# FAILURE PREDICTION CAPABILITIES OF COMPOSITE FAILURE CRITERIA UNDER OUT-OF-PLANE LOADS

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**Abstract**
*In this study, failure behavior of fiber-reinforced composites under out-of-plane loads is investigated by means of four – point bending tests. Four – point bending tests are modeled analytically using the classical lamination theory (CLT) and numerically using finite element method (FEM). Considering unidirectional* $[\theta_6]_s$ *as well as balanced symmetric* $[\theta_3/-\theta_3]_s$ *composite laminates, the maximum allowable moment resultant predictions of Tsai-Wu, maximum stress, maximum strain, Hashin, Tsai-Hill, Hoffman, quadric surfaces, modified quadric surfaces and Norris failure criteria, as a function of fiber orientation angle, θ, are obtained. Experiments are conducted for 0°, 5°, 15°, 30°, 45°, 60°, 75°, and 90° fiber angles and the differences between the model predictions and experimental results are discussed.*

## 1. Introduction

Composite materials have long been used because of their superior specific strength and specific stiffness properties as compared to classical engineering materials, e.g. metals. One can tailor the laminate configuration to achieve desired characteristics. For the safe use of composite plates, one should use reliable failure theories during design stage that can correctly predict failure under given loading conditions for any chosen laminate configuration.

There are many studies in the literature [1-8] on the prediction of macro scale failure in fiber reinforced laminated composites; yet, these criteria have not been fully examined. Although the predictive capabilities of the most frequently used criteria are extensively tested under in-plane loading [9-11]; however, they have not been investigated adequately under out-of-plane loads. Studies on the predictive capabilities of composite failure criteria under out-of-plane loads are limited in number [12-15].

## 2. Analytical and finite element model of the problem

Analytical and finite element model of the problem is the same with the study of Sonmez et al. [15]. The only difference is that five additional criteria, Tsai-Hill, Hoffman, maximum strain, modified quadric surfaces and Norris failure criteria are included in addition to Tsai-Wu, maximum stress, quadric surfaces and Hashin criteria.

## 3. Failure criteria

### *3.1. Tsai-Hill criterion*

Hill [3] proposed a stress based, quadratic, failure mode independent criterion which accounts for stress interaction. Considering that the material is transversely isotropic, the failure criteria simplifies to the following under plane stress condition

$$\left(\frac{\sigma_{11}}{X}\right)^2 + \frac{\sigma_{11}\sigma_{22}}{X^2} + \left(\frac{\sigma_{22}}{Y}\right)^2 + \left(\frac{\sigma_{12}}{S_{12}}\right)^2 \geq 1 \tag{1}$$

wherein X and Y are either tensile or compression strengths depending on the sign of respective stresses.

### *3.2. Hoffman criterion*

Hoffman [4] introduced a failure criterion by adding linear terms to Hill's criterion. Just like Tsai-Hill failure criterion, Hoffman criterion is a stress based, quadratic, failure mode independent criterion which accounts for stress interaction. It is stated as

$$-\frac{\sigma_{11}^{\;2}}{X_t X_c} + \frac{\sigma_{11}\sigma_{22}}{X_t X_c} - \frac{\sigma_{22}^{\;2}}{Y_t Y_c} + \frac{X_t + X_c}{X_t X_c}\sigma_{11} + \frac{Y_t + Y_c}{Y_t Y_c}\sigma_{22} + \frac{\tau_{12}^{\;2}}{S_{12}^{\;2}} \geq 1 \tag{2}$$

### *3.3. Maximum strain criterion*

Maximum strain criterion is a strain based criterion. It is also linear and failure mode dependent; yet, it does not account for interaction between strains.

$$X_{\varepsilon_c} \leq \varepsilon_{11} \leq X_{\varepsilon_t} \tag{3}$$

$$Y_{\varepsilon_c} \leq \varepsilon_{22} \leq Y_{\varepsilon_t} \tag{4}$$

$$\left|\varepsilon_{12}\right| \leq S_{\varepsilon} \tag{5}$$

where $X_{\varepsilon_t}$ and $X_{\varepsilon_c}$ are the maximum tensile and compression strains in the 1-direction, $Y_{\varepsilon_t}$ and $Y_{\varepsilon_c}$ are the maximum tensile and compression strains in the 2-direction and $S_\varepsilon$ is the maximum shear strain in the 1-2 plane, respectively. $M_{max}$ is found for a lamina by substituting the strain components in the principal material coordinates, $\varepsilon_{11}$, $\varepsilon_{22}$, and, $\varepsilon_{12}$, into Eqs. 3-5 for the equality cases.

