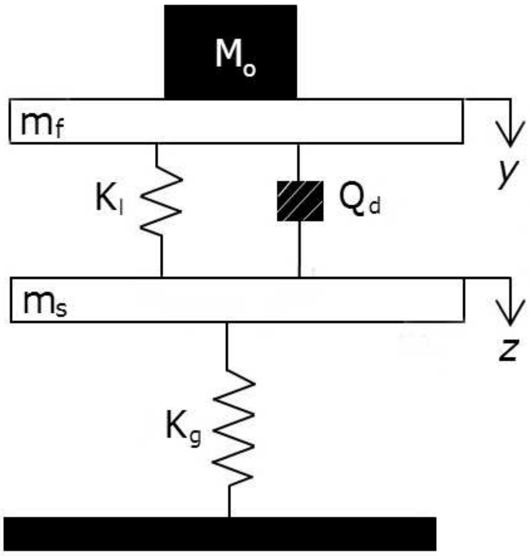

Figure 1. Low-velocity impact two-degree-of-freedom mass-spring model.

In Figure 1, M_o is the mass of the striker, whereas the inertia of the upper skin and the inertia of the sandwich beam are represented by two effective masses m_f and m_s in the analysis. The global stiffness of the beam is represented as a linear spring K_g and the core crushing load is symbolised by Q_d. The local contact between the upper skin and the striker is represented by a non-linear local spring defined as K_l. The local displacement of the upper skin and the global displacement of the whole sandwich structure are represented by *y* and *z*, respectively.

The dimensionally independent units used in the dynamic response analysis are: the striker mass, the thickness of the upper skin, and the thickness of the sandwich beam. The time variable and the contact force are dimensionless through use of a characteristic time and a characteristic force, respectively. The non-dimensional equations of motion formulated as a function of the resulting dimensionless groups, symbolised as Π_i, are given by:

$$\begin{gathered}(1+\Pi_1)\cdot\left(\ddot{\hat{z}}+\Pi_3\cdot\ddot{\hat{y}}\right)+\Pi_6\cdot\hat{y}^{3/4}+2\cdot\Pi_5=0\\ \Pi_6\cdot\hat{y}^{3/4}+2\cdot\Pi_5=\Pi_2\cdot\ddot{\hat{z}}+\Pi_4\cdot\hat{z}\end{gathered}\tag{1}$$

The resulting system shown in Eq. (1) is non-linear and it cannot be solved analytically, therefore it is required to use numerical methods. In this work, the Runge-Kutta method is used to solve the equations of motion.

Each dimensionless group has a physical meaning pertinent to the problem. The group Π_1 represents the relationship between the effective mass of the upper skin and the mass of the striker, whereas Π_2 represents the relationship between the effective mass of the sandwich beam and the mass of the striker. Π_3 is the dimensionless thickness, which connects the thickness of the composite upper skin with the thickness of the sandwich beam. The group Π_4 is defined as the dimensionless global stiffness, which relates the equivalent bending stiffness with the equivalent shear stiffness of the sandwich beam. Both equivalent stiffnesses are calculated using the classical Strength of Materials Theory. Π_5 represents the relationship between the dynamic core crushing load, and the equivalent bending stiffness of the sandwich beam. Π_6 represents a dimensionless parameter which connects the non-linear relationship between the indentation force and the local displacement with the equivalent bending stiffness of the beam.

In addition, the initial conditions for the equation of motion are also formulated in dimensionless form, as shown in Eq (2). The only non-zero initial condition is defined as a dimensionless initial velocity (Π_7). This group links the initial velocity of the striker with the first mode of vibration of a simply-supported beam:

$$\hat{z}(0)=0 \qquad \dot{\hat{z}}(0)=\Pi_7 \qquad \hat{y}(0)=0 \qquad \dot{\hat{y}}(0)=0 \tag{2}$$

2.1. Dimensionless model validation

Composite sandwich beams with honeycomb core (50 mm in width and 24 mm in thickness) were tested in an instrumented drop-weight tower at different impact velocities. The striker mass and the nose radius were 3.96 kg and 20 mm, respectively. The low-velocity impact tests were recorded by a high-speed camera, and contact force versus time curves were provided by the test machine.

