

## FRACTURE MECHANICS ON MICROSCALE OF COMPOSITE PLYS UNDER TRANSVERSE LOADING

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### Abstract

*The energy release rate delivered during the debonding of a fiber inside a composite ply is analysed using a representative volume element. Two different fiber volume fractions are studied by a linear elastic fracture mechanical analysis by using the finite element method. The dependency of the mixed mode energy release rate on the length of the crack increment is analysed. It is shown that the same general behaviour found in case of a straight interface crack is valid also for circular cracks. As a result the discrimination of the energy release rate into mode I and mode II portions is questionable.*

### 1. Introduction

#### *1.1 Debonding of fibers in composite material*

The failure of fiber reinforced materials is initiated by failure processes taking place on the microscale. The prevailing failure mechanism is the debonding of fibers under transverse and shear loading. This so called inter fiber failure is due to the stress concentrations between the neighbouring fibers which becomes more and more dominant with growing fiber content. A variety of studies was performed in order to investigate the interfacial failure process on different levels. Experiments on single fibers loaded under different off-axis angles were performed [1-4] as well as theoretical analyses based on fracture mechanics using numerical methods such as the boundary element method or the finite element method [5-9]. The interfacial debonding is simulated by a circumferentially propagating interface crack around a single fiber. The fracture process is characterised by the energy release rate during crack propagation. In particular the two different modes of the energy release rate, this is, mode I and mode II are analysed. Further studies deal with the debonding of fibers embedded in a composite material [10-11]. In the present paper the meaning of the mode I and mode II energy release rate in case of a curved interface crack is studied. In particular the dependence of the mode ratio on the length of the crack increment is inspected.

#### *1.2 Phenomena in bimaterial interfaces*

Analysing interface cracks between dissimilar media one is confronted with some phenomena unfamiliar in classical fracture mechanics of single materials. While in single material problems, e.g. for a straight crack, the external loading can be chosen such that a crack

propagation under pure mode I or pure mode II occurs, in case of an interface crack between two dissimilar media the two failure modes are necessarily coupled. This is, even in case of an external loading which causes pure mode I failure in a single material the same loading generates a simultaneous mode II part in case of an interface crack. This is for example caused by the constrained lateral contraction of the two phases in the vicinity of the interface, leading to normal tensile and compressive stresses parallel to the interface in both phases, respectively, as well as shear stresses in the vicinity of the interface. If the crack develops the two materials can release stress by moving parallel to the interface in opposite directions. This causes a mode II energy release rate as a result of the shear stresses in the interface. There is, however, a second phenomenon occurring in bimaterial fracture mechanics which seems rather uncommon. While a closed form solution for a straight crack in an unbounded single material (Griffith problem) exists, no unambiguous solution is known for a crack between two dissimilar media. The common solution exhibits oscillations of the stresses at the crack tip, even though this is limited to a very small zone [12-14]. The drawbacks of the common solution are addressed by Comninou who proposed an alternative solution, following an idea of Malyshev and Salganik [15] by assuming a small contact zone near the crack tip [16]. This solution is free of oscillations, it requires, however the numerical solution of an integral equation and accordingly cannot be regarded as a fully analytical solution. A comprehensive overview over the principal problems arising in the mathematical description of interface cracks is given by Comninou [17].

The oscillations inherent in the standard solution prevent the determination of the mode I and mode II energy release rates in terms of a limit process for the crack increment approaching zero. As a result the mode I and mode II energy release rates depend on the length of the crack increment and lose their meaning as physical terms. The consequences for the interpretation of the energy release rate results are discussed in the following. Since the analytical solution is not free of contradictions it is questionable if the loss of meaning of the mode I and mode II part of the energy release rate is an artefact of this specific solution or if it is inherent in bimaterial fracture mechanics. A possibility to check this is the use of numerical methods which are free of the respective artefacts at the crack tip. Finite element solutions however suffer from the strong dependency on the element size in zones of high stress gradients.

In this respect the dependency of the mode I and mode II part of the energy release rate was studied by comparison of finite element results with the analytical solution by Sun and Jih [18] for a straight crack between two different linear elastic materials. They concluded from their results that with decreasing crack increment length the two parts of the energy release rate become more and more equal, this is, they tend to one half of the total energy release rate. The phenomenon of the oscillating stresses near the crack tip was also addressed, among others, by Mantic and Paris [19].

