STOCHASTIC MODELLING OF THE EFFECT OF YARN PATH AND LAYER SHIFT VARIABILITY ON STIFFNESS AND STRENGTH OF TEXTILE COMPOSITES

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Abstract
Structural variability in textile composites was analysed experimentally at meso- and macro-scales. A statistical model based on experimental data was applied to generate models of the composite with variability of yarn path. Monte Carlo numerical analysis was employed for predictions of the distributions of the Young's modulus and strength of the composite with variability of yarn paths and layer shift. Results of simulations were in good agreement with experimental results.

1. Introduction

Variabilities of structure at all scale levels, namely micro-, meso- and macro-, are inevitable in composite materials. Examples of such variabilities are fibre arrangement at the micro-scale, and variation of yarn paths and layer shift at the meso-scale [1-3]. Each of these variabilities has effects on the mechanical properties and manufacturing process of composite materials. This study focuses on structural variability at the meso-scale i.e. level of textile reinforcement unit cell.

The effect of in-plane waviness of yarn paths on the mechanical properties of laminate was studied by Crookston et al [4] for the case of a non-crimp fabric laminate. The gap between yarns was described in form of a summation of periodic functions as proposed by Endruweit and Long [5]. The Monte Carlo (MC) approach combined with Classical Laminate Theory yielded the Young's modulus and strength within 15% of experimental values and coefficients of variation (CoV) of 1% and 2.5%, respectively, which were lower than experimental values of 2.5% and 3%.

A Markov Chain approach was employed by Blacklock et al [6] for description of yarn path variability in a textile reinforcement. Data acquired with μCT scanning were analysed in terms of systematic and stochastic variation which represent weaving pattern and randomness. The model was calibrated using experimental data and used for modelling of yarn path variability. This methodology in conjunction with a unit cell approach and MC method was applied by Vanaershot et al [7] for prediction of mechanical properties of textile composites. It was found that the average Young's modulus of a unit cell was lower than that of an ideal composite and CoV of the modulus was found to be less than 1%. However, this study relied on data measured within a single unit cell i.e. it did not include any long-range variations.

Skordos and Sutcliffe [8] proposed to describe a textile reinforcement as a Gaussian and Markovian random field. The Ornstein-Uhlenbeck (OU) sheet [9] was fitted to experimental data and used to generate a model of the reinforcement. This approach was applied for predictions of the Young's modulus of a twill weave textile composite with yarn path variability [10]. The distribution of the Young's modulus was found to be non-symmetric with CoV of similar order of magnitude as the experimental value. However, studies were performed for the composite with no layer shift which is an artificial configuration for a textile composite.

The effect of layer shift on properties of a textile composite was studied by Woo and Suh [3]. It was demonstrated that laminates with no layer shift were always less stiff than laminates with random layer shift. The distributions of the Young's moduli were asymmetric and tended to narrow when the number of layers increased. Nonetheless, this study did not assess the effect of layer shift on nonlinear behaviour of laminates. The effect of layer shift on strength was studied analytically and experimentally by Ito and Chou [11]. Both the analytical approach and experiments showed that a laminate with no shift was less stiff and possessed higher strain to failure than a laminate with maximum layer shift. However, the analytical approach did not allow to predict damage propagation through the composite.

2. Experimental study

2.1. Specimen manufacturing

A 2×2 twill weave textile manufactured by Carr Reinforcements with an overall areal density of 660 g/m^2 was used for the current experimental studies. The textile consisted of 12K Grafil 34-700 carbon fibre yarns woven together with a density of 4.0 picks/ends per cm. It was used to manufacture two types of 6-layer laminates using different layer stacking procedures and the vacuum assisted RTM process. Three panels (#1, #2 and #3) were manufactured following the conventional manufacturing procedure, which resulted in laminates with random layer shift, and two panels (#4 and #5) were manufactured enforcing layers to have no relative shift between each other i.e. replicating periodic stacking. All the panels had thickness of 4 mm and a nominal fibre volume fraction of 55%. Gurit Prime 20LV epoxy mixed with Prime 20 Slow Hardener in ratio 100:26 by weight was used as a resin system. Panels were cured at 65°C after infusion.