The core was made of 3003 alloy hexagonal aluminium honeycomb, with a cell-size of 4.8 mm, and a core height of 20 mm. The sandwich skins were made of plain woven laminate of carbon fibre and epoxy resin (AS4-8552), with a thickness of 2 mm. The main characteristics of the composite skins are shown in Table 1.

Property	**Composite Skins**
E_1	68.9 GPa
E_2	68.9 GPa
G_{12}	9 GPa
ν_{21}	0.22
ρ_f	1600 kg/m^3

Table 1. Main properties of the composite skins.

An example of the analytical and experimental contact force-time curves obtained is presented in Figure 2. The analytical and experimental curves show a similar trend.

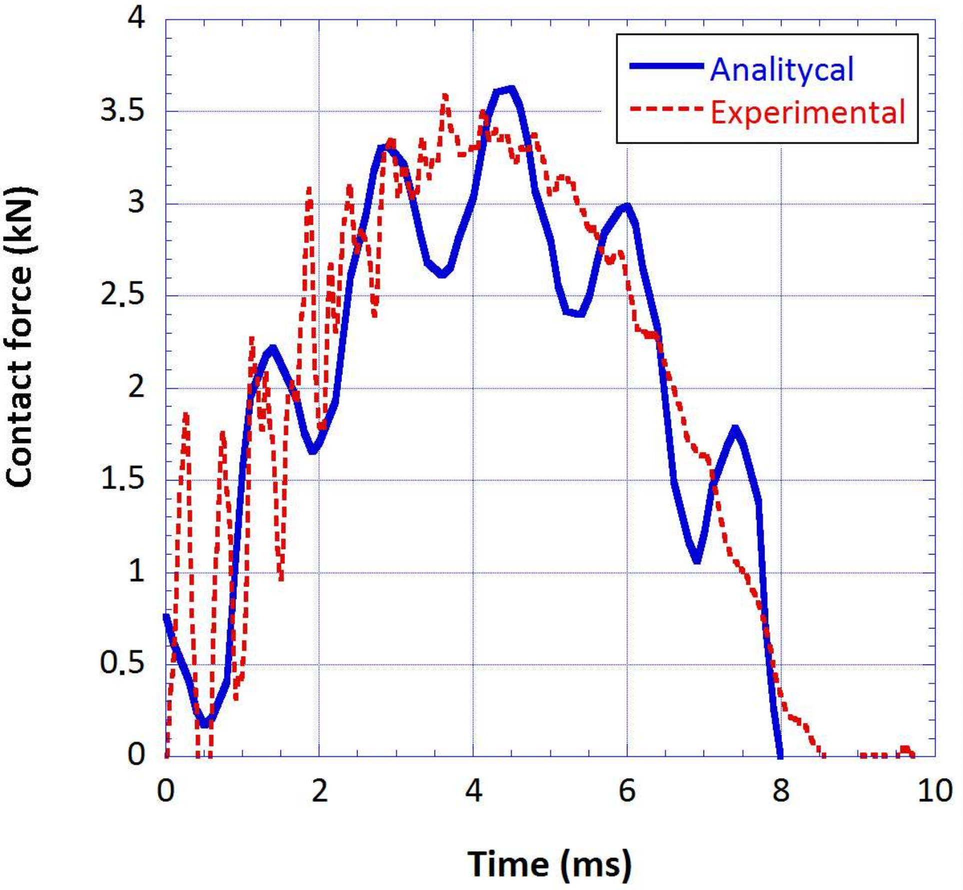

Figure 2. Experimental and analytical contact force versus time curve: impact energy of 13.6 J.

The validation was carried out for impact energies for which no visible failure of the composite upper skin occurs (between 8.2 J and 17.8 J). The impact energy was calculated using the initial velocity of the striker, which was measured with the high-speed camera recordings. The difference between the predicted results and the experimental measurements for three impact energies is presented in Table 2.

Impact energy (J)	Maximum contact force difference (%)	Maximum contact time difference (%)
8.2	3.5	2.0
13.6	7.8	5.2
15.2	3.3	4.3

Table 2. Differences between the experimental data and the non-linear model results.

The experimental and the predicted maximum contact force differences is less than 8% and in terms of contact time, the difference is less than 5.5%; thus, the comparison shows that the analytical results are within a reasonable range of prediction.