## **2. Micromechanics of failure in fiber reinforced composites**

### *2.1 Representative volume element, Finite Element model*

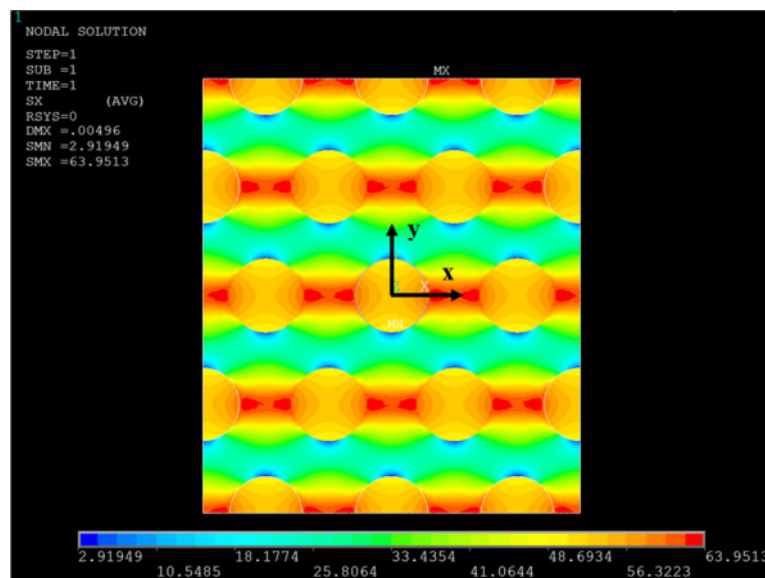
The stress field around a fiber which is surrounded by a composite material differs significantly from that of a single fiber embedded in pure matrix material. The change of the stress field caused by the neighbouring fibers results in a change of the energy released during debonding which, as a matter of fact, grows with increasing fiber content. In order to determine the influence of the fiber volume fraction the debonding process of a single fiber

inside a composite material is investigated using a representative volume element of a unidirectional composite. The composite material is comprised of glass fibers embedded in an epoxy matrix. Linear elastic behaviour is presumed for both materials. Plane strain conditions are applied because they represent a reasonable approximation of the stress/strain state within the composite away from the edges. Therefore they are appropriate for most of the composite material. The model under investigation is a representative volume element of a regular hexagonal 12-fiber array. On the edges of the model the respective symmetry boundary conditions are prescribed. The load is applied by prescribing constant displacements in x-direction on the right edge.

Material	E-Modulus [MPa]	Poisson ratio
fiber	72000	0.21
matrix	2800	0.35

**Table 1.** Elastic constants of glass fiber and epoxy matrix

In the interface between fiber and matrix the adjacent elements of fiber and matrix possess individual nodes, this is, two lines of nodes exist having identical coordinates. The bonding between fiber and matrix elements is realised by coupling the displacements of the respective nodes. During crack propagation the coupling is continuously resolved. This is an alternative method to the well known cohesive zone element technique for modelling crack propagation. On both sides of the interface contact elements are applied since the crack faces may come into contact during crack extension. The crack is assumed to develop in the interface of the central fiber starting at the point where the maximum radial tensile stresses occur, this is, at y equal to zero. The crack is assumed to propagate simultaneously clockwise and counterclockwise. The energy release rate is calculated by the classical virtual crack closure method (VCC) introduced by Rice [20].



**Figure 1.** Representative volume element of hexagonal fiber array, stresses in x-direction.

## 2.2 Stress field around the debonding fiber

The stresses in the direction of the external load, this is  $\sigma_{xx}$ , occurring in the representative volume element of an hexagonal fiber array before onset of interfacial failure are shown in

fig. 1. The fiber volume fraction is 70%. In the matrix zone between the neighbouring fibers a concentration of the  $\sigma_{xx}$  stresses occurs. This is due to the large difference in stiffness of the fibers compared to the matrix. Since fiber and matrix are arranged more or less in line basically the matrix has to carry almost all of the deformation in loading direction resulting in high matrix strains and stresses as well.

