BUCKLING ANALYSIS OF COMPOSITE PLATES UNDER COMBINED LOADING, WITH ACCOUNT OF BENDING-TWISTING COUPLING

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Abstract

Flat rectangular composite plates made of carbon fiber/epoxy unidirectional tape layers are considered. The plates have symmetric lay-up, with D16 and D26 elements of stiffness matrix being not neglected. Buckling under combined loading is considered. The main purpose of the paper is to explore and reveal the peculiarities inherent in the case. The characteristics of the buckling curves such as convexity and piecewise-smoothness are discussed. The points of slope discontinuity are treated as the points of duplicated buckling eigen values, where corresponding switch of buckling modes participating in the load/modes interaction occurs. It is also indicated how the bending-twisting terms are influencing the curves. Some formulas for calculating characteristics of the curves are derived. Numerical example of a thin rather long composite CFRP (carbon fiber-reinforced plastic) plate is presented.

1. Introduction

Buckling of composite plates under combined loading (shear, compression, in-plane bending) was considered by many authors in the framework of the Classical Laminate Plate Theory, both for the case of specially-orthotropic material and for the case of the so-called flexural orthotropy (when the bending-twisting coupling terms in the buckling equation proportional to D16 and D26 elements of the bending stiffness matrix are assumed to be neglected). As papers in the field we indicate [1-5], where corresponding references may be found.

In [3] the design formulas for a long composite plate made of specially-orthotropic material are presented. The formulas are derived basing on regression analysis of buckling data obtained from a very accurate Rayleigh-Ritz method.

As papers, where the bending-twisting coupling is also taken into account, we mention [1, 2, 4, 5]. In the numerous papers of Nemeth (see [1-2] and papers referenced there) the influence of the flexural anisotropy to buckling interaction is numerically explored in detail. The paper [4] details a series of tests carried out to investigate the behavior of a number of optimized fiber composite plates of differing geometry, simply supported along two edges and built in along the other two, subject to a varying combination of shear and in-plane bending, for which no theoretical solution exists, and assesses the suitability of analytical techniques and finite element analysis to predict this behavior.

In [5] the Balabukh formula is indicated, approximating the shear-compression buckling interaction curve in case of flexural anisotropy. The formula is based on quadratic interpolation of the interaction curve using compression-only and shear-only buckling limits. In the case of shear there are two limit values, depending on direction of shear loading.

The book [6] (chapter 2) gives a review of the theoretical status of the problem in general, without considering peculiarities of composite materials. Some theoretical results of the book create a basis for generalization of them to the composite structure case. In particular, the Papkovich theorem (see [7]) states a useful feature of the interaction curve, namely the set of points corresponding to stable structures is a convex one.

The present paper presents some theoretical consideration of the buckling interaction phenomena and a numerical example illustrating, for the case of composite materials, the features and proposed approaches.

2. Buckling interaction curves and their creation.

As it has been indicated in the Introduction, several approaches are used for creation of the buckling interaction curves for rectangular plates.

Basing on the expression for the buckling eigen value, the formulas for slopes of the buckling interaction curves are derived. Below we denote by N_x, N_y, N_{xy} the normal force flows in *x* and *y* directions and the shear flow in *x*-*y* plane. Using the regular loading sign convention of [8], the formulas are:

shear – compression case
$$\frac{dN_x}{dN_{xy}} = -\frac{\int dS \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}}{\frac{1}{2}\int dS\left(\frac{\partial w}{\partial x}\right)^2}, \tag{1}$$

where the integrals are calculated over the plate surface (the coordinate axes are parallel to the corresponding lateral sides of the plate), the plate is compressed in *x* direction, *w* is the deflection,

shear – in-plane bending case
$$\frac{dN_x^{(0)}}{dN_{xy}} = -\frac{\int dS \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}}{\frac{1}{2}\int dS\,\frac{y}{2b}\left(\frac{\partial w}{\partial x}\right)^2}, \tag{2}$$

where it is supposed that the in-plane bending at plate sides parallel to y axis is created by the distributed normal force flow

$$N_x = N_x^{(0)}\frac{y}{2b} \tag{3}$$

where *b* is the dimension of the plate in y direction, the origin is supposed to be at the center of the plate. Using the above formulas for derivatives acting at isolated cases (i.e., compression-only and shear-only), the following formula for the compression-shear case is derived (in fact, this is a step ahead from the Balabukh formula for including the interaction curve derivatives at separate loadings):

