EFFECT OF ALTERNATE CHANGE IN STRESS RATIO ON FATIGUE STRENGTH OF WOVEN FABRIC CFRP LAMINATE AND LIFE PREDICTION USING THE ANISOMORPHIC CFL DIAGRAM

M. Kawai[a*], K. Yang[a], S. Oh[a]

[a]*Department of Engineering Mechanics and Energy, University of Tsukuba, Tsukuba 305-8573, Japan*
**e-mail : mkawai@kz.tsukuba.ac.jp*

Keywords: Fatigue, *R*-ratio, Constant life diagram, Variable loading.

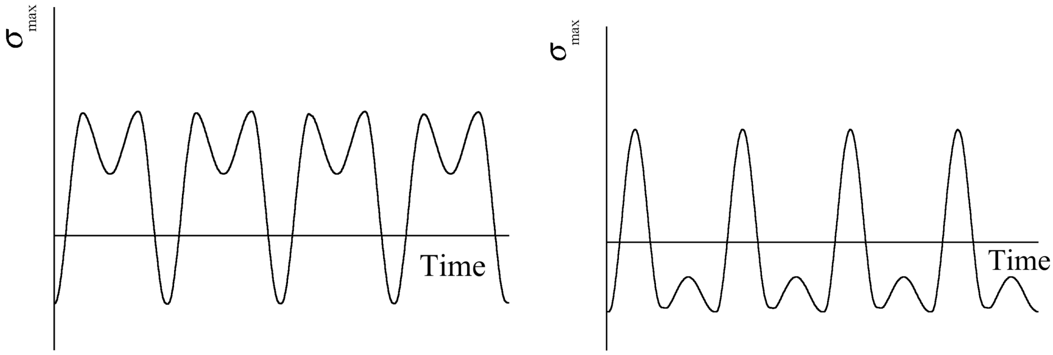
Abstract

Effect of variable fatigue loading with alternate R-ratios on the fatigue life of a quasi-isotropic woven fabric CFRP laminate has been studied. Variable R-ratio fatigue tests are carried out during which stress ratio of fatigue loading alternates between two values keeping the level of maximum (or minimum) fatigue stress constant. Fatigue life under variable R-ratio loading is examined not only for different pairs of stress ratios, but also for different levels of fatigue stress. The prediction of fatigue life of the CFRP laminate subjected to alternate R-ratio loading is attempted using the anisomorphic constant fatigue life diagram in conjunction with a representative cumulative fatigue damage rule. It is demonstrated that the Miner rule is satisfactory for prediction of fatigue failure under the alternating R-ratios loading tested in the present study with an accuracy of a factor of two.

1. Introduction

For fatigue life analysis of structures under service loading conditions, it is required to identify dominant factors contributing to fatigue failure under variable fatigue loading, and to model the accumulation of damage during variable fatigue loading. To understand fatigue behavior under variable loading conditions, two-step fatigue loading tests are often performed in which the maximum stress of constant amplitude fatigue loading increases or decreases stepwise during fatigue loading. In a more realistic context, block-loading tests are performed in which a block of different waveforms characterized by different values of stress amplitude and stress ratio is repeated until failure occurs. In the practice of engineering fatigue analysis, on the other hand, it would be efficient and appreciable if the fatigue life of a composite for a given variable loading can be connected with the fatigue lives for constant amplitude loadings contained by the variable loading. In this context, the Palmgren-Miner hypothesis of linear damage accumulation [1, 2] is often assumed to predict fatigue life under variable loading conditions as well as to quantify the effect of sequence of loading.

The present study aims to examine the effect of alternating *R*-ratios on fatigue life of a quasi-isotropic woven fabric CFRP laminate. To establish a baseline for this end, constant amplitude fatigue tests are first performed on coupon specimens at different stress ratios. Then, variable *R*-ratio fatigue tests are carried out in which the stress ratio of fatigue loading alternates between two different values, R_1 and R_2, under the condition that the maximum or