3. Results

The non-linear model can reproduce the dynamic bending response of composite sandwich beams subjected to low-velocity impact when the damage on the upper skin is not extensive. This type of damage is barely visible and potentially dangerous for composite sandwich structures. In this section, a dimensional analysis in terms of maximum contact force and contact time is performed by varying every dimensionless group presented in Eq. (1).

The upper and lower limits of the studied variation range for the dimensionless groups are between an order of magnitude above and below the validation experimental value. For the sake of brevity, the results of three out of seven dimensionless groups are presented: Π_1, Π_2 and Π_4.

Π_1 (Figure 3) and Π_2 (Figure 4) are the dimensionless effective masses, corresponding to the upper skin and the sandwich beam, respectively. The variations observed in the studied ranges are above 10%, in terms of maximum contact force and maximum contact time, thus their influence on the dynamics of the system should not be considered negligible.

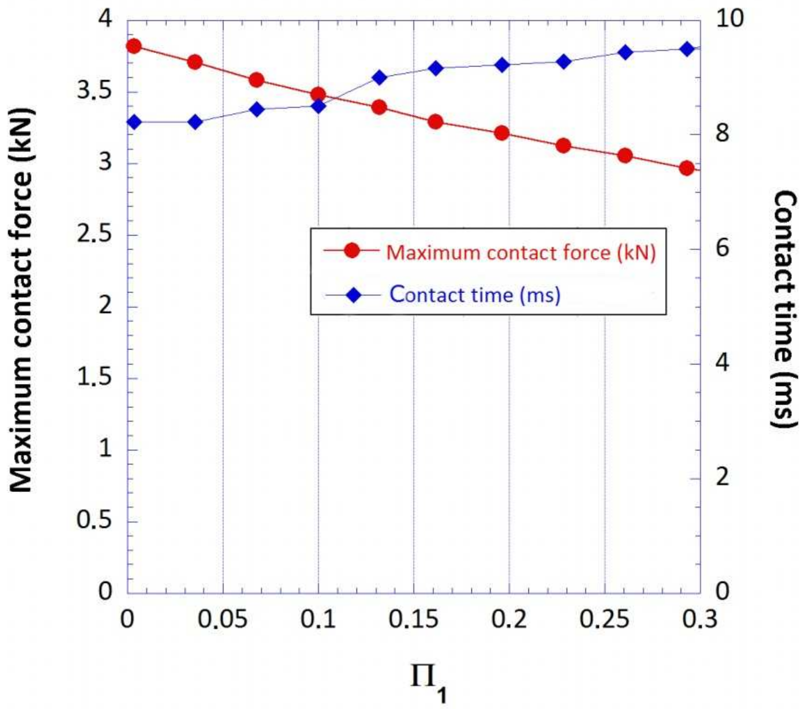

Figure 3. Maximum contact force and contact time for Π_1 variation.

Between the selected limits, the variation of the maximum contact force as a function of Π_1, and Π_2 gives a straight line, which depicts a linear trend in the predicted data. The contact time has a more irregular trend, although in general increasing maximum contact force, results in decreasing contact time.

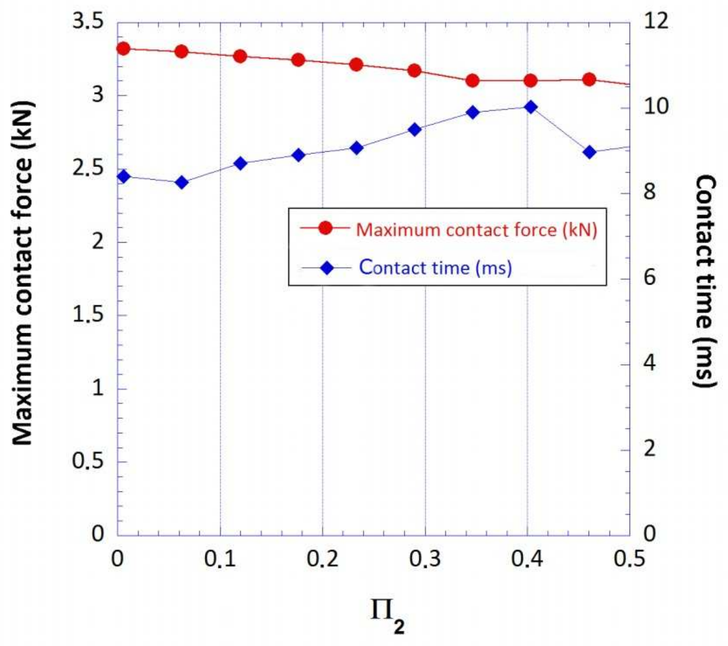

Figure 4. Maximum contact force and contact time for Π_2 variation.