$$N_x \big/ N_{x0} = \frac{1}{q}\left(\frac{N_{xy}}{N_{xy01}} - 1\right)\left(\frac{N_{xy}}{N_{xy02}} - 1\right)\left[\left(N_{xy}\right)^2 - p\left(N_{xy}\right) + q\right] \tag{4}$$

where the subscript 0 means the eigen value for one loading, 01 corresponds to the shear eigen value in one (considered) direction, 02 corresponds to the shear eigen value in case of loading in opposite direction (it has the "-" sign). In fact, (4) gives N_x as a fourth power polynomial of N_{xy}. In (4) the quantities *p* and *q* are determined as follows:

$$q = -\frac{N_{xy01}^2 N_{xy02}}{N_{xy01} + 2N_{xy02} + \dfrac{f_0^{/} N_{xy01} N_{xy02}}{N_{x0}} - \dfrac{f_1^{/} N_{xy01} N_{xy02}^2}{N_{x0}\left(N_{xy01} - N_{xy02}\right)}} \tag{5}$$

$$p = -q\left(N_{xy01} + N_{xy02} + \frac{f_0^{/} N_{xy01} N_{xy02}}{N_{x0}}\right) \tag{6}$$

where $f_0^{/}, f_1^{/}$ are the derivatives of the buckling interaction curve $N_x(N_{xy})$ at points $N_x = 0$ (pure positive shear) and $N_{xy} = 0$ (pure compression), respectively.

Formulas similar to (5) may be easily derived for other combinations of loadings.

One more option for determining the buckling interaction curve is to use the buckling deflection fields of separate loadings for Galerkin-type approach. For example, for compression-shear loading one may take the buckling modes for pure compression $w_1(x, y)$ and for pure positive shear $w_2(x, y)$. The modes satisfy both essential and natural boundary conditions. Due to that the Galerkin approach is applicable. Making usual Galerkin analysis with these two modes only, one may obtain the corresponding quadratic algebraic equation for eigen values. Then, taking the lowest solution of the equation, one obtains the eigen value in case of interaction. The only information necessary for such analysis is the eigen modes/values for separate loadings. At the below chapter there will be an example of the approach presented, the approach is mentioned as the "special Galerkin-type approach".

It should be also noted that the modes interacting under combined loading may be different at different portions of the buckling interaction curve. This case may occur when there is a multiple eigen value for some combination of the loadings. At such a point there is a switch of the interacting modes and the curve slope may not be continuous. The example of the below chapter will demonstrate the feature.

Observation of the known Papkovich theorem leads to a conclusion, that it is valid for composite plates also.

3. Numerical results

The proposed approaches have been used for a plate of t300/5208 tape material with symmetric +/-45° lay-up. The tape thickness has been 0.125 mm, with the total plate thickness being 2 mm. The plate dimensions have been 200*600 mm. The elastic modules of the material have been taken from http://composite.about.com/library/data/blc-t300-5208.htm , they have been of the following values:

T300/5208 Carbon/Epoxy Unidirectional Prepreg

Manufacturer	Hexcel

Fiber	**Matrix**	**Form**
T300 Carbon	5208 Epoxy	UD

Vf	**rho**	**tply**	**Temp**	**Cond**
0.70	1.60		22.2	Dry
	0.058		72.0	

E11	**E22**	**G12**	**nu12**
181	10.3	7.17	0.28
26.3	1.49	1.04	

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Table 1. Properties of the T300/5208 material.

Room temperature conditions have been considered. All eigen values have been normalized using the value of 50 N/mm. High-accuracy numerical solution has been used for comparison with proposed approach and calculation corresponding derivatives. The plate has been loaded by compression in *x*-direction and by shear.

The simple compression eigen value has been equal to 1.23, with the derivative $f_0^{/}$ being equal to -0.169. The lowest shear eigen value has been equal to 1.39, with the derivative $f_1^{/}$ being equal to -1.665. The lowest shear eigen value for the case of opposite loading has been equal to 1.80 (it should be taken with "-" in the above formulas).

The buckling modes 1 and 2 for pure compression (out-of-plane displacements) are shown in the Figs. 1 and 2, respectively. The maximal positive displacement is red, the maximal negative displacement is white.