2.2. Specimen analysis

Three samples with size of 14x14mm were cut from panels #1, #3 and #5. The μCT scanning of samples was performed using a Phoenix Nanotom CT System with energy of 60 keV and a resolution of 15μm. Examples of the specimen cross-sections are presented in Figure 1. The scans and microscopy showed that nesting between layers in the specimens from Panels #4 and #5 was not uniform in the through thickness direction as was intended during manufacturing. Four out of six and five out of six layers, respectively, were well aligned with no significant layer shift, while other layers were shifted. This was a result of error in lay-up or was caused by the manufacturing process (i.e. moving preform or closing tool could shift the layer). Yarn width, thickness, and position of the yarn's centre and the orientation of its cross-section were manually measured from μCT images for every yarn in every layer with spacing of 0.9 mm (15 measurements for every yarn). These data made possible to reconstruct the actual geometry of the laminates i.e. yarn paths and mutual layer orientation.

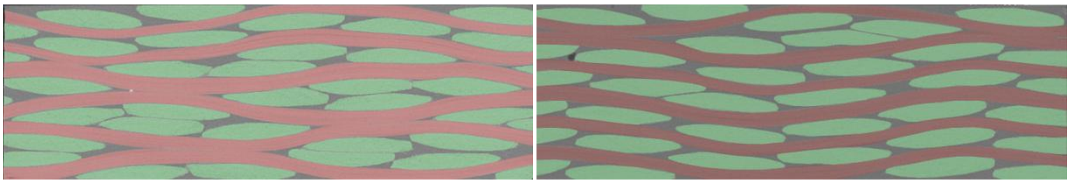

Figure 1. μ-CT images of manufactured composite with highlighted yarns: Panel #3 (left), from Panel #5 (right)

In-plane and out-of-plane components of yarn path were analysed in terms of systematic and stochastic variations following the methodology proposed by Blacklock [6]. Systematic and stochastic variations (i.e. deviation from systematic) were determined using experimental data. Systematic out-of-plane yarn path averaged through all layers and yarns is shown in Figure 2a. The distribution of stochastic variation from the average path, which is shown in Figure 2b, was found to be close to normal with a standard deviation of 22 μm. The deviation of the average in-plane yarn path from a nominal straight line was within 40μm, while the standard deviation of individual yarn paths from the average path was 25μm. This means that the in-plane yarn path is virtually straight on the span of a unit cell length. Results for all the analysed specimens were very similar and no meaningful differences were found.

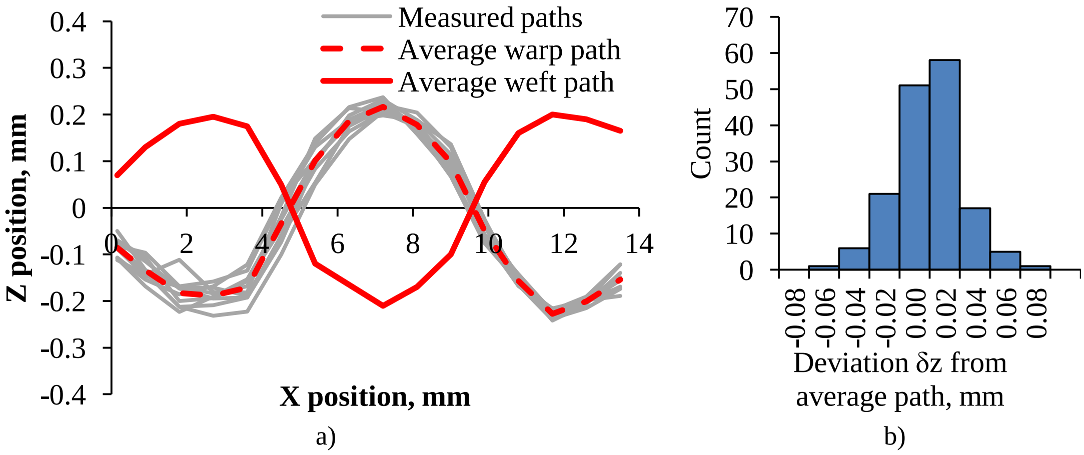

Figure 2. a) Out-of-plane yarn paths in Panel #5; b) Distribution of deviation from average out-of-plane path