### 2.3 Energy release rate during debonding of single fiber

A typical course of the energy release rate developing in the representative volume element under transverse tension is shown in fig. 2. The crack increment is 1° in clockwise and counterclockwise direction, i.e. 2° in total. With the initiation of the crack the total energy release rate rapidly increases. This indicates a highly unstable crack propagation. The maximum of the total energy release rate is reached around a crack angle of 30°. The energy release rate decreases and shows a plateau around 120°. This is the position where the next neighbouring fibers are located. With further crack propagation the influence of the stiffening due to the neighbouring fibers is levelled off and the energy release rate decreases again and diminishes at around 270°. Interestingly the fibers located at 240° have no remarkable influence on the energy release rate. One has to consider that the course of the energy release rate beyond a crack of 270° is of no practical meaning because the crack could not propagate beyond this point. This is because the energy release rate vanishes even if the external load would be increased, accordingly the critical energy release rate could never be reached.

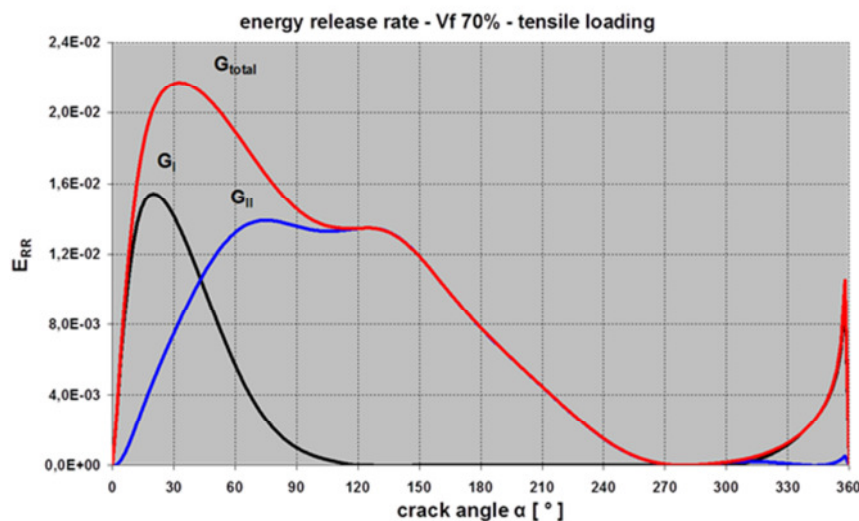
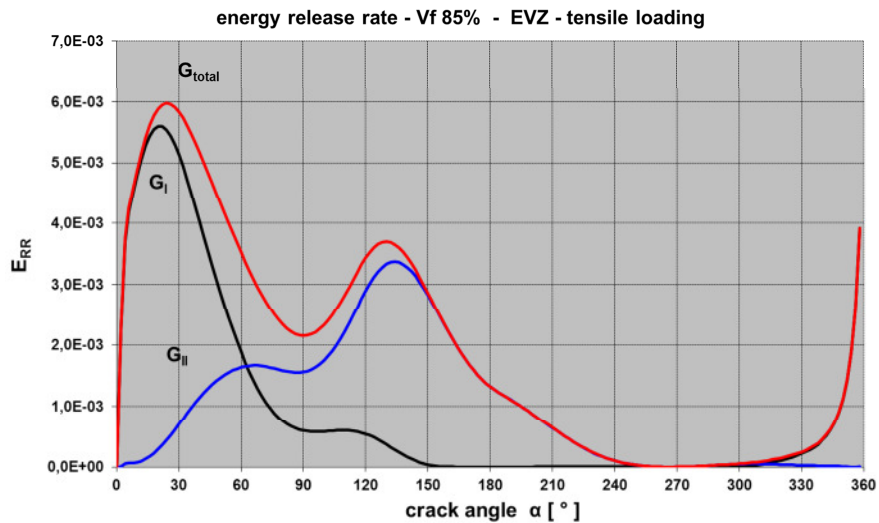


Figure 2. Energy release rate, mode I - mode II energy release rates, Vf=70%.

The mode I part of the energy release rate rises very rapidly and is dominating the initial phase of the crack (fig. 2). In this phase the external load is more or less perpendicular to the interface, this is, a typical mode I loading in the classical sense. Already at a crack extension of 10° in both directions the mode I part reaches the maximum and decreases steeply. At an angle of 120° the mode I part has vanished. This behaviour is roughly to be expected from the stress distribution before crack initiation, even though the stress field changes during crack propagation. The mode II part develops somewhat delayed and distinctly slower. A plateau forms between 70° and 130°. The further crack propagation is fully driven by shear stresses.

