Statistical Prediction of Tensile Creep Failure Time for Unidirectional CFRP

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Abstract
This paper is concerned with the statistical prediction of creep failure time under the tension loading along the longitudinal direction of unidirectional CFRP based on the viscoelasticity of matrix resin. It was cleared that the statistical creep failure time under the tension loading along the longitudinal direction of unidirectional CFRP can be predicted by using the statistical static tensile strengths of carbon mono filament and unidirectional CFRP and the viscoelasticity of matrix resin based on Rosen's model for unidirectional CFRP and Christensen's model of viscoelastic crack kinetics.

1. Introduction
Carbon fiber reinforced plastics (CFRP) has been used for the primary structures of airplanes, ships, automobiles and others, in which the high reliability should be kept during the long-term operation. Therefore, it is strongly expected that the accelerated testing methodology for the long-term life prediction of CFRP structures exposed under the actual environmental temperature, water and others will be established.

The mechanical behavior of matrix resin of CFRP exhibits time and temperature dependence, called viscoelastic behavior, not only above the glass transition temperature T_g but also below T_g. Thus, it can be presumed that the mechanical behavior of CFRP significantly depends on time and temperature [1-6]. We have proposed the formulation for the statistical static strength of CFRP based on the viscoelasticity of matrix resin in our previous papers [7, 8].

The tensile strength along the longitudinal direction of unidirectional CFRP is one of the important data for the reliable design of CFRP structures. The authors developed the test method for the creep and fatigue strengths as well as the static strength at elevated temperatures for the resin impregnated carbon fiber strand (CFRP strand) combined with T300-3000 and epoxy resin [9-11]. Recently the authors have developed the test method for the CFRP strand of T800-12000 and epoxy resin with highly reliable co-cured tab and the temperature dependent tensile strength of this CFRP strand was successfully evaluated [12, 13].

This paper is concerned with the prediction of statistical creep failure time under the tension loading along the longitudinal direction of unidirectional CFRP. The CFRP strands of T800-

12000 and epoxy resin for high reliable static and creep tests were employed in this study. First, the static tensile tests for carbon mono filament were carried out at four levels of specimen length by using 50 specimens for each length. Second, the static tensile tests for CFRP strands were carried out at three levels of constant temperature by using 50 specimens for each temperature. Third, the statistical creep failure time at a constant load and temperature was predicted using the statistical results of static tensile strengths at three temperatures and the viscoelastic behavior of matrix resin. Finally, the validity of predicted results was cleared by comparing with the experimental results obtained by the creep tests for CFRP stands of 41 specimens.

2. Statistical Prediction of Creep Failure Time of Unidirectional CFRP

We have proposed the formulation for the statistical static strength σ_s of CFRP based on the viscoelasticity of matrix resin as shown in the following equation in our previous paper [8],

$$\log \sigma_s(P_f, t, T) = \log \sigma_0(t_0, T_0) + \frac{1}{\alpha_s}\log[-\ln(1-P_f)] - n_R \log\left[\frac{D^*(t,T)}{D_c(t_0,T_0)}\right] \tag{1}$$

where, P_f is the failure probability, t is the failure time, t_0 is the reference time, T is the temperature, T_0 is the reference temperature, σ_0 and σ_s are the scale parameter and the shape parameter on Weibull distribution of static strength, n_R is the viscoelastic parameter, and D_c and D^* are the creep and viscoelastic compliances of matrix resin. The viscoelastic compliance D^* for the static load with a constant strain rate is shown by the following equation.

$$D^*(t,T) = D_c(t/2, T) \tag{2}$$

The failure probability of unidirectional CFRP under static load with a constant strain rate can be shown by the following equation.

$$P_f = 1 - \exp(-F), \ \log F = \alpha_s \log\left[\frac{\sigma_s}{\sigma_0}\right] + \alpha_s n_R \log\left[\frac{D_c(t/2, T_0)}{D_c(t_0, T_0)}\right] \tag{3}$$

It was cleared in our previous paper that the viscoelastic parameter n_R in equations (1) and (3) is shown by the following equation for the unidirectional CFRP based on Rosen’s model [11].

$$n_R = \frac{1}{2\alpha_C} \tag{4}$$

where, α_c is Weibull shape parameter for the tensile strength of carbon fiber mono-filament.

