NUMERICAL SIMULATION OF THE LIQUID COMPOSITE MOLD FILLING PROCESS WITH VOID FORMATION USING TWO-PHASE POROUS MEDIA THEORY

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Abstract
In order to minimize final void content in composite parts manufactured by Liquid Composites Molding (LCM) processes, the study of unsaturated flows in dual scale (microscopic pores between the filaments in the fiber tows and macroscopic pores between the tows) at the macroscopic level is becoming increasingly necessary. Even the continual progress made in the last decades, there still exist many unresolved issues and limitations with current numerical approaches \cite{1}. By one hand, modeling coupled flow (Darcy’s law) and transport (saturation) in LCM processes with void formation remains a mathematical challenge and, by other one, the choice of higher order schemes for numerical simulation of the saturation in LCM processes is very important for effective simulations of the mold-filling process. In this work, we present a procedure to simulate the LCM filling process with void formation. To validate the proposed model, numerical predictions for the saturation were compared with experimental data obtained on a glass RTM mold under controlled manufacturing conditions \cite{2}.

1. Introduction

In the manufacturing of composite parts by Liquid Composite Molding complete saturation of the fibrous reinforcement is key. Incomplete saturation leads to voids within the fibers which cause failure of the final product. Thus, understanding of the formation of voids is necessary for proper molding of composite parts. In order to analyze the formation of voids during the resin impregnation process, a one-dimensional solution based on two-phase flow through a porous medium, is proposed. This model leads to a coupled system of a nonlinear advection-diffusion equation for saturation and an elliptic equation for pressure and velocity. The permeability is assumed to be a function of saturation, and then the continuity equation that governs the pressure distribution includes a source term which depends on the saturation. A key part of our work is the choice of the relative permeability model and the mathematical formulation for saturation equation proposed to determine the saturation curves.

Essential to the optimum process design in LCM is the mathematical model and the numerical simulation of the modified saturation equation. In general, the saturation equation is a non-
linear advection-diffusion equation which includes the capillary pressure effect and it reduces to a purely advection transport equation when capillary effects are neglected. The hyperbolic nature of the saturation equation and its strong coupling through relative permeability represent a challenging numerical issue. Common numerical methods used to solve these equations suffer from nonphysical oscillations, numerical dispersion or a combination of both. The technique here used for solving the equation which governs the evolution of the degree of saturation of porous media is based on an essentially non-oscillatory fixed mesh strategy and was described in [3]. For the ENO schemes, interpolation polynomials of one order less than the order of accuracy required in the solution are computed and these polynomials are a good approximation to the values of the numerical flux function at the cell walls. The key idea in the \( r \)th-order ENO schemes is to use the “smoothest” stencil among \( r \) possible candidates to approximate the fluxes at cell boundaries to high-order accuracy and at the same time to avoid spurious oscillations near shocks.

The approach presented here combines relative permeability models [4] with the saturation equation in order to find an optimal mathematical model to simulate the saturation in LCM process. In this work, the relative permeability in the elliptic equation depends on the saturation and on the air fraction in a unit volume of resin component. This parameter has been assumed to be linearly dependent on pressure. Some preliminary numerical results are presented and compared with experimental results in order to validate the proposed mathematical model and the numerical scheme.

2. Governing Equations

The mathematical formulation of the saturation in LCM takes into account the interaction between resin and air as it occurs in a two phase flow. Combining equations that describe mass conservation and Darcy’s laws for resin and air phases as described in [3], the resulting equation for the saturation in its most general form gives

\[
\phi \frac{\partial S}{\partial t} + \nabla \cdot (q f(S)) = - \nabla \cdot \left( D_{cf}(S) \nabla S \right)
\]  

where

\[
f(S) = \frac{\lambda_g(S)}{\lambda_g(S) + \lambda_r(S)} \quad \text{and} \quad D_{cf}(S) = f(S) \lambda_g(S) \frac{\partial P}{\partial S}
\]

Here, \( D_{cf}(S) \) is the nonlinear diffusivity coefficient due to capillary pressure \( P_c \), defined as \( P_c = P_g - P_r \); \( q \) is the total velocity, \( S \) is the degree of saturation of the reinforcement by the liquid resin,

\[
\lambda_j(S) = \frac{K_{sat}K_j(S)}{\mu_j}
\]

is the phase mobility, with \( K_j(S) \) the relative permeability of the phase \( j \), \( \mu_j \) the viscosity of phase \( j \) and \( K_{sat} \) the saturated permeability.
To simulate the flow of the resin phase, we can derive the governing equations combining, for the resin phase, Darcy’s law,

\[ q_R = -\frac{K_{sat}K_R(S)}{\mu} \nabla p \]

with mass conservation

\[ \nabla \cdot q_R = -\phi \frac{\partial S}{\partial t} \]