(a) (χ, 0.5) (b) (χ, 2)
Figure 1. Schematic illustration of (R_1, R_2) alternating *R*-ratios loading waveforms

minimum fatigue stress level is kept constant. Such alternating *R*-ratios fatigue tests are performed for different pairs of stress ratios and for different constant levels of maximum or minimum fatigue stress, respectively. By comparing the fatigue lives under constant amplitude and variable *R*-ratio loading conditions, the effect of alternation between given *R*-ratios on fatigue life of the woven CFRP laminate is elucidated. The effect of alternation in stress ratio on the Miner's sum is examined by means of the constant amplitude fatigue data established in this study. Making use of the Miner's sum, the applicability of the hypothesis of linear damage accumulation is evaluated. Finally, an engineering fatigue life analysis methodology that uses the anisomorphic CFL diagram [3-5] to predict the fatigue lives of a given composite for constant amplitude loading at any stress ratios in conjunction with the linear damage accumulation rule to predict fatigue failure under variable loading is established, and tested for alternating *R*-ratios loading of the quasi-isotropic woven fabric CFRP laminate.

2. Material and testing procedure

2.1. Specimen

The material used in this study was a woven carbon/epoxy quasi-isotropic laminate [(±45)/(0/90)]4S fabricated using a prepreg QFC133-6E01A (TORAY). The cure temperature of laminates was 350°F (176.7°C). The thickness of laminates was about 3.2 mm. Since the experimental program includes tension-compression and compression-compression fatigue loadings, short specimens were used for all tests in this study to reduce a risk of the buckling of specimen due to compressive stress. The dimensions of the short specimen were gauge length L_G = 10 mm and width W = 10 mm. Rectangular-shaped aluminum alloy tabs were glued on both ends of the specimens with epoxy adhesive in order to protect their gripped portions.

2.2. Testing Procedure

To establish reference fatigue data for this study, constant amplitude fatigue tests were performed under load control at room temperature. Fatigue load was applied to specimens in a sinusoidal waveform with a constant frequency of 10 Hz. Most specimens were fatigue tested for up to 10^6 cycles, and the fatigue tests that lasted over this limit were terminated prior to fracture. Constant amplitude fatigue tests were performed for five kinds of stress ratio: R = 0.1, 0.5, 10, −1 and χ to elucidate the effect of stress ratio on fatigue life of the composite. A particular value of stress ratio denoted by χ, which is called the critical stress ratio [3-5], is

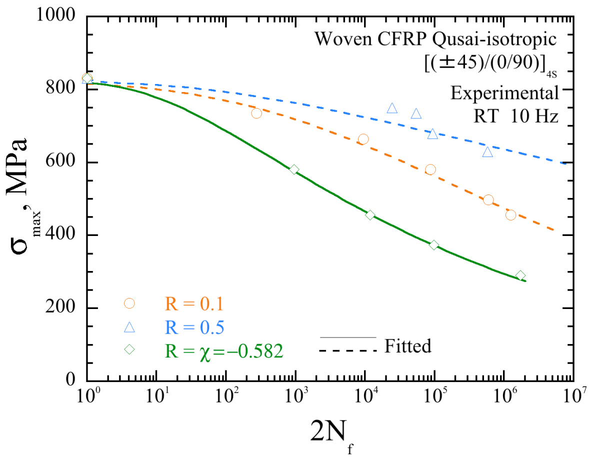


Figure 2. S-N relationships tension-dominated fatigue loading

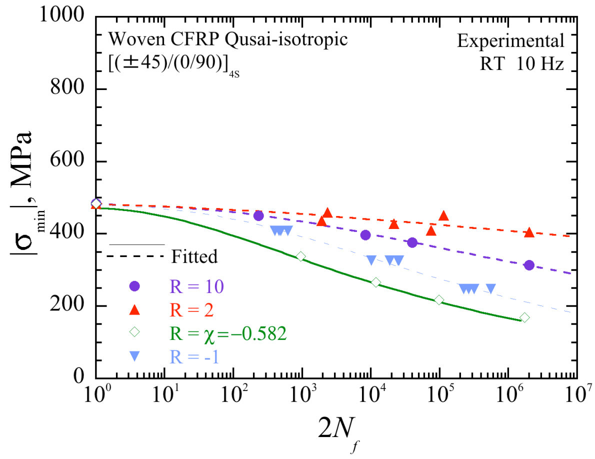


Figure 3. S-N relationships for compression-dominated fatigue loading

defined as the ratio of compressive strength σ_C (< 0) to tensile strength σ_T (> 0); i.e. $\chi = \sigma_C / \sigma_T$. Alternating stress-ratios fatigue tests that may be considered as two-unit block loading tests in which two waveforms of different stress ratios R_1 and R_2 alternate are performed under the following combinations of stress ratios. Alternating R-ratios tests for different pairs of stress ratios, which are designated explicitly as (R_1, R_2) alternating R-ratios tests, are schematically illustrated in Fig. 1.