If the fiber volume fraction is increased up to 85% which can be regarded rather as an extreme case the dominance of the mode I energy release rate is even enlarged (fig. 3). Now the

influence of the next neighbouring fibers is stronger leading to a small plateau even for the mode I part. The maximum of the mode II energy release rate is shifted and reached at a crack angle of 130°.



**Figure 3.** Energy release rate, mode I - mode II energy release rates,  $V_f=85\%$ .

The results show that the course of the energy release rate strongly depends on the fiber volume fraction. In particular the mode I and mode II parts develop in very different ways. The question is what the calculated courses of the energy release rate tell about the interfacial failure process in a composite ply under transverse loading inside a laminate. The extremely steep increase of the energy release rate after crack initiation shows that under linear elastic conditions the debonding of a fiber would develop highly unstable. If the Griffith criterion is applied a crack would start to propagate if the energy release rate equals the critical energy release rate of the material or of the interface. Even if a short initial crack is present in the interface, e.g. due to a non perfect bonding between fiber and matrix, and crack extension would start if the critical energy release rate is met, the energy release rate developing during further crack propagation supersedes the critical energy release rate by far. This means that a surplus of energy develops resulting in a shock wave which may advance the crack also into regions with decreasing energy release rate. Experiments with single glass fibers under tension transverse to the fiber axis show that the interface crack indeed has an unstable initial phase [4]. However, the crack stops already at a crack angle of about 130°. Further crack propagation occurs only when the load is increased. The early stop of the crack reveals that a remarkable part of the released energy must be compensated by non elastic processes, this is, by viscoelastic and viscoplastic processes.

### 3. Dependency of the mode I / mode II energy release rate on the crack increment

As shown, among others, by Sun and Jih [19] the partitioning of the energy release rate into mode I and mode II part depends on the length of the crack increment. By varying the length of the increment for a given initial crack length they showed that in phases where the total energy release rate is approximately constant the total energy release rate is independent of the crack increment length. This correlates to the common solution where the total energy release rate is free of oscillations. The mixed mode ratio however changes with the length of the crack increment. While the mode I part decreases with decreasing increment length the mode II part simultaneously increases. Strictly speaking not the absolute length of the crack increment is relevant but the ratio of crack increment over the total crack length.

The failure of curved interface, e.g. in case of the debonding of a fiber, is a more complex problem because of the curvature of the crack. The curvature causes a strong variation of the ratio of normal to shear stresses around the crack tip during crack propagation. The influence of the crack increment on the mixed mode energy release rate evolving during the debonding process is studied on a 70% fiber volume fraction model composite. The length of the crack increment is varied between maximum 4° (2° in each direction) and minimum 0.2° (0.1° in each direction). This is, for a fiber diameter of 10µm the crack increment varies between 0.70µm and 0.035µm. In any case the length of the crack increment equals the length of a single element. This is, different meshes were used for the analyses. This implies the drawback that the element mesh has some influence on the results. Studies with different meshes however showed that this influence is moderate.

With decreasing crack increment the mode I energy release rate decreases distinctly (fig. 4). The maximum slightly shifts to shorter crack lengths and is decreased to about 82% in case of the shortest increment. In addition the mode I part of the energy release rate levels off earlier for shorter increments. The general course however does not change remarkably. In contradiction to the mode I part the mode II energy release rate increases with decreasing increment length (fig. 5). Again, the maximum shifts to lower crack lengths.

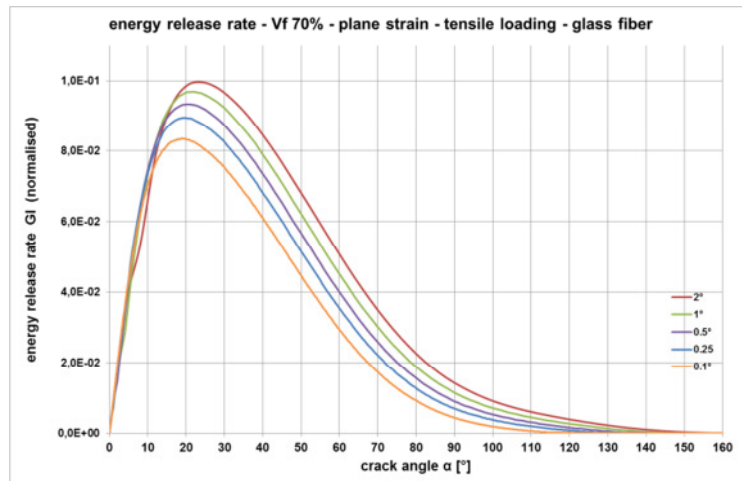


Figure 4. Mode I energy release rate – variation of increment length, Vf=70%.