The layer shifts and layer orientations in the laminates were characterised using μCT. However, the limited number of specimens (two only) with random layer shift did not allow to draw any particular conclusions about the type of layer shift distribution or any of its features but made it possible to reconstruct the exact geometry of the laminate for further analysis. All the studied cases of composites with this particular reinforcement were found to be not significantly variable at the meso-scale with standard deviations comparable with the resolution of μCT images i.e. close to the measurement error. Therefore, the collected data cannot really be used for statistical modelling of the reinforcement at the meso-scale.

Images of three samples of dry reinforcements and outer surfaces of the Panel #2 - #5, 270×210 mm each, were acquired using a flatbed scanner with resolution of 1200dpi (i.e. one pixel is 21 μm) and then yarn paths were extracted automatically. The results of macro-scale image analysis made it possible to reconstruct the geometry of the reinforcement of a large size and analyse it using statistical descriptors i.e. average path and deviations. It was found that individual yarns can have a variation of up to 0.7mm from a nominal straight line. An example of deviations of individual yarns and their average paths are shown in Figure 3a. The distributions of stochastic variations from the average path were fitted with normal distributions with standard deviations ranging from 0.08mm to 0.1mm for various samples.

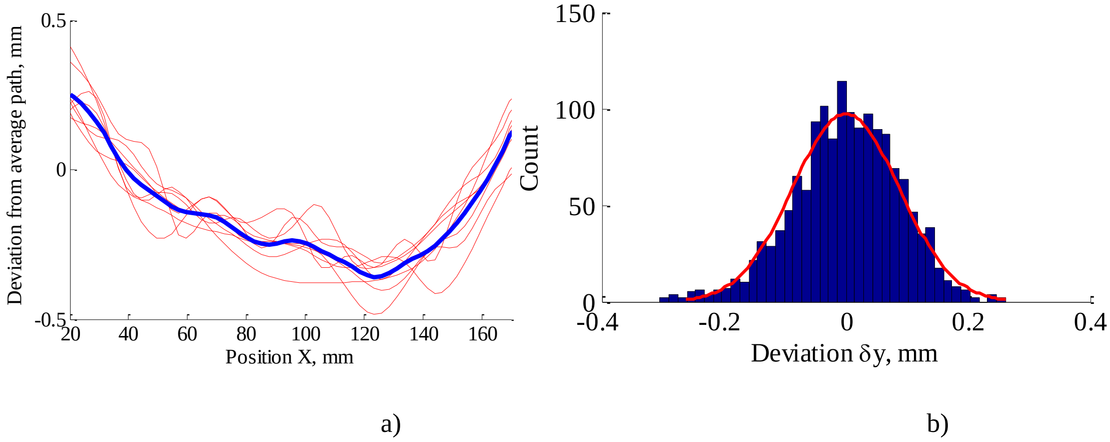

Figure 3. a) Yarn paths in the reinforcement; b) Distribution of weft yarn path deviations from nominal straight line in dry textile

Correlation and autocorrelation properties were also analysed for the reinforcement using processed data. It was found that both warp and weft yarns are strongly correlated as shown in Figure 4. The high correlation means than not only adjacent yarn paths are similar to each other but also yarn paths spaced apart tend to be similar. This behaviour is explained by very tight weaving of the textile with virtually no gaps between yarns. Autocorrelation of yarn path decays fast and becomes insignificant at a length of 3-4 unit cells as illustrated in Figure 4. This shows that there is no obvious periodicity in yarn paths.