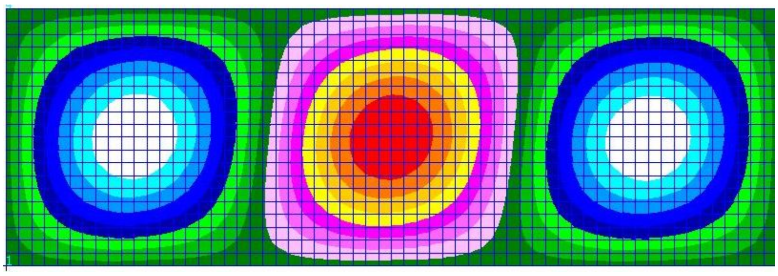

Fig. 1. First mode for pure compression.

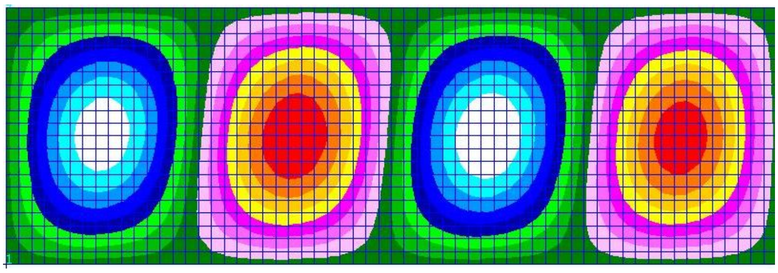

Fig. 2. Second mode for pure compression.

The buckling modes for pure shear are shown in the Figs. 3 and 4, respectively.

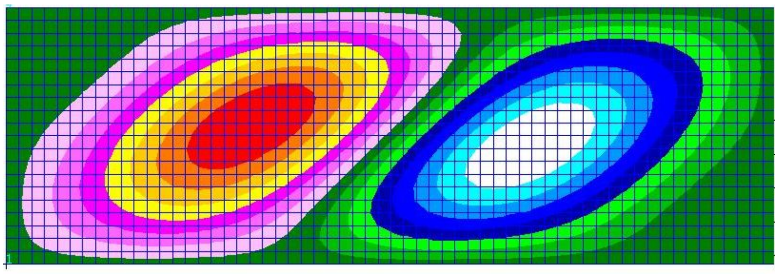

Fig. 3. First pure shear mode.

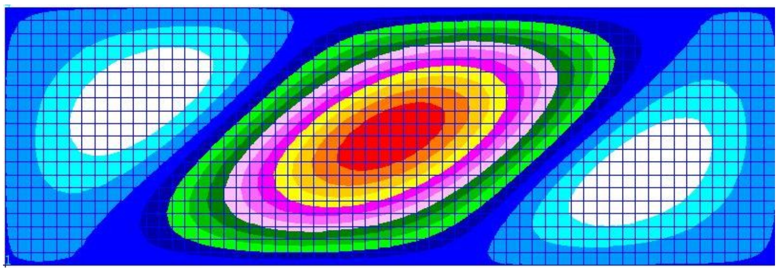

Fig. 4. Second pure shear mode.

It has been identified that for Nx/Nxy = 0.80 the lowest eigen value 1.14 is duplicated (see Figs. 5 and 6). Calculating the slope of the curve Nx(Nxy) according to (2), we obtain two values for them, namely -1.05 and -1.19 (13.3% difference). The first value corresponds to the mode of Fig. 6, the second value corresponds to the mode of Fig. 5.

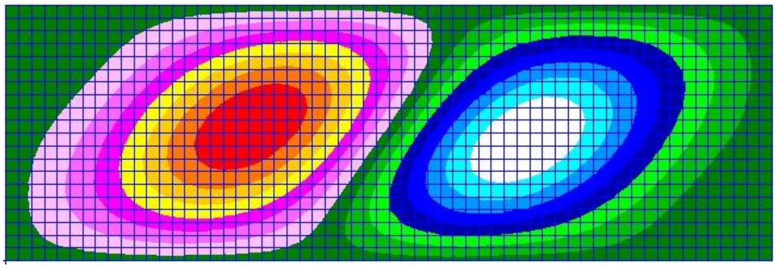

Fig. 5. First duplicated mode with eigen value 1.14.

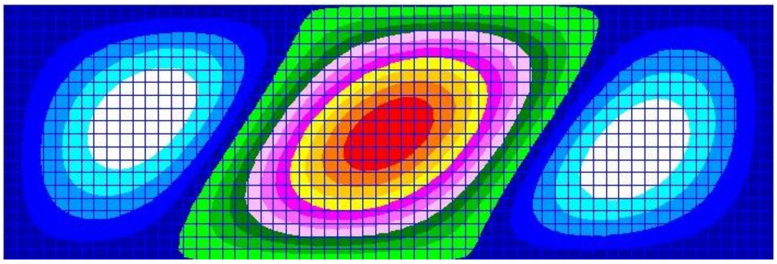

Fig. 6. Second duplicated mode with eigen value 1.14.