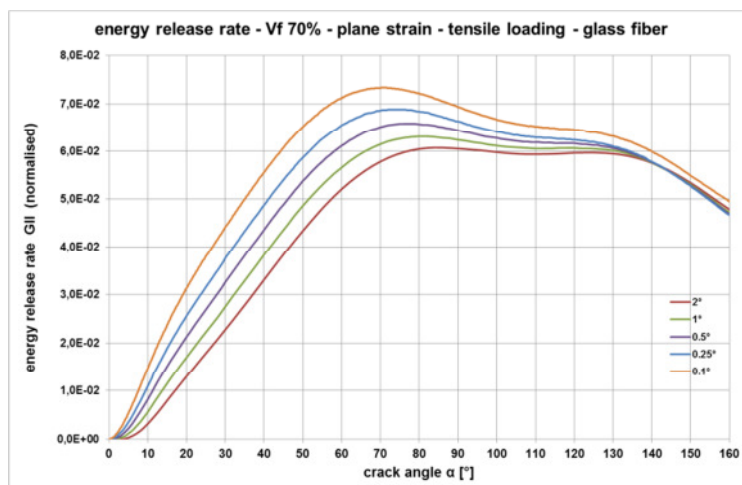
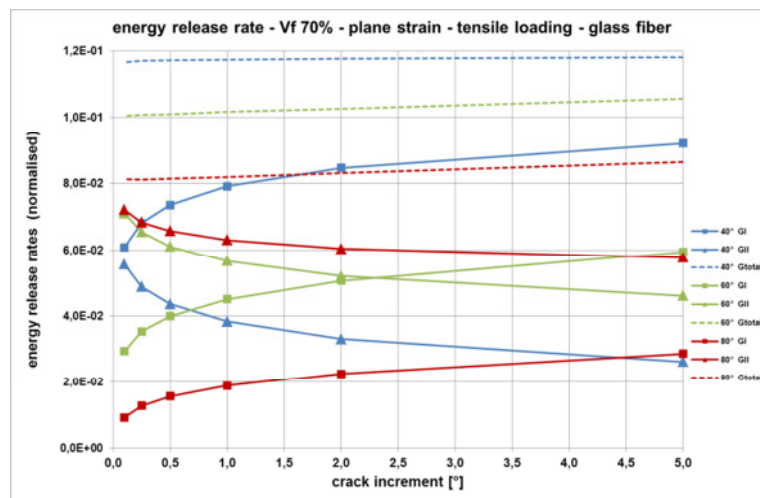


Figure 5. Mode II energy release rate – variation of increment length, Vf=70%.

The dependency of the portions of the energy release rate is studied at three specific crack lengths. For the largest crack increment the mode I part is dominating at a crack angle of 40°, at 60° both parts are almost equal and at 80° the mode II part is dominating. In each case the same general behaviour is encountered: the mode I part decreases with decreasing crack increment while at the same time the mode II part increases (fig. 6). In case of the shortest initial crack (40°) both parts of the energy release rate converge with decreasing increment length. For a 60° initial crack the slight dominance of the mode I part at a large increment changes to a dominance of the mode II part with diminishing increment length. In case of the largest initial crack (80°) the dominance of the mode II energy release rate is further enlarged when the increment is decreased. The general behaviour, that is, the increase of mode II and the decrease of mode I is identical to that of a straight interface crack. Furthermore it is not depending on the initial crack length.