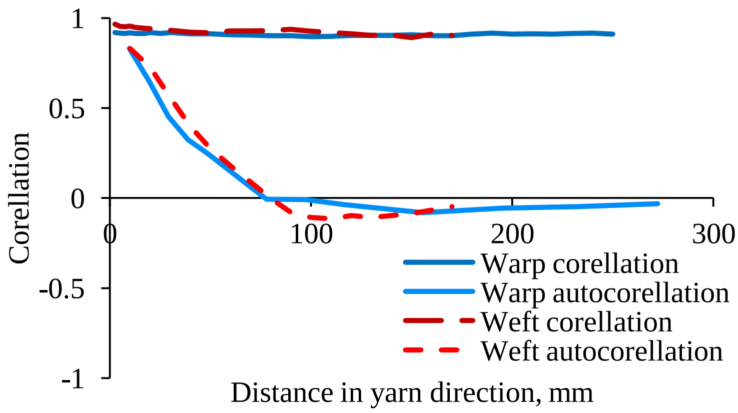

Figure 4. Correlation and autocorrelation of warp and weft yarns in dry textile

2.3. Mechanical testing

Specimens from each panel (see Section 2.1.1 for description) were tested in tension in the warp direction according to ISO 527 (equivalent to ASTM D638) using an Instron 5985 machine with 250kN load cell at a test speed of 2mm/min. The specimens were tabbed with aluminium tabs using epoxy adhesive to prevent early damage in the jaws. A DANTEC Q400 digital image correlation (DIC) system was used to monitor displacements (and therefore strains) at the surface of the composite. A summary of experimental results is given in

Table 1. Typical stress-strain curves are shown in Figure 5. Experimental results generally agree with published studies [3, 11].

	# of tested samples	Young's modulus, GPa	Poisson's ratio	Strength, MPa	Final strain, %
Panel #1	6	55.74 (1.38)	0.069 (0.007)	571.0 (20.5)	1.11 (0.1)
Panel #2	6	55.36 (2.13)	0.082 (0.016)	484.6 (31.0)	1.37 (0.26)
Panel #3	12	55.96 (1.65)	0.054 (0.008)	582.2 (17.6)	1.35 (0.24)
Panel #4	9	54.89 (1.02)	0.073 (0.012)	595.9 (25.5)	1.38 (0.13)
Panel #5	8	53.26 (1.23)	0.129 (0.007)	644.6 (74.8)	1.46 (0.27)

Table 1. Experimental results, values in parenthesis are standard deviations

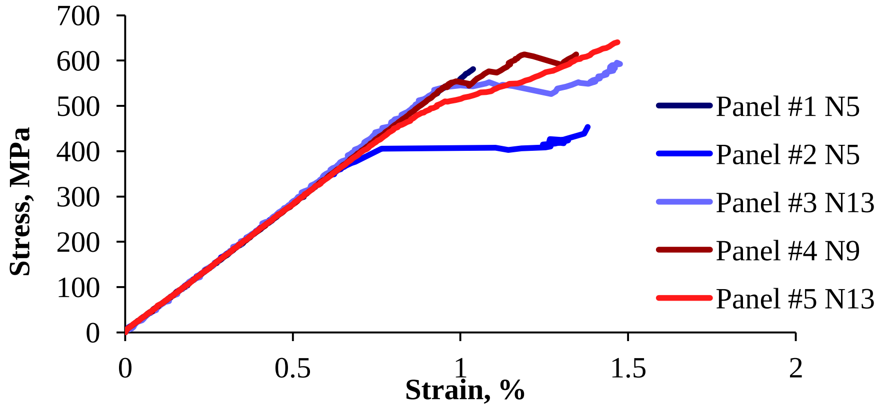

Figure 5. Typical stress-strain curves for tested specimens

Strain monitoring with DIC highlighted qualitative differences between strain fields for the two types of laminates. Longitudinal strains for three samples at applied strain of 0.4% are shown in Figure 6. Longitudinal strains in the laminates with no layer shift possessed very regular patterns which corresponded to the textile pattern. Regions with high strain corresponded to transverse yarns and low strain regions corresponded to longitudinal yarns. Strain fields in laminates with random nesting did not possess this degree regularity.

