MEASUREMENT OF THE INTRALAMINAR FRACTURE TOUGHNESS AND R-CURVE OF POLYMER COMPOSITES LAMINATES USING THE SIZE EFFECT LAW

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Abstract

This paper presents a new methodology to measure the crack resistance curve associated with the longitudinal and the shear failure of polymer composites reinforced by unidirectional fibres. Rather than using standard test methods the identification of the size-effect law of is used to obtain the crack resistance curve. Special emphasis is placed on the appropriate calculation of the stress intensity factor of the specimens when using quasi-isotropic or cross-ply laminates. For this purpose, both analytical closed-form solutions and numerical methods are investigated.

1. Introduction

The most recent analysis methods that predict fracture of polymer composite materials require not only the value of the fracture toughness, but also its relation with the increment of the crack length, i.e., the crack resistance curve. Taking the thickness of the individual ply as the representative length scale it is possible to formulate 'mesomodels' that account for both delamination (interlaminar cracking) and ply failure mechanisms (intralaminar cracking) [1, 2, 3]. The softening constitutive relation that simulates longitudinal failure, where the fracture plane is approximately perpendicular to the fibre direction, requires the fracture toughness to regularize the numerical solution [3]; however, the crack resistance curve must also be measured to identify the different regions of the softening constitutive relation so that the failure mechanisms acting at the crack tip and along the wake of the crack are properly accounted for [4].

Recently, Finite Fracture Mechanics models that use the laminate thickness as the representative length-scale have been developed to predict fracture of multidirectional composite laminates in the presence of stress concentrations [5, 6, 7]. These methods are typically used for the preliminary design and optimization of composite structures, and are based on the simultaneous fulfillment of a stress-based criterion, which requires a stress allowable, and of an energy based criterion, which requires the fracture toughness [5, 6, 7] or the crack resistance curve [8].

Based on the above observations, it becomes apparent that reliable test methods for the measurement of the intralaminar fracture toughness of composite laminates and of the corresponding crack resistance curve (R-curve) are required. While a strong emphasis has been placed on the use of compact tension test specimens [9], recent results have shown that using the current geometry of the compact tension test specimen it is not possible to measure the fracture toughness of modern resin systems that result in high values of the fracture toughness [10]. For example, in previous attempts to measure the fracture toughness of cross-ply Hexcel's T800/M21 carbon-epoxy laminates using the geometry proposed in [9] the region of the specimen subjected to compressive stresses buckled [10]; such an elastic instability renders the test results meaningless.

There is also the need to measure the fracture toughness and the corresponding R-curve associated to the propagation of a kink-band, which shows a crack-like behaviour [11, 12, 13] with an R-curve that results from the broadening of the damage height [12]. It is considered here that the compact compression test specimen is inadequate to measure the compressive crack resistance curves of polymer composite materials. In fact, the correction factor used in the data reduction method of the compact compression test method to calculate the energy release rate is the same as that used in the compact tension method. However, the contact tractions that occur on the crack faces during a compact compression test render the data reduction method inaccurate. This was demonstrated in a previous investigation [14] where the J-integral around the crack tip was computed using digital image correlation. It was shown [14] that the R-curve of the compact test specimens using the J-Integral and the data reduction method proposed by Pinho et al. [9] are virtually the same for tension but not for compression.

It should also be noted that the compact compression specimen triggers diffused damage during the propagation of the kink-band, artificially increasing the value of the measured fracture toughness, and that it is not possible to identify the location of the tip of the kink band [14]. Therefore, it is considered that while the compact compression test method may be used to measure the initial value of the fracture toughness it does not provide reliable information for the generation of the R-curve.

The experimental difficulties in drastically increase when measuring the R-curve in mode II. In this case no tests method exists and, to the authors' best knowledge, no relevant work has been done using fibre-reinforced plastics. However, the measurement of the R-curve in mode II could be very valuable because it would enable the generalization of several analysis models developed for mode I propagation (e.g., FFMs and progressive damage models).

Therefore, the objective of this paper is to develop a new methodology to obtain the R-curve of polymer composite laminates reinforced by unidirectional fibres. The main idea put forward here, which follows Bažant's seminal work [15], is to relate the size effect law with the crack resistance curve of the composite material.