Observing the Figs. 1-6, one may say that Fig. 5 could be some combination of Figs. 2-3, and Fig. 6 could be some combination of Figs. 1, 4. The deflections of the indicated pairs have been used for the above-described special Galerkin-type approach.

In the Fig. 7 the results for different approaches are shown. It is seen that there is a considerable difference in interaction curves, when the flexural anisotropy (bending-twisting coupling) is taken and not taken into account. The use of (4)-(6) gives approximately two times closer results to high-accuracy numerical solution ones, as compared to the Balabukh formula. The (4)-(6) approach gives accuracy about 1%. Both Balabukh and (4)-(6) approaches give the curves coming below the high-accuracy solution. The special Galerkin-type approach gives critical force flows slightly higher (up to 1-3%) than for the high-accuracy solution. The level of closeness to the high-precision numerical curve is higher for interaction of mode1-compression and mode1 – shear, rather than for the mode2-compression and mode1 – shear, especially close to the switching point.

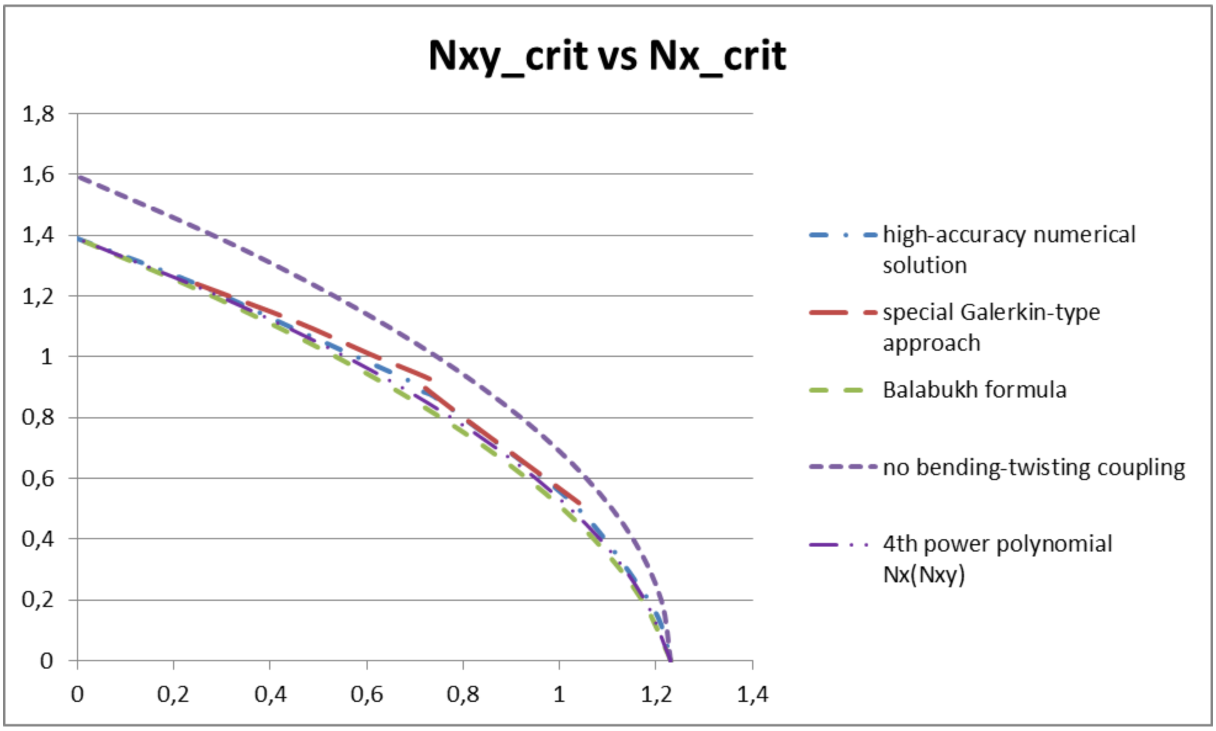

Fig. 7. Buckling interaction curves.

4. Conclusions

The presented results demonstrate some new ways of analysis of predicting buckling of composite plates under combined loading. The approaches allow creating the buckling interaction curves using the information on separate loading solutions only.

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