In the classical formulation of two-phase flow, the total velocity \( q \) in Eqn. (1) is constant in space. Numerical simulations indicated that this technique is not a good alternative for solving the saturation equation. For this work, isothermal conditions are assumed and Eqn. (1) has been coupled with the elliptic equations for the pressure and velocity of the resin phase where the relative permeability has been calculated considering resin and air as a mixture. Then the velocity total \( q \) in Eqn. (1) is obtained using the elliptic equation for the resin where the saturation \( S \) in relative permeability in Eqns. (4)-(5) has been replaced by

\[ \overline{S} = S + R_S (1 - S) \]

Here \( R_S \) denotes the air fraction in a unit volume of resin component. In this work, \( R_S \) has been assumed to be linearly dependent on pressure.

The simulation of the filling process involves the following operations at each time step:

1. Calculate the pressure distribution by applying a standard finite element discretization to Equation

\[ \nabla \cdot \left( K_R(\overline{S}) \nabla p \right) = \frac{\phi \mu}{K_{sat}} \frac{\partial S}{\partial t} \]

where the relative permeability and the term on the right side depend on the saturation degree. In this work we use the power law model for relative permeability, which is a particular case of the often called the Corey model [4],

\[ K_R(\overline{S}) = \left[ S + R_S \right]^{\frac{1}{m}} (1 - S)^m \]

where the relative permeability for the resin has been replaced by considering resin and air as a mixture [3]. Standard choices for \( K_R \) are linear (\( m=1 \)) and quadratic (\( m=2 \)).

2. Calculate the velocity field from Darcy’s law

\[ q = -\frac{K_R(\overline{S})K_{sat}}{\mu} \nabla p \]
3. Update the saturation distribution by integrating Eqn. (1) using a fourth-order ENO technique described in [3]. In the general case, the saturation equation depends on the phase mobility, and therefore, we use

\[
\phi \frac{\partial S}{\partial t} + \nabla \cdot (\mathbf{q} f(S)) = 0 \quad \text{with} \quad f(S) = \frac{k_1 S^m}{k_1 S^m + M k_2 (1 - S)^m}
\]

where \(M\) denotes the resin-air viscosity ratio. We take \(S\) for the saturation of the resin phase and \((1-S)\) for the air phase. The coefficients \(k_1\) and \(k_2\) are the endpoint relative permeabilities of each fluid. When \(m=1\) in Eqns. (8)-(10) we consider a linear model and if \(m=2\) we have a quadratic model. The treatment of the diffusivity coefficient will be more widely investigated in the future.

The best results in this work have been obtained using a quadratic model for the relative permeability.

The boundary conditions are given by: the pressure gradient in the normal direction to the mold walls vanishes, the flow rate is specified on the inflow boundary and the pressure is zero in the empty part of mold.

3. Numerical results

In order to test and evaluate the ability of commonly used relations for two phase flow predictions in RTM, results of saturation simulation are compared to experimental data [2]. Three experiments were compared. The geometry considered in the current study consists of a mold cavity RTM with 36mm of length, 105mm of width and 3.175mm of thickness. The analyzed area has been composed by 21 long representative elementary volume (REV) by 5 wide REV, which have been used to analyze the saturation. The saturation value has been calculated in the centre of each REV and an average value has been performed in the width of the piece. Each REV has a dimension of 15 mm by 15 mm, so after the port before injection and before the vent port, there are two small bands of 22.5 mm long by 105 mm wide which are not analyzed for the saturation. Three injections were performed by changing only the injection rate: 0.025 ml/s, 0.1 ml/s and 0.3 ml/s. The saturated permeability \(K_{sat}\), resin viscosity and the porosity have been set to \(7.9 \times 10^{-10}\) m², 0.4788 Pa.s and 0.614, respectively.