2. Analytical method

In a two–dimensional orthotropic body, taking x_1 and x_2 as the preferred axes of the material, the mode I component of the energy release rate for crack propagation in the x_1-direction, $\mathcal{G}_I$, reads [16]:

$$\mathcal{G}_I = \frac{1}{\acute{E}}\,\mathcal{K}_I^2 \tag{1}$$

where $\mathcal{K}_I$ and $\acute{E}$ are respectively the stress intensity factor and the equivalent modulus. The equivalent modulus reads:

$$\acute{E} = \left(s_{11}s_{22}\frac{1+\rho}{2}\right)^{-1/2}\,\lambda^{1/4} \tag{2}$$

where s_{lm} are the components of the compliance matrix calculated in the x_1-x_2 coordinate system, and λ and ρ are two dimensionless elastic parameters defined as:

$$\lambda = \frac{s_{11}}{s_{22}}, \qquad \rho = \frac{2\,s_{12} + s_{66}}{2\,\sqrt{s_{11}s_{22}}} \tag{3}$$

The stress intensity factor in equation (1) is a function of ρ, of the remote stress σ, and of the shape and size of the specimen. Using the orthotropic rescaling technique [16, 17], the stress intensity factor can be written as:

$$\mathcal{K}_I = \sigma\,\sqrt{w}\,\phi(\alpha,\rho) \tag{4}$$

where $\alpha = a/w$ is the shape-parameters, and ϕ is the correction factor that depends on the geometry and orthotropy of the material.

Replacing (4) in (1) the energy release rate reads:

$$\mathcal{G}_I = \frac{1}{\acute{E}}\,w\,\sigma^2\phi^2 = \frac{1}{4\,w\,\acute{E}}\left(\frac{P\phi}{t}\right)^2 \tag{5}$$

where t is the thickness of the specimen and P is the applied load. Assuming that ϕ is an increasing function of the crack length (the specimen has a *positive geometry*), the size effect method can be used to measure the fracture toughness of the material [15]. Equation (5) can be re-written as:

$$\mathcal{G}_I\,(\Delta a) = \frac{P^2}{4\,w\,t^2\,\acute{E}}\,\phi^2\!\left(\alpha_0 + \frac{\Delta a}{w},\rho,\zeta\right) \tag{6}$$

where α_0 is the initial value of the shape parameter, $\alpha_0 = a_0/w$. For different sizes, w_n, the crack driving-force curves $\mathcal{G}_I$ corresponding to the peak loads, P_{un}, are tangent to R-curve, and this fact can be used to measure the R-curve, $\mathcal{R}$. Mathematically the peak load, P_u, or the ultimate nominal stress, $\sigma_u = P_u/\,(2wt)$, can be obtained from the following system of equations:

$$\begin{cases} \mathcal{G}_I(\Delta a) = \mathcal{R}(\Delta a) \\ \dfrac{\partial \mathcal{G}_I(\Delta a)}{\partial \Delta a} = \dfrac{\partial \mathcal{R}(\Delta a)}{\partial \Delta a} \end{cases} \tag{7}$$

Assuming that the size effect law is known, $\sigma_u = \sigma_u(w)$, using (5) in the first of equation (7) yields:

$$\mathcal{R}(\Delta a) = \frac{1}{\acute{E}}\, w\, \sigma_u^2\, \phi^2 \tag{8}$$

This equation is valid for every w. Following [15], differentiating (8) with respect to w, under the hypothesis that geometrically similar specimens are tested (α_0 and ξ are not functions of the width, w) and remembering that the R-curve does not depend on the size of the specimen w ($\partial\mathcal{R}/\partial w = 0$), the following equation is obtained:

$$\frac{\partial}{\partial w}\left(w\, \sigma_u^2\, \phi^2\right) = 0 \tag{9}$$

Equation (9) can be solved for $w = w(\Delta a)$. Replacing w in (8) yields the R-curve, $\mathcal{R}(\Delta a)$.

The size effect law should be determined by testing geometrically similar specimens. Following Bazant and Planas [15], three different kinds of fitting are generally used: i) the bilogarithmic regression; ii) the linear regression I; iii) the linear regression II. The regression fits normally used, and the corresponding formula for the calculation of the length of the fracture process zone, l_{fpz}, and for the fracture toughness at propagation, $\mathcal{R}_{ss}$, are reported in Table 1 where $\phi_0 = \phi|_{\alpha=\alpha_0}$ and $\acute{\phi}_0 = \partial\phi/\partial\alpha|_{\alpha=\alpha_0}$.