The relationship between the creep failure time and the static failure time can be shown by Fig.1 [14]. This figure shows the creep strength and the static strength versus failure time. The creep strength curve can be obtained by shifting horizontally the static strength curve. The shifting amount log A determined by the slope of the static strength curve is shown by the following equation.

$$\log A = \log(1 + 1/k_R) \tag{5}$$

Figure 2(a) shows the slope m_R of the creep compliance of matrix resin against time. Figure 2(b) shows the slope k_R of the static strength of CFRP against failure time. The slope k_R can be obtained from the following equation.

$$k_R = n_R m_R = \frac{m_R}{2\alpha_C} \tag{6}$$

The failure probability against failure time at an arbitrary constant load under static loading (constant strain rate) is predicted by using equations (5) and (6) as shown in Fig.3 and the creep failure probability against failure time can be predicted by shifting horizontally that of the static loading.

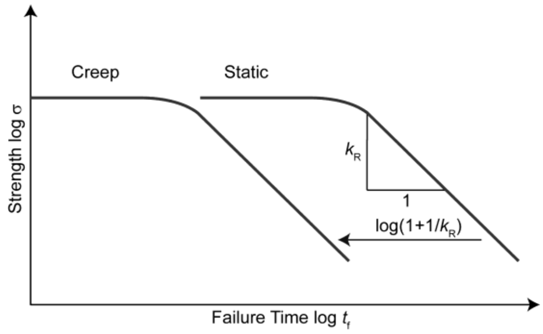


Figure 1. Time shifting between static strength and creep strength.

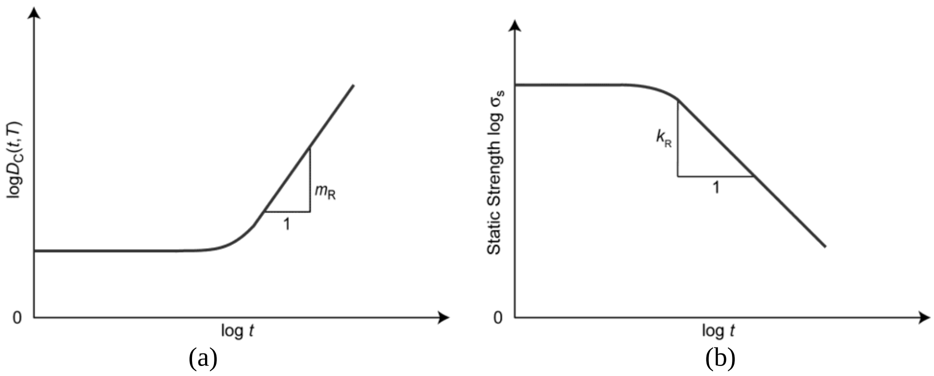


Figure 2. Slope against time. (a) Creep compliance of matrix resin. (b) Static strength of CFRP.

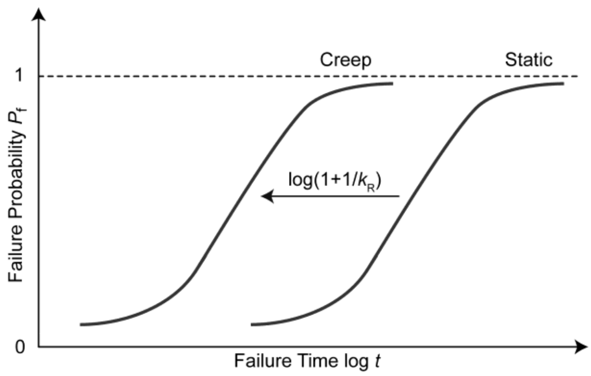


Figure 3. Time shifting between failure probability of static and creep.