The validation of the mathematical model and the numerical technique for the saturation was based on the experimental RTM injection at constant flow rate of 0.1 ml/s. In this case capillary effects and the air residual have been ignored. Figure 1 shows experimental saturation results in function of time for three REV (V3, V10 and V19) compared with the numerical solution. The graphic illustrated at the left is based on a linear model of Corey permeability \((m=1\) in Eqns. (8)-(10)), and the graphic at the right is based on a quadratic model of Corey permeability \((m=2\)). In linear and quadratic cases, results depend on the value of \(R_s\). We tested a relative permeability model with \(R_s\) linearly variable with the pressure.

\[
R_s(p) = \alpha + \beta \frac{p}{2 \cdot p_{MAX}} \quad , \quad 0 < p < p_{MAX} \quad , \quad 0 < R_s(p) < 1
\]
In general, numerical results using a quadratic model for the relative permeability shows trends that are in good agreement with experimental results for the case of RTM injection at constant flow rate of 0.1 ml/s. In this case capillary effects and the air residual have been ignored. However, linear models for relative permeability do not seem to match with experimental observations.

Power law models for relative permeabilities are commonly used in numerous applications. If the disturbance of the flow of one phase is only due to the restriction of available pore volume caused by the presence of the other fluid, a linear correlation for the relative permeability can be applied. In reality, one phase usually not only influences the flow of another phase just by the restriction in available volume, but also by additional interactions between the fluids. It explains that the quadratic power law model for relative permeability yields better numerical results that the linear power law model, as we can see in numerical results.

In order to analyze the cases with low and high resin injection rate we use the quadratic power law Corey model for the relative permeability. Figure 2 shows experimental saturation results in function of time for three REV (V3, V10 and V19) for the injection at constant flow rate of 0.025 ml/s compared with the numerical solution. The graphic illustrated at the left ignores capillary terms. For this case, the presence of capillary forces is very important. Simulations at the right have been carried out incorporating the diffusive term due to the capillary pressure, using the following capillary pressure-saturation relations

\[ p_c(S) = p_{at}(1-S)^{-1/2}, \quad p_{at} = \frac{\sigma \cos(\theta)}{\sqrt{K/\phi}} \]  

This change improves numerical results but doesn’t change the numerical behaviour of the saturation, which always decreases with the time, whereas the experimental results show a change in the behaviour of the variation of the saturation with the time for this case (first, increases and then decreases).

Figure 3 shows experimental saturation results in function of time for the test of injection at 0.3 ml/s constant flow rate compared with the numerical solution, using a quadratic power law model for the relative permeabilities and the linear function of pressure for \( R_s \). Numerical
Simulations have been carried out including a constant residual air saturation of 0.3, which can be identified as the saturation of the air located in the immobile bubbles.

Figure 2. Curves of saturation as a function of time for three REV (V3, V10 and V19) for the injection at constant flow of 0.025 ml/s (numerical solution is represented by straight lines)

Figure 3. Curves of saturation as a function of time for three REV (V3, V10 and V19) for the injection at constant flow of 0.3 ml/s (numerical solution is represented by straight lines)

4. CONCLUSIONS

In LCM processes, distinctions are made between saturated flow region, where preforms have been wetted so that only single phase fluid (resin) need to be considered, and unsaturated region, where dry spots or voids exist and dual phase fluid (resin and air) should be considered. For single phase flow in a homogeneous porous medium, Darcy’s law is an expression of momentum conservation at the macroscopic scale, but when two or more fluid phases are present, the permeability in Darcy’s original equation is replaced by an effective value to accommodate the presence of other phases. In this case, the permeability experienced by one phase depends on the degree of saturation of the reinforcement. So, the saturation equation and the relative permeability are based on a two phase flow description.

A detailed analysis for different relative permeability models combined with the saturation equation is performed to assess the saturation profiles in LCM. Numerical results using a FEM code have been compared with measured saturation profiles in time for three
experiments at different injection flow rates. In order to numerically solve the saturation equation for the LCM process a high order essentially non-oscillatory (ENO) technique proposed in [3] has been used. Models based on the linear Corey relative permeability did not agree as well with the experimental data as those obtained with the quadratic Corey model for the relative permeability. Quadratic model for relative permeability yielded predictions that closely matched the experimental data in the experiments when a ENO technique for the calculation of the saturation was applied.

The proposed model with Rs varying linearly with the pressure improved the results in all the cases, and yields predictions that closely matched the experimental data in the test corresponding to injection flow rate of 0.1ml/s. The results also demonstrate that the model can predict numerical saturation in LCM and to recreate the pressure for dual-scale porous medium. Results for injection flow rates lower or higher than 0.1 ml/s showed different behaviours that need deeper study.

The numerical examples presented in this paper demonstrate that the proposed procedure is versatile and robust and produces numerical results which are in excellent agreement with the experimental solutions, but it is necessary modeling the hysteresis phenomena for the lower flow rate injection cases and an adjustment of the residual air-saturation for the microvoids formation. This formulation opens up new opportunities to improve LCM flow simulations and optimize injection molds.

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References