### *3.4. Modified quadric surfaces criterion*

The modified quadric surfaces [6] failure criterion for the composite materials is a modification of quadric surfaces criterion. The difference between them is that coefficients of in-plane and shear coupling terms are assumed to be zero in the latter one.

$$\frac{a}{X^2}\sigma_{11}^{\;2} + \frac{a}{Y^2}\sigma_{22}^{\;2} + \frac{a}{S_{12}^{\;2}}\tau_{12}^{\;2} + \frac{b}{XY}\sigma_{11}\sigma_{22} + \frac{c}{X}\sigma_{11} + \frac{c}{Y}\sigma_{22} + \frac{c}{S_{12}}\tau_{12} \geq 1 \tag{6}$$

### *3.5. Norris criterion*

Norris [7] proposed a failure theory for orthotropic materials based on the Henky-von Mises energy theory. It is a non-linear, stress-based criterion. The criterion accounts for stress interaction; however, it does not account for failure mode. According to Norris, the onset of failure occurs if at least one of the following equations is satisfied:

$$\left(\frac{\sigma_{11}}{X}\right)^2 + \left(\frac{\sigma_{22}}{Y}\right)^2 - \frac{\sigma_{11}\sigma_{22}}{XY} + \left(\frac{\sigma_{12}}{S_{12}}\right)^2 \geq 1 \quad (7)$$

$$\left(\frac{\sigma_{11}}{X}\right)^2 \geq 1 \quad (8)$$

$$\left(\frac{\sigma_{22}}{Y}\right)^2 \geq 1 \quad (9)$$

One may refer to reference [15] for Tsai-Wu, maximum stress, quadric surfaces, and Hashin criteria. In order to calculate the maximum allowable moment, $M_{max}$, the four-point-bending test is simulated. The deflection caused by the upper supports in the plate that makes the maximum failure index equal to 1.0 according to the above-mentioned failure criteria is found, iteratively. Then, the reaction forces at the supports are calculated, and then the moment due to these forces.

## 4. Experiments

Experimental studies are conducted in two stages: Manufacturing of composite plates and four-point bending tests. The plates were manufactured by stacking individual AS4/8552 unidirectional prepregs, a carbon-fiber-reinforced epoxy, in a 118 × 190 [$mm^2$] mold with desired stacking sequence and cured in accordance with the prescription of the manufacturer. A typical prepreg's thickness is 0.184 [mm] and fiber volume fraction is 57.42%. Two or three specimens with 48 × 96 × 2.21 [$mm^3$] dimensions were cut from each plate. The test setup, which is designed with the help of FEM analyses, is manufactured from forged steel at Bogazici University machine workshop. The supports are made of carbon steel. The test machine is an electric controlled Zwick/Roell with 10 [kN] maximum loading capacity.

For each 15° of fiber orientation angle, θ, from 0° to 90°, five samples were tested. Some of the chosen failure criteria predict a slight increase in strength as the fiber angle is varied from 0 to 3 - 5 degrees. $[5_6]_s$ plates are also tested to observe the correlation between the experimental results and predictions. The samples were cut off from at least two different plates by means of a power saw to prevent consistent error due to manufacturing defects in a given sample.

## 5. Results and discussion

The material chosen for the simulations is AS4/8552. Mechanical and thermal properties of the material are given in the previous study [15]. Abscissas of the figures below are drawn in logarithm base two in order to show the failure trend prediction of the criteria more clearly. Figures 1-2 illustrates the analytical and finite element predictions of Tsai-Wu, maximum stress, maximum strain, Hashin, Tsai-Hill, Hoffman, quadric surfaces, modified quadric surfaces and Norris failure criteria for unidirectional off axis $[\theta_6]_s$ laminates as a function of fiber orientation angle, θ, and their comparison with the experimental results. Figures 3-4, on the other hand, are similar with Figures 1-2. The only difference is that they belong to symmetrically balanced $[+\theta_3/- \theta_3]_s$ laminates. In symmetrically balanced laminates, residual stresses are also included in the model predictions.