In the variable R-ratio fatigue tests, it is assumed that R_1 takes a constant value of χ and it is paired with a different constant value of stress ratio R_2. It is referred to as (χ, R) alternating R-ratios tests. The waveforms of two different stress ratios $R_1 = \chi$ and $R_2 = R$ are combined into a new waveform, as schematically illustrated in Fig. 1(a) and Fig. 1(b), and it is repeated until fatigue failure occurs. In this study, four different values of R_2 are chosen as $R_2 = R = 0.1$, 0.5, 10, and 5, which allows us to perform alternating R-ratio loading tests for four pairs of stress ratios: (R_1, R_2) = (χ, 0.1), (χ, 0.5), (χ, 10), and (χ, 2). In the alternating R-ratios tests where tension-compression cycle with $R_1 = \chi$ alternates with tension-tension cycle R_2 (= 0.1 or 0.5), it is assumed that the maximum fatigue stresses for R_1 loading and R_2 loading are of equal value, as illustrated in Fig. 1(a). Each of these tests are performed in two different levels of maximum fatigue stress σ_{max} = 440 MPa and 551 MPa. In the case of combinations with compression-compression cycle, i.e. (R_1, R_2) = (χ, 10), (χ, 2), the minimum fatigue stresses for R_1 and R_2 loadings are assumed to agree to each other, as illustrated in Fig. 1(b). Two values of minimum fatigue stress are chosen as σ_{min} = –259 MPa and –319 MPa. Four to eight specimens are used for each of the alternating *R*-ratios tests, while one specimen is allocated in principle to each of the additional second and third kinds of alternating *R*-ratios tests.

3. Experimental Results and Discussion

3.1. Constant amplitude fatigue behavior and R-ratio dependence

The fatigue data obtained from constant amplitude loading at different stress ratios are shown in Fig. 2 for tension-dominated loading (R = 0.1, 0.5, χ) and in Fig. 3 for compression-dominated loading (R = 10, 2, –1, χ). From Fig. 2 and Fig. 3, it is seen that the S-N relationship for the woven CFRP laminate greatly depends on stress ratio. The overall features in the sensitivity to mean stress are similar to those reported so far for different composites, e.g. [3-5]. The sensitivity to fatigue becomes highest under T-C loading at $R = \chi$, suggesting that a larger value of alternating stress amplitude in fatigue loading has a more degrading effect on the fatigue of the composite. A detailed observation reveals that the S-N data for stress ratios in the range $\chi < 0 \le R < 1$ can approximately be described by means of the smooth dashed curves that can be extrapolated back to the level of the tensile strength. By contrast, the S-N curves fitted to the fatigue data for T-C loading ($R = -1 < \chi$) and C-C loading (R = 2, 10) can smoothly be connected with the compressive strength. Specimens failed in a compressive mode under completely reversed loading at R = –1, since $\chi = -0.55 > -1$, and thus the distance from the minimum fatigue stress to the compressive strength $|\sigma_C - \sigma_{min}|$ is less than the distance from the maximum fatigue stress to the tensile strength $|\sigma_T - \sigma_{max}|$ under fatigue loading at R = –1.

3.2. Constant fatigue life (CFL) diagram

Identification of constant amplitude fatigue behavior of a given material is equivalent to identification of its CFL diagram. With a CFL diagram, the effect of stress ratio on fatigue life can more clearly be understood. In this study, the full shape of CFL diagram for the woven CFRP laminate is identified on the basis of the S-N relationships observed by experiment. The experimental CFL diagram is compared with a theoretical anisomorphic CFL diagram [3, 4] that has been validated for different types of CFRP laminates. The main reason for involving the theoretical anisomorphic CFL diagram in this context is that a combination of the CFL diagram approach to predict constant amplitude fatigue lives for any constituent waveforms contained by variable fatigue loading with a certain cumulative damage rule, e.g. the Miner rule, to predict fatigue failure under variable loading yields an engineering fatigue life prediction methodology that prompts us to test it for the alternating *R*-ratios loading in this study.