Regressions fit	Formula	Fitting parameters	$\mathcal{R}_{ss}$	l_{fpz}
Bilogarithmic	$\ln\sigma_u = \ln\dfrac{M}{\sqrt{N+w}}$	M, N	$\dfrac{\phi_0^2}{\acute{E}}M^2$	$\dfrac{\phi_0}{2\,\acute{\phi}_0}N$
Linear regression I	$\dfrac{1}{\sigma_u^2} = A\,w + C$	A, C	$\dfrac{\phi_0^2}{\acute{E}}\dfrac{1}{A}$	$\dfrac{\phi_0}{2\,\acute{\phi}_0}\dfrac{C}{A}$
Linear regression II	$\dfrac{1}{w\sigma_u^2} = \acute{A}\dfrac{1}{w} + \acute{C}$	$\acute{A}, \acute{C}$	$\dfrac{\phi_0^2}{\acute{E}}\dfrac{1}{\acute{C}}$	$\dfrac{\phi_0}{2\,\acute{\phi}_0}\dfrac{\acute{A}}{\acute{C}}$

Table 1. Size effect law fits [15].

The mathematical treatment developed before is rigorous for mode I in tension. However, for the other kind of crack propagation (mode I in compression and shear) the mathematical treatment is analogous. The reader is referred to [18, 19, 20] for details (see Table 2).

Failure mechanism	spec.	ϕ	ref.
Fibre fracture tension	double edge cracked	ϕ^+	[18]
Fiber compression (kink band)	double edge cracked	ϕ^-	[19]
Shear fracture	modified Iosipescu	ϕ^s	[20]

Table 2. tests

3. Determination of ϕ

The dimensionless function $\phi(\alpha, \rho)$ can be defined for the problem under consideration using the Finite Element Method (FEM). For this purpose a parametric Finite Element model was created using Python [21] together with Abaqus 6.8-3 Finite Element code [22].

The correction factor for tension, ϕ^+, reads [18]:

$$\phi^+ = \sqrt{\tan\frac{\pi\alpha}{2}} \sum_i \sum_j \Phi^+_{ij}\, \alpha^{i-1} \rho^{j-1} \tag{10}$$

where Φ^+_{ij} is the element of the matrix $\mathbf{\Phi}^+$ of indexes i and j. The matrix $\mathbf{\Phi}^+$ reads:

$$\mathbf{\Phi}^+ = \begin{bmatrix} 1.7482487564 & -0.053754159533 & 0.0040142704949 & -9.8480085881\text{E-}5 \\ -0.76896688866 & -0.0068632911438 & 0.0029984681658 & -0.00010108691939 \\ 0.85633404777 & 0.23922363475 & -0.023289123198 & 0.00062358861997 \\ -0.67470597429 & -0.25334178248 & 0.022297779266 & -0.00056784694513 \\ 0.18495379886 & 0.084067007027 & -0.0068989066533 & 0.00016783852495 \end{bmatrix} \tag{11}$$

The correction factor associated with the propagation of a kink band, ϕ^-, is [19]:

$$\phi^- = \sqrt{\frac{\alpha}{1-\alpha} \sum_{i=1}^{M} \sum_{j=1}^{N} \Phi^-_{ij} \rho^{j-1} \alpha^{i-1}} \tag{12}$$

where Φ^-_{ij} is the element of the matrix $\mathbf{\Phi}^-$ at the row i and at the column j, and M and N are the number of rows and columns of $\mathbf{\Phi}^-$ respectively. The matrix $\mathbf{\Phi}^-$ reads:

$$\mathbf{\Phi}^- = \begin{bmatrix} 4.315050777 & -0.1833177904 & 0.01642021976 & -4.829962430\text{E-}4 \\ -5.148136502 & -0.3554678337 & -9.974634025\text{E-}4 & 4.975387379\text{E-}4 \\ 2.385888075 & 1.339974300 & -0.05966399650 & 7.544565390\text{E-}4 \\ -0.2810124370 & -0.8040552990 & 0.04491874691 & -7.869467548\text{E-}4 \end{bmatrix} \tag{13}$$

Finally for the mode II propagation (in shear) the correction factor ϕ^s is computed using the following polynomial:

$$\phi^s = \frac{\alpha}{1-\alpha} \sum_i \sum_j \Phi^s_{ij} \alpha^{i-1} \rho^{j-1} \tag{14}$$

where Φ^s_{ij} is the element, with the indexes i and j of the matrix $\mathbf{\Phi}^s$ defined as:

$$\mathbf{\Phi}^s = \begin{bmatrix} -0.03644 & 0.22363 & -0.00437 & 2.21749\text{E}{-5} \\ 5.23616 & 1.44575 & -0.16601 & 0.00449 \\ -7.39060 & -6.89756 & 0.63003 & -0.01629 \\ 1.55840 & 9.24044 & -0.79486 & 0.02025 \\ 0.84958 & -4.05275 & 0.33844 & -0.00855 \end{bmatrix} \tag{15}$$