The dimensionless groups with more influence within the studied range on the dynamic response are Π_4, Π_6 and Π_7. These groups correspond to the dimensionless global stiffness, the dimensionless local relationship between the indentation force and the local displacement, and the dimensionless impact velocity, respectively.

Decreasing the dimensionless global stiffness Π_4 of the structure reduces the maximum contact force and increases the contact time (Figure 5); however, the results showed that this group is not significant in the dynamic response of the sandwich beam for values above approximately the unity. With the increasing dimensionless global stiffness of the structure, the response tends to stabilise both in maximum contact force and contact time values.

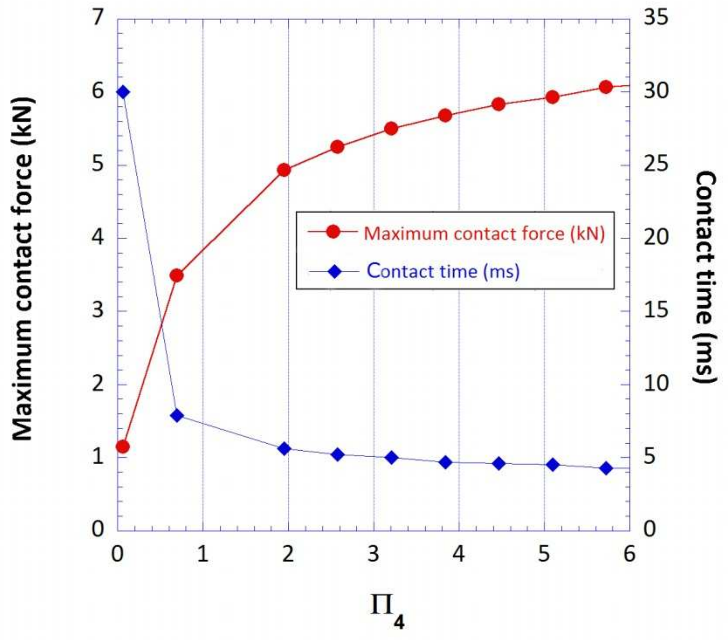

Figure 5. Maximum contact force and contact time for Π_4 variation.

The group Π_6 has a significant effect on both maximum contact force and contact time results below approximately the experimental value. Increasing this dimensionless parameter, results in a rapid decrease of the contact time, until it stabilises around a constant value. On the contrary, the maximum contact force shows a strong increase until reaching the stable value.

Π_7 shows more influence on the maximum contact force than on the contact time results. The contact time remains almost constant for the studied range. However, increasing the dimensionless initial velocity causes noticeable increase in the maximum contact force, which was adequately represented by linear regression.

Finally, an increasing dimensionless thickness Π_3 shows a slight decrease in the maximum contact force. However, the contact time shows a completely different behaviour and it remains almost constant until the dimensionless group reaches the experimental value. From this value, the contact time experiences an increasing trend.

4. Conclusions

The low-velocity impact response of composite sandwich beams, for no extensive damaged area on the upper skin, was studied by developing a non-linear two-degrees-of-freedom mass-spring model. The model predictions are in good agreement with the experimental results. The dimensionless formulation of the model allowed determining the key dimensionless groups used to evaluate the dynamic response.

In the studied range, the groups with more influence in terms of maximum contact force and contact time are: the dimensionless global stiffness, the dimensionless non-linear relationship between the indentation force and the local displacement, and the dimensionless impact velocity.

The dimensionless effective mass of the upper skin and the dimensionless effective mass of the whole sandwich beam have less influence on the system response; however, they should not be considered negligible in the analysis of the dynamic response.

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