Figure 1 shows that finite element model based predictions of the Tsai-Wu, Tsai-Hill, Hoffman, quadric surfaces, modified quadric surfaces and Norris criteria are very close to the experimental results. Failure trend predictions of Tsai-Wu, Tsai-Hill, Hoffman and Norris criteria are very similar for unidirectional laminates as expected, because these criteria have similar characteristics. In general, predictions of these criteria for unidirectional laminates are either very close to the average value of the experimental results or lower. The only exceptions are 45˚ and 75˚ in which the strength of the material is over estimated. As it is shown in the figures, average strength of specimens with 75˚ specimens is minimum and even less than the average strength of specimens with 90˚ fiber orientation angle. All of these three criteria predict this failure trend well. Finite element model based predictions of quadric surfaces criterion are more coherent with the experimental results as compared to modified quadric surfaces criterion. As it is shown in Figure 1, quadric surfaces criterion predicts the minima at 75˚ orientation angle better than all of the chosen criteria for unidirectional laminates. Analytical predictions of these criteria are smooth. The only exception is the modified quadric surfaces criterion in which there is a sharp change around 40˚.

Figure 2 shows the predictions of maximum stress, maximum strain, and Hashin criteria. All of these criteria are physically based, i.e. they predict the failure mode of a laminate. These criteria predict a slight increase in strength in first few degrees of the orientation angle, which is difficult to explain physically. At it is seen from the figure, they overestimate the average strength of laminates with 5˚ orientation angle. From 0˚ to 30˚, finite element model based predictions of maximum stress and maximum strain criteria correlate better with the experimental results as compared to analytical model; however, from 45˚ to 60˚, analytical model based predictions seems more realistic. Additionally, the finite element model based predictions predicts a decrease at strength at 75˚ orientation angle, the analytical model does not show this trend.

According to Figure 3, finite element model and analytical model based predictions of the Tsai-Wu, Tsai-Hill, Hoffman, quadric surfaces, modified quadric surfaces and Norris criteria are very close to each other. All of these criteria underestimates the strength of the laminates especially from 15˚ to 45˚ orientation angle. Analytical model based predictions of quadric surfaces, modified quadric surfaces and Norris criteria are not smooth which is different than Tsai-Wu, Tsai-Hill and Hoffman criteria. All of these criteria predicts the decrease in strength about 60˚ orientation angle.

Maximum stress, maximum strain and Hashin criteria predicts an increase in strength in first few degrees for symmetrically balanced laminates as well, as it is demonstrated in Figure 4. Analytical model based predictions of maximum stress criterion is more successful at the prediction of failure strength of the laminates as compared to the other criteria. Hashin criterion is the least successful among these three criteria which is shown in Figure 4.