3. Experiments

3.1. Specimens

CFRP strand which consists of high strength type carbon fiber T800-12000 (Toray Industries Inc.) and a general purpose epoxy resin jER828 (Mitsubishi Chemical Corp.) was molded using filament winding method developed by authors [12]. The composition of epoxy resin and the cure condition of CFRP strand are shown in Table 1. The diameter and the gage length of CFRP strands are approximately 1mm and 200mm, respectively. The glass transition temperature T_g of the epoxy resin is approximately 150˚C. The fiber volume fraction of CFRP strand is approximately 50%. The tensile strength of the CFRP strand σ_s is defined by

$$\sigma_s = \frac{P_{max}}{t_e}\rho \tag{7}$$

where P_{max} is maximum load [N]. ρ and t_e are the density of the carbon fiber [kg/m^3] and the tex of the carbon fiber strand [g/1000m].

Table 1 Carbon fiber strand and resin system

Carbon fiber strand	Composition of resin (weight ratio)	Cure schedule
T800-12000	Epoxy: jER828 (100) Hardener: MHAC-P (103.6) Cure accelerator: 2E4MZ (1)	100°C×5h + 150°C×4h + 190°C×2h

3.2. Tensile Strength of Carbon Fiber Mono-filament and Creep Compliance of Matrix Resin

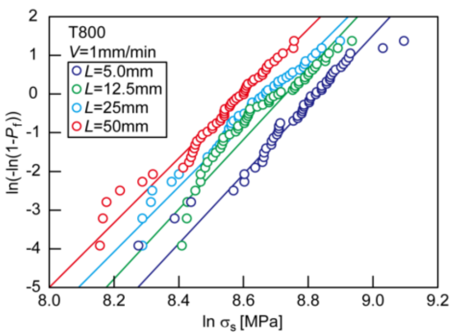
In our previous paper [13], tensile tests of carbon fiber T800 mono filament for several lengths of filament were conducted at the room temperature to determine the Weibull shape parameter for the tensile strength of the carbon fiber T800 mono filament. The tensile test speed was 1 mm/min. The Weibull distributions of the tensile strength of the carbon fiber T800 mono filament are shown in Fig.4.

The scale parameter β_c* for an arbitrary fiber length L is shown by the following equation based on one dimensional chain model.

$$\beta_c^*(L) = \left(\frac{L_0}{L}\right)^{\frac{1}{\alpha_c}} \beta_c(L_0) \tag{8}$$

The statistical static strength of carbon fiber T800 shows Weibull distribution based on the one dimensional link model because the shape parameter α_c is constant for various fiber lengths L and the experimental scale parameter β_c agrees well with the predicted one β^*_c for the same fiber length. The shape parameter of carbon fiber T800 can be fixed to $\alpha_c = 8.0$.

The dimensionless creep compliance D_c/D_{c0} measured at various temperatures is shown in the left of Fig.5 and the long-term D_c/D_{c0} at T=120°C is obtained by shifting horizontally those at various temperatures as shown in the right of Fig.5. The reference temperature and time are selected as T_0=25 °C and t_0=1min in this study. Creep compliance at reference temperature and reference time D_{c0} is 0.33(GPa)$^{-1}$. The dashed curve is the dimensionless viscoelastic compliance D* of matrix resin under the constant strain rate at T=120°C.



L[mm]	αc[-]	βc[MPa]	β'c[MPa]
5.0	7.8	6,870	6,870
12.5	8.1	6,140	6,123
25.0	8.0	5,890	5,490
50.0	8.0	5,430	5,152

Figure 4. Weibull distributions of the tensile strength of T800 mono filament.

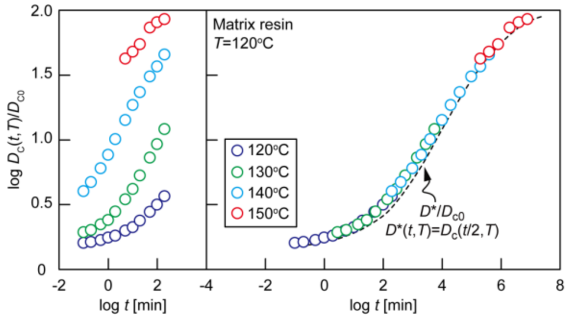


Figure 5. Dimensionless creep compliance of matrix resin at T=120^{o}C.