3.3. Comparison between the experimental and theoretical CFL diagrams

The anisomorphic CFL diagram for the woven CFRP laminate is shown in Fig. 4 by dashed lines. From Fig. 4, it is seen that the anisomorphic CFL diagram for the $[(\pm45)/(0/90)]_{4S}$ woven CFRP laminate agrees well with the experimental CFL data in the tested range of fatigue life. Therefore, this observation proves that the anisomorphic CDL diagram approach is valid for the woven CFRP laminate as well as to the non-woven CFRP laminates [26-28]. The experimental results obtained from this study elucidate that the CFL diagram for the

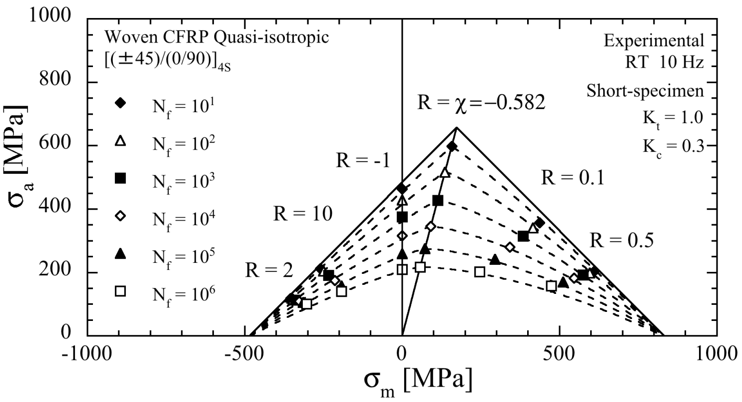


Figure 4. Anisomorphic constant fatigue life diagram identified for the woven CFRP laminate

woven CFRP laminate is asymmetric about the alternating stress axis, and the peaks of the CFL envelopes for different constant values of life appear under fatigue loading at a stress ratio close to the critical stress ratio $\chi = -0.582$. These features of the CFL diagram for the woven CFRP laminate are similar to those for the non-woven CFRP laminates that were observed in the previous studies [3-5].

4. Results of alternating *R*-ratios fatigue tests

The experimental results of alternating *R*-ratios tests in which stress ratio alternates between the critical stress ratio $R_1 = \chi < 0$ and another stress ratio $R_2 = 0.1$ or 0.5 under the same constant value of maximum fatigue stress are first observed. Fig. 5 shows the fatigue lives obtained from the alternating *R*-ratios tests for the pairs of stress ratio $(R_1, R_2) = (\chi, 0.1)$ and $(\chi, 0.5)$, respectively, with the constant maximum fatigue stress σ_{max} = 551 MPa. The mean values of fatigue lives obtained from alternating *R*-ratios tests were as follows: $N_f^{(R_1,R_2)} = N_f^{(\chi,0.1)} = 2029$ cycles and $N_f^{(R_1,R_2)} = N_f^{(\chi,0.5)} = 2264$ cycles. The associated constant amplitude fatigue lives for the respective stress ratios, $R = \chi$, 0.1 and 0.5, at the same constant value of maximum fatigue stress were evaluated using the S-N curves fitted to the fatigue data obtained for these stress ratios, respectively. From comparison between the alternating *R*-ratios fatigue life $N_f^{(\chi,R_2)}$ and the associated constant amplitude fatigue lives for χ and R_2, it can be found that $N_f^{(\chi,R_2)} \ll N_f^{(R_2)}$ and $N_f^{(\chi)} < N_f^{(\chi,R_2)} < 2.2 N_f^{(\chi)}$.

Next, the experimental results from the alternating *R*-ratio tests in which tension-compression cycle with the critical stress ratio $R_1 = \chi < 0$ and compression-compression cycle with $R_2 = 10$ or 2 alternate keeping the minimum fatigue stresses constant are observed. The alternating *R*-ratios test results for σ_{min} = –319 MPa are shown in Fig. 6. The average lives calculated in six samples were as follows: $N_f^{(R_1,R_2)} = N_f^{(\chi,10)} = 2525$ cycles and $N_f^{(R_1,R_2)} = N_f^{(\chi,2)} = 2015$ cycles.