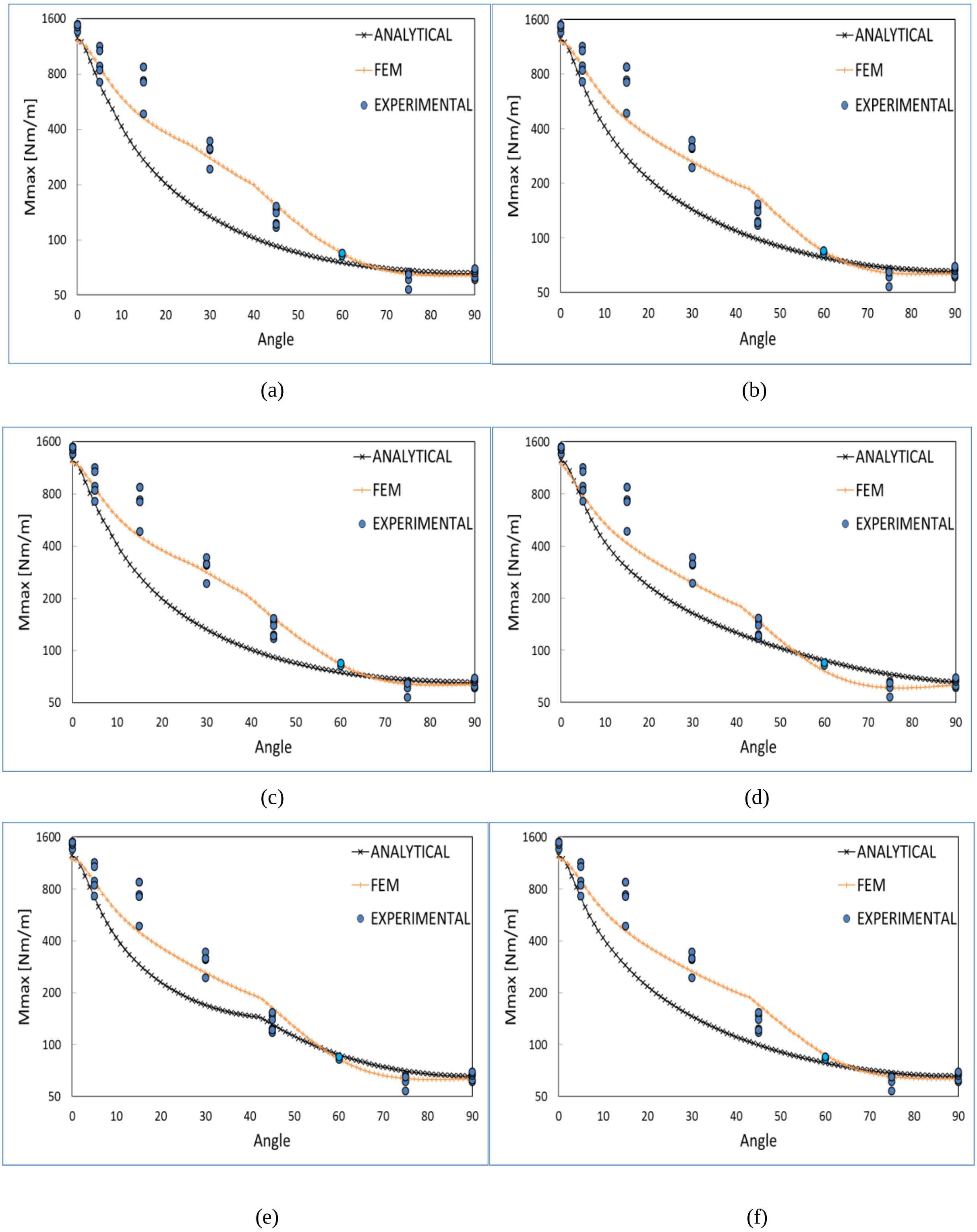