3.3. Static Strength of CFRP Strand

In our previous paper [13], the static strength tests for CFRP strand were conducted at 3 levels of temperature, 25°C, 120°C, and 150°C with cross-head speed 2mm/min. The Weibull distributions of the static strength of CFRP strand at three temperatures are shown in Fig.6. α_s is the shape parameter and β_s is the scale parameter of CFRP strand in this figure. While the scale parameter decreases according to the temperature raise, the shape parameter keeps almost a constant value.

Figure 7 shows the dimensionless static strength of CFRP strand σ_s/σ_0 against the dimensionless viscoelastic compliance of matrix resin D^*/D_{c0} at the same time and temperature. The scale parameter of static strength at the reference temperature 25°C and the reference failure time 1min σ_0 is 5,608MPa. The slope agrees well with the solid line predicted by Rosen's model.

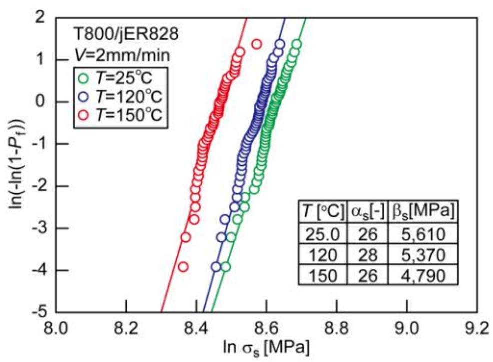


Figure 6. Weibull distributions of static tensile strength of CFRP strand at three temperatures.

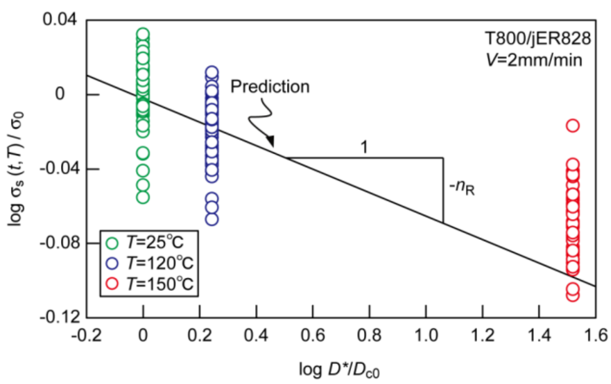


Figure 7. Static strength of CFRP strand versus viscoelastic compliance of matrix resin.

3.4. Creep Failure Tests of CFRP Strand

Creep failure tests of T800 CFRP stand were conducted by using the specially designed creep failure testing machine shown in Fig.8. The test condition is shown in Table 2. The results of the creep failure tests are shown in Fig.9.

3.5. Statistical Prediction of Creep Failure Time of CFRP Strand

As shown in Fig.9, the statistical static failure time were obtained by Equations (3) and (4) and the statistical creep failure time were obtained by shifting horizontally the statistical static failure time by the amount log A calculated by substituting the parameters on Table 3 in Equations (5) and (6). The predicted statistical creep failure time agrees well with the experimental data.

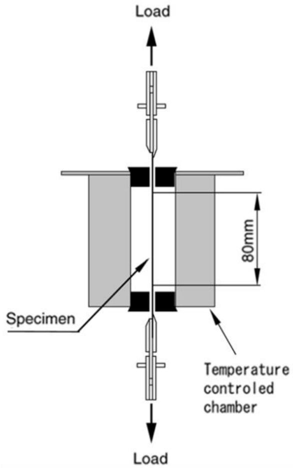


Figure 8. Creep failure testing machine for the CFRP strand specimen.