5. Prediction of variable loading fatigue lives using the Miner rule

Once the anisomorphic CFL diagram is constructed for a given composite, we can use it to predict fatigue life for constant amplitude fatigue loading at any stress ratio *R*. This means that if the component waveforms of constant amplitude involved by a given variable fatigue loading are identified, we can predict the fatigue lives for those component constant *R*-ratio loadings using the anisomorphic CFL diagram for any stress ratios, and thus calculate the

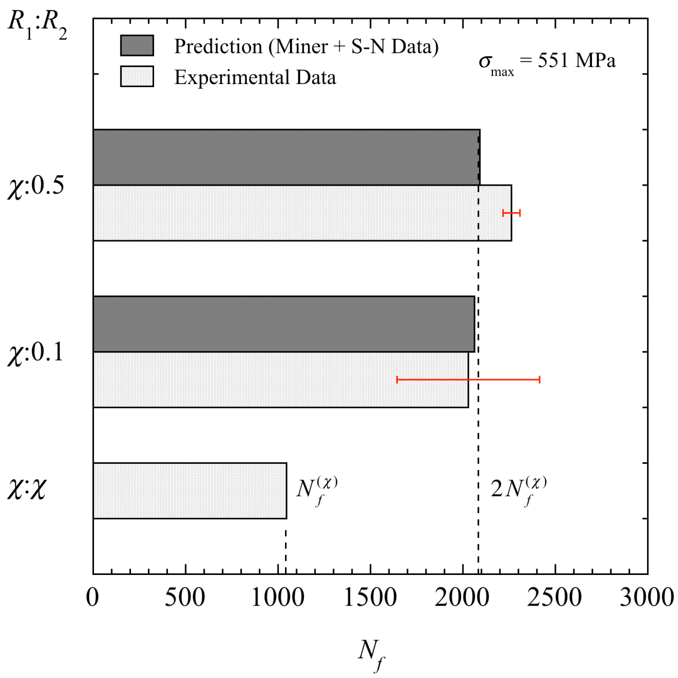


Figure 5. Results of alternating R-ratios loading experiments for the pairs (χ, R) = (χ, 0.1) and (χ, 0.5) with σ_{max} = 551 MPa

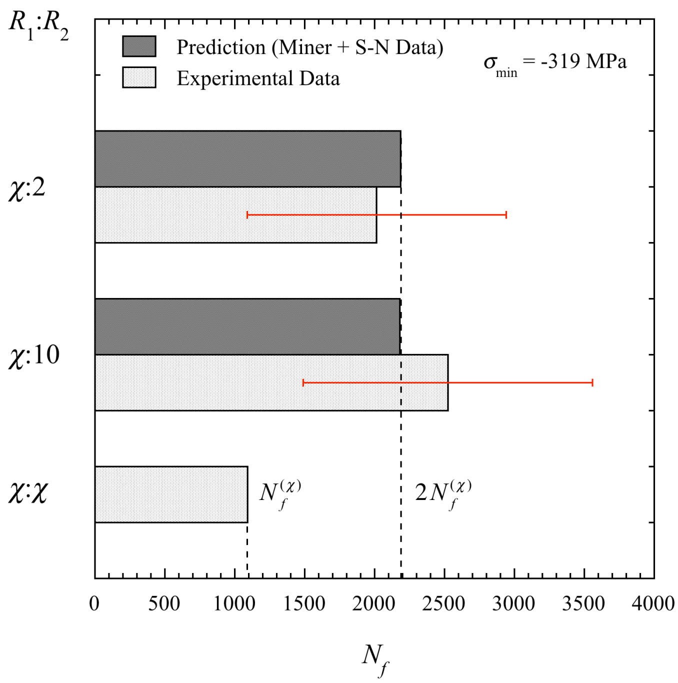


Figure 6. Results of alternating R-ratios loading experiments for the pairs (χ, R) = (χ, 2) and (χ, 10) with σ_{min} = – 319 MPa

damage fractions (cycle ratios) at the time of given cycles for any component cycles identified. Accordingly, it is suggested that a fatigue life prediction methodology for variable loading of composites can be established by means of a cycle-ratio based cumulative fatigue damage theory in conjunction with the anisomorphic CFL diagram approach.