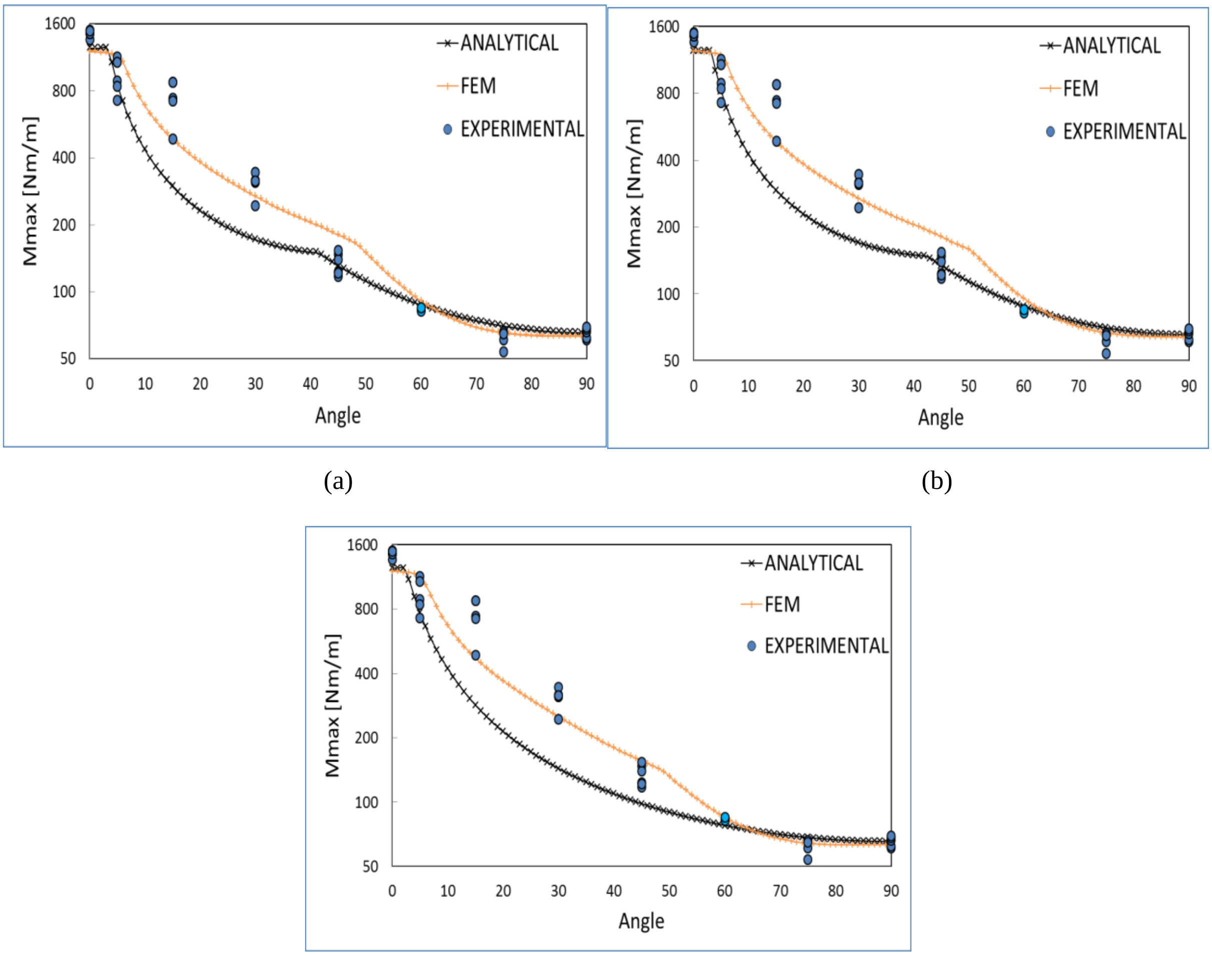