Table 2. Condition of creep failure test of T800 CFRP strands

Temperature (°C)	Stress (MPa)	Number of specimens
120	4,564 *	41

* 81.4% of scale parameter of static strength at 25°C

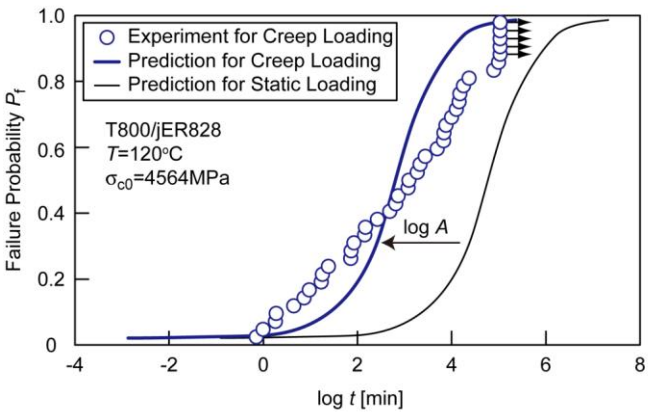


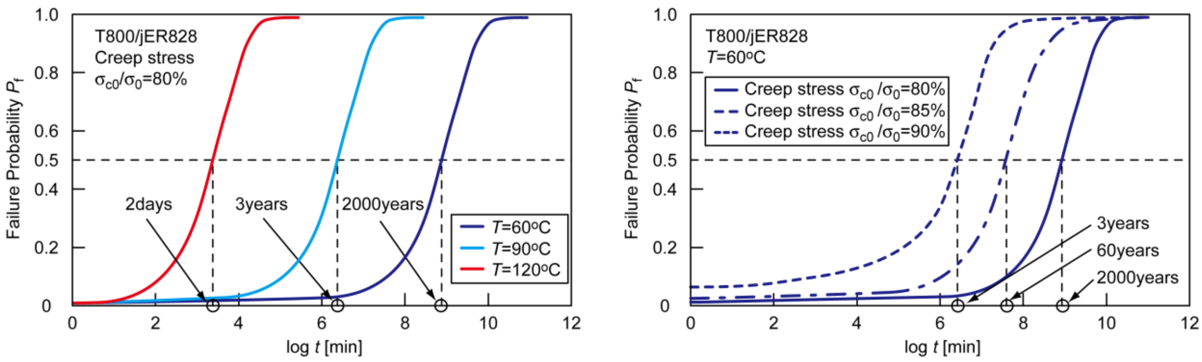
Figure 9. Probability of creep failure time of T800 CFRP strand.

Table 3. Parameters for statistical creep failure time prediction

Shape parameter of static strength of CFRP strand: α_s	26
Shape parameter of static strength of carbon fiber mono-filament: α_c	8.0
Viscoelastic parameter of matrix resin: n_R	0.063
Slope of viscoelastic compliance of matrix resin: m_R	0.28
Slope of static strength of CFRP strand against failure time: k_R	0.018
Logarithmic time shifting factor: $\log A$	1.8

4. Probability of creep failure time

The probability of creep failure time at various temperatures for a constant creep stress is shown in Fig.10 and that at various creep stress for a constant temperature is shown in Fig.11 from substituting the parameters of Table 3 into Equations (3)~(6). It is cleared from these figures that the creep failure time of CFRP strand drastically change with temperature and creep stress.



(a) various temperatures for a constant creep stress (b) various creep stresses for a constant temperature

Figure 10. Probability of creep failure time at various temperatures for a constant creep stress

5. Conclusions

We proposed the prediction method for statistical creep failure time under the tension loading along the longitudinal direction of unidirectional CFRP using the statistical static tensile strengths of carbon mono filament and CFRP strand and the viscoelasticity of matrix resin based on Rosen's model for unidirectional CFRP and Christensen's model for viscoelastic crack kinetics. The applicability of prediction method can be confirmed through the following steps.

1. The statistical static strength of carbon fiber T800 shows Weibull distribution based on the one dimensional link model. The shape parameter is independent of fiber length and the scale parameter decreases with increasing of fiber length.
2. The statistical static strength of T800 CFRP strand shows Weibull distribution based on Christensen's model of viscoelastic crack kinetics. The shape parameter is independent of the creep compliance of matrix resin. The scale parameter decreases with increasing of the creep compliance of matrix resin based on Rosen's model for unidirectional CFRP.
3. The statistical creep failure time at a constant load and temperature predicted using the statistical static tensile strengths of T800 CFRP strand and the viscoelasticity of matrix resin based on Christensen's model for viscoelastic crack kinetics agrees with the experimental results obtained by the creep rupture tests for T800 CFRP stands.

4. It is cleared from the statistical prediction that the creep failure time of CFRP strand drastically change with temperature and creep stress.

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