4. Measurement of the R–curve

Knowing the size effect law, the R-curve is calculated as described in section 1. This is equivalent to obtain the R-curve as envelope of the crack driving force curves. Figure 1 shows the fracture toughness for the ply in the longitudinal direction for the IM7/8552 material system. The experimental points obtained from the compact tension (CT) specimens are also reported using the FEM based data reduction method proposed in [9].

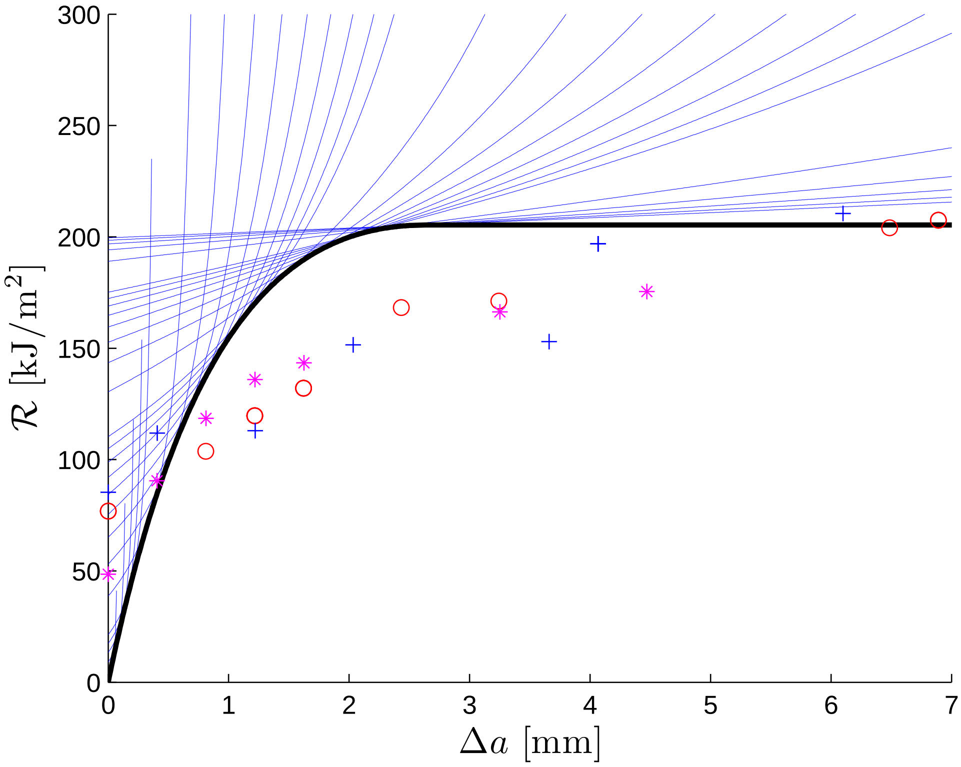

Figure 1. R-curve of the 0° ply for IM7/8552 (in black) obtained as envelope of the driving force curves (in blue) and comparison with experimental results obtained using CT specimens (every marker a different specimen).

Analogous R-curves are obtained for the case of compression and shear.

5. Conclusions

Using the size effect law measured in composite laminates with two edge cracks it is possible to obtain the crack resistance curve, both for the multiaxial laminate tested and for the 0° ply. The methodology proposed here circumvents both the need to perform complex post-processing analysis based on Finite Elements and the need to measure the crack length during the test.

The stress intensity factor used in the model can be easily obtained using a polynomial approximation of the results of the application of the Virtual Crack Closure technique in parametric Finite Element models of specimens with two edge cracks loaded in tension. It is concluded that this is the preferred method to calculate the stress intensity factor for general lay-ups and geometries.

The methodology proposed here provides a robust way to measure the steady-state value of the R-curve for fibre-reinforced composites, if compared with the CT specimen, for which the determination of the steady-state value may be ineffective.

The information generated in this paper will be used in the definition of the constitutive relations of the analysis models that aim to predict the mechanisms of crack initiation and propagation of composite structures.

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