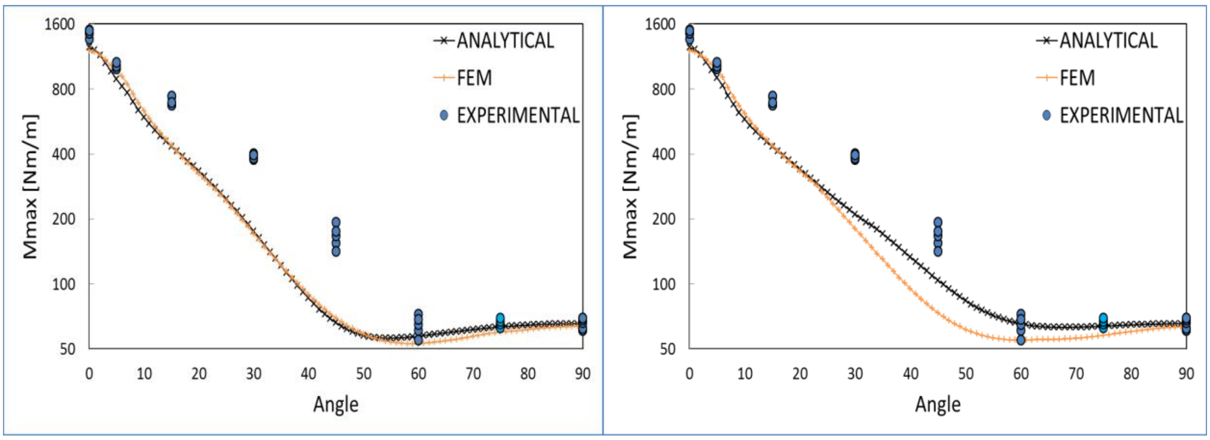

Figure 1. Comparison of the analytical and finite element $M_{max}$ predictions obtained using (a) Tsai-Wu, (b) Tsai-Hill, (c) Hoffman, (d) the quadric surfaces (e) the modified quadric surfaces (f) Norris criteria for unidirectional off-axis $[\theta_6]_s$ specimens with the experimental results.



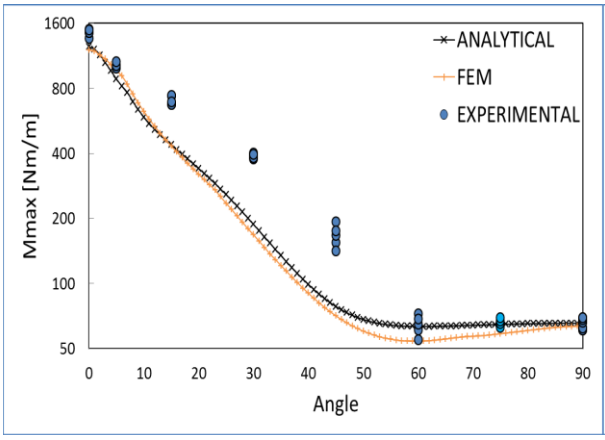

(a) (b)

(c)

Figure 2. Comparison of the analytical and finite element $M_{max}$ predictions obtained using (a) maximum stress, (b) maximum strain, (c) Hashin criteria for unidirectional off-axis $[\theta_6]_s$ specimens with the experimental results.




(a) (b)

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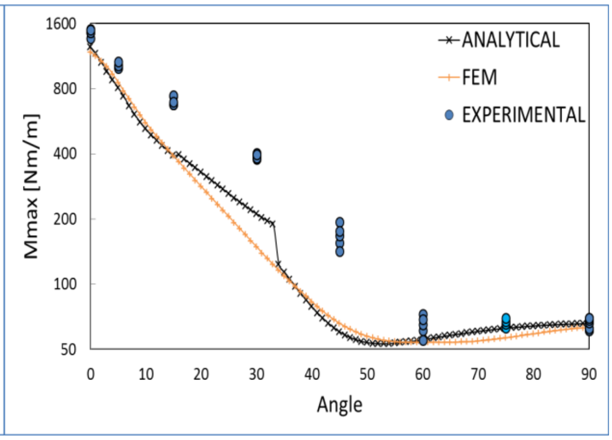
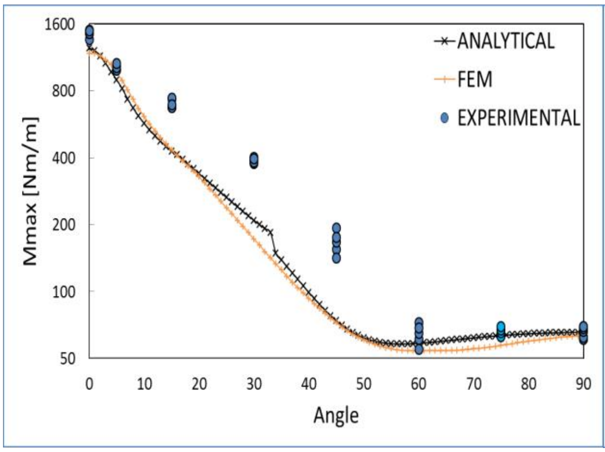



(d)