As observed above, we have confirmed that the Miner rule can approximately be used for evaluation of cumulative fatigue damage during alternating R-ratios loading. Therefore,

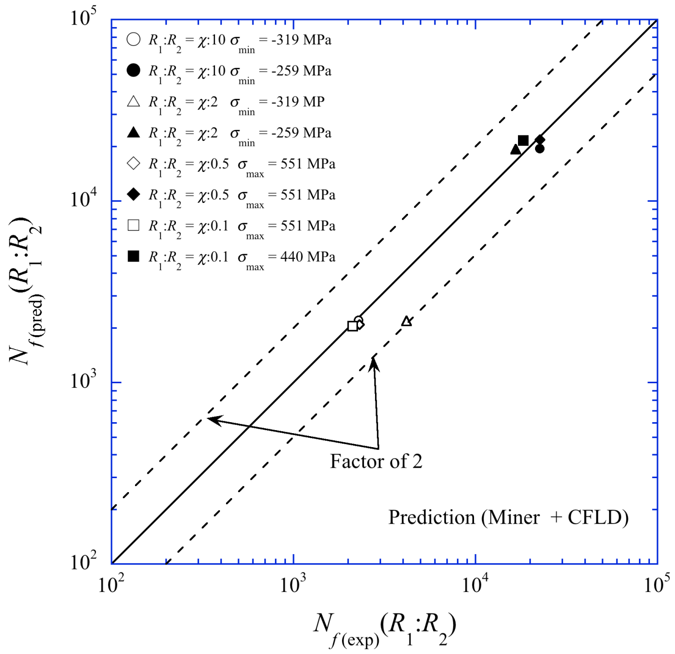


Figure 7. Comparison between the predicted and observed fatigue lives for (χ, *R*) alternating *R*-ratios loading

assuming the Miner rule for variable *R*-ratio loading and the anisomorphic CFL diagram approach for constant *R*-ratio loading, we can predict the fatigue life for alternating *R*-ratios loading $N_{f(pred)}^{(R_1,R_2)}$ by means of the following formula:

$$N_{f(pred)}^{(R_1,R_2)} = \frac{2}{\dfrac{1}{N_{f(pred)}^{(R_1)}} + \dfrac{1}{N_{f(perd)}^{(R_2)}}} \tag{1}$$

where $N_f^{(R_1)}$ and $N_f^{(R_2)}$ are the constant amplitude fatigue lives for R_1 and R_2, respectively, that are predicted using the anisomorphic CFL diagram.

Fig. 7 shows comparison of $N_{f(pred)}^{(R_1,R_2)}$ predicted using the methodology mentioned above and $N_{f(\exp)}^{(R_1,R_2)}$ observed by the alternating *R*-ratios loading tests. From this figure, it can be confirmed that the predicted fatigue lives for alternating *R*-ratio loading agree with the experimental results with an accuracy of a factor of two. The observation in these figures that the Miner rule is satisfactory for failure prediction in alternating *R*-ratio loading is just the rephrase of what we observed in the discussion of experimental results. Additional importance of these successful correlation of the predicted and experimental results is that the anisomorphic CFL diagram approach succeeds in accurately predicting the fatigue lives for the stress ratios paired in the alternating *R*-ratios loading tests, and thus it can efficiently be combined with the Miner rule.

References

[1] A.Z. Palmgren. Die Lebensdauer vin Kugellagern, Zeitschrift des Verein Deutcher Ingenieure, 68:339-341, 1924.
[2] M.A. Miner. Cumulative Damage in Fatigue, ASME J. Appl. Mech., 12:A159-A164, 1945.

[3] M. Kawai. A method for identifying asymmetric dissimilar constant fatigue life diagrams of CFRP laminates. Proceedings of the Fifth Asian-Australasian Conference on Composite Materials (ACCM-5), Hong Kong, November 27-30, 2006.
[4] M. Kawai and K. Koizumi. Nonlinear constant fatigue life diagrams for carbon/epoxy laminates at room temperature. Composites Part A, 38:2342-53, 2007.
[5] M. Kawai and T. Murata. A three-segment anisomorphic constant life diagram for the fatigue of symmetric angle-ply carbon/epoxy laminates at room temperature. Composites Part A, 41:1498-1510, 2010.