**Figure 6.** Mode I and mode II energy release rate at different stages of interface crack

### *Discussion of the results*

Due to the strong variation of the total energy release rate with total crack length and specifically the mixed mode ratio the situation of a curved interface crack is far more complicated compared to a straight interface crack. Still the same general feature occurs, that is, the mixed mode ratio depends on the length of the crack increment. As a result, the discrimination of the energy release rate into mode I and mode II is questionable. The crack increment is in the first instance an artificial term arising in the numerical procedure with no physical meaning. An ideal fracture process is assumed to be continuous with no finite crack increment length. As a consequence the length of the crack increment tending to zero would be the appropriate description of the ideal fracture process. This counts for the numerical and the analytical solution as well. As stated above the common analytical solution does not possess a limiting value due to the oscillation of the stresses. The numerical solution neither has a limit, as a matter of fact. The results for varying increment lengths show an increase of the absolute value of the slope with decreasing increment size for both parts of the energy release rate (fig. 6). This indicates that with further downsizing of the increment the mode I part even could vanish while the total energy release rate becomes pure mode II. To answer this question a further refinement of the increment size must be conducted.

Another question is the course of an debonding process in a real composite material. The interfacial fracture process may not be fully continuous but may occur in small finite steps,

not detectable without a high speed recording. If this would be true the next question arises, namely if all steps are of equal or of varying length. As long as these questions are not answered by experiments the evaluation of fracture mechanical analyses of interface cracks should be restricted to the total energy release rate.

## References

- [1] Tandon G.P., Kim R.Y., Bechel V.T. Evaluation of interfacial normal strength in a SCS-0/epoxy composite with cruciform specimens. *Comp Sci Tech* 2000;60:2281-2295.
- [2] Ogihara S, Koyanagi J. Investigation of combined stress state failure criterion for glass fiber/epoxy interface by cruciform specimen method. *Comp Sci Tech* 2010;70:143-50.
- [3] E. Correa, E. K. Gamstedt, F. Paris, V. Mantic "Effects of the presence of compression in transverse cyclic loading on fibre-matrix debonding in unidirectional composite plies". *Composites Part A* 2007; 38: 2260-2269.
- [4] Marotzke C. Influence of the interface on the strength of fiber reinforced polymers. *Proceedings of the ECCM 14, Budapest, Hungary, 2010.*
- [5] Paris F., Cano J.C., Varna J. The fiber -matrix interface crack - a numerical analysis using boundary elements. *Int. J. Fracture* 1996;82:11-29.
- [6] Varna J., Berglund L.A., Ericson L. Transverse single fibre test for interfacial debonding in composites. *Composites Part A* 1997;28A:317-326.
- [7] Paris F., Correa E., Canas J. Micromechanical view of failure of the matrix in fibrous composite materials. *Comp Sci Tech*, 63, pp 1041-1052, 2003.
- [8] Correa E, Mantic V, Paris F. Numerical characterisation of the fibre-matrix interface crack growth in composites under transverse compression. *Eng Fract Mech.* 2008;75:4085-4103.
- [9] Correa E., Mantic V., Paris F. A micromechanical view of inter-fibre failure of composite materials under compression transverse to the fibres. *Comp Sci Tech* 2008; 68:2010-2021.
- [10] Hobbiebrunken T., Hojo M., Adachi T., de Jong C., Fiedler B. Evaluation of interfacial strength in CF/epoxies using FEM and in-situ experiments. *Composites Part A* 2006;37:2248-56.
- [11] Marotzke, C. Feldmann, T. Failure of composites under transvers loading - a fracture mechanical approach. *Proceedings of ECCM15, Venice, Italy, 2012.*
- [12] Williams M.L. The stresses around a fault or crack in dissimilar media. *Bull. Seismological Soc. America* 1959;49:1999-204
- [13] England A.H. A crack between dissimilar media. *J. Appl. Mech.* 1965;32:232-237.
- [14] Rice J. R. Sih G.C. Plane problems of cracks in dissimilar media. *J. Appl. Mech.* 1965;32:418-423
- [15] Comninou M. The interface crack. *J. Appl. Mech.* 1977;44:631-636.
- [16] Malyshev B.M., Salganik R.L. The strength of adhesive joints using the theory of fracture. *Int. J. Fracture Mech.* 1965;1:114-128.
- [17] Comninou M. An overview of interface cracks. *Eng Fract Mech* 1990; 37.1:197-208.
- [18] Sun C.T., Jih C.J. On strain energy release rates for interfacial cracks in bi-material media. *Eng. Fract. Mech.* 1987;28:13-20.
- [19] Mantic V., Paris F. Relation between SIF and ERR based measures of fracture mode mixity in interface cracks. *Int. J. Fracture* 2004;130:557-569.
- [20] Rice J. Elastic fracture mechanics concepts for interfacial cracks. *J. Appl. Mech.* 1988;55:98-203