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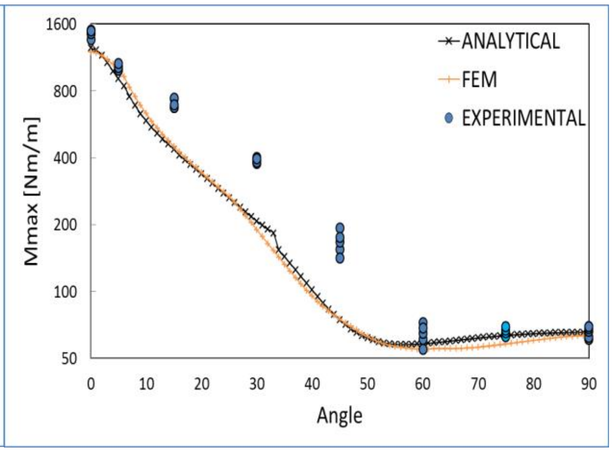
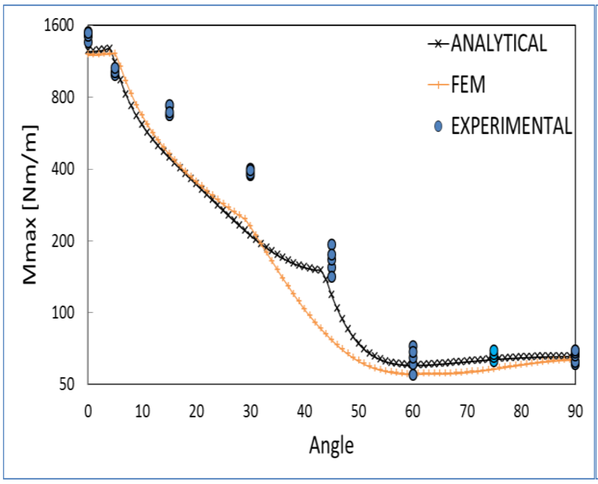
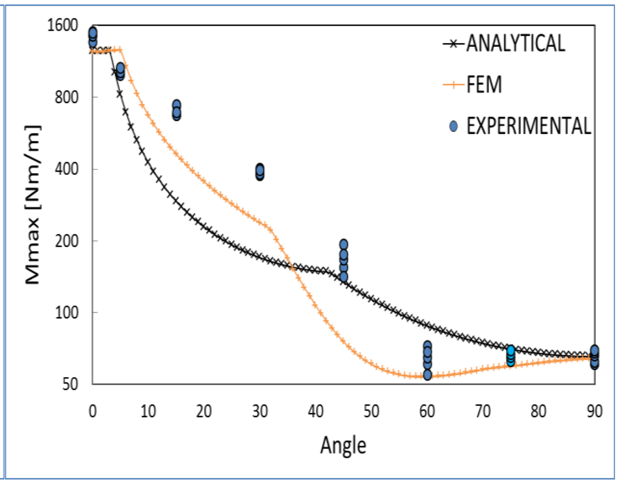



(f)

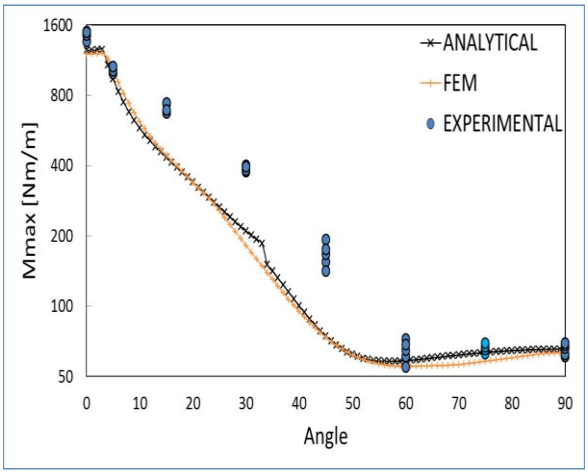
Figure 3. Comparison of the analytical and finite element $M_{max}$ predictions obtained using (a) Tsai-Wu, (b) Tsai-Hill, (c)Hoffman, (d) Quadric Surfaces (e) Modified Quadric Surfaces (f) Norris criteria for multidirectional $[+\theta_3/-\theta_3]_s$ specimens with the experimental results.




(a)




(b)



(c)

Figure 4. Comparison of the analytical and finite element $M_{max}$ predictions obtained using (a) Maximum Stress, (b) Maximum Strain, (c) Hashin criteria for multidirectional $[+\theta_3/-\theta_3]_s$ specimens with the experimental results.

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