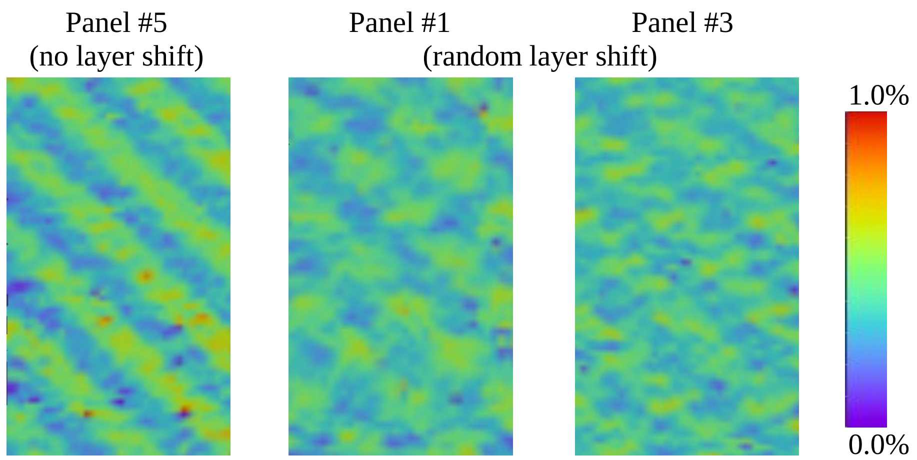

Figure 6. Longitudinal component of strain at average applied strain of 0.4%

3. Variability modelling

3.1. Idealised unit cell model

A textile pre-processor TexGen was used to prepare a unit cell model of the idealised textile using average dimension of the reinforcement: unit cell length 10 mm, yarn spacing and width 2.5 mm, yarn thickness 0.33 mm. Both a single layer model and a six layer model with no layer shift were prepared. The model was meshed with voxels and yarn properties were defined using Chamis' formulae [12] as shown in Table 2. A validated phenomenological continuum damage mechanics model [13] was implemented in an Abaqus subroutine and utilised for non-linear analysis.

	E_1, GPa	E_2=E_3, GPa	$v_{12} = v_{13}$	v_{23}	G_{12}=G_{13}, GPa	G_{23}, GPa
Matrix (Prime 20LV)	3.1	3.1	0.35	0.35	1.15	1.15
Fibre (Grafil 34-700)	234	15 [14]	0.20 [12]	0.25 [12]	13.0 [15]	6.0 [12]
Yarn (V_f = 72%)	169.3	9.48	0.24	0.37	5.07	2.75

Table 2. Material elastic properties

Imposed periodicity of the unit cell allows periodic boundary conditions to be applied in in-plane directions for both models and through thickness for the single layer model (free surface was assumed for six layer model). However, Dirichlet boundary conditions were applied in in-plane directions in order to make possible comparison with a non-idealised model where periodic boundary conditions are not appropriate.

3.2. Stochastic modelling

Following the methodology proposed by Skordos and Sutcliffe [8] variability of the reinforcement was modelled using an Ornstein-Uhlenbeck sheet i.e. Gaussian random field with covariance matrix defined as:

$$\Sigma = \sigma^2 \exp\left(-\gamma_1 |x_1 - x_2| - \gamma_2 |y_1 - y_2|\right) \tag{1}$$

Its parameters were fitted using a maximum loglikelihood estimator and found to be σ =0.2 cm, γ_1=0.09 cm^{-1}, γ_2=0.01 cm^{-1}.

Random fields for warp and weft yarn were generated separately and then used to produce models of the reinforcement within TexGen automatically. Every layer was generated independently and then assembled in a laminate with an arbitrary number of layers and arbitrary layer shifts. This approach made it possible to create a model of arbitrary size but only sizes multiple to single unit cells i.e. 1×1, 2×2 and 5×5 unit cells were chosen for the study. The models were meshed with voxels and Dirichlet boundary conditions were applied in in-plane directions.

3.3. Results of stochastic simulations

Single layer and six layer models with no layer shift and no yarn path variability were the subjects of non-linear numerical analysis. Figure 7a shows results of numerical simulations which are in a good qualitative agreement with experimental results. The numerical models were able to capture a kink in the stress-strain curve related to the transverse damage of yarns and matrix and consequent yarn straightening due to lack of stiffness. The variability of the Young's modulus was investigated on single layer models of various sizes and the six layer model of 1×1 unit cell size. Resulting cumulative distribution functions are shown in Figure 7b. It can be seen that the distributions narrows with increase of the number of unit cells included in the model. The standard deviation of the widest distribution (1×1 model) was 0.55 GPa which is half of the standard deviation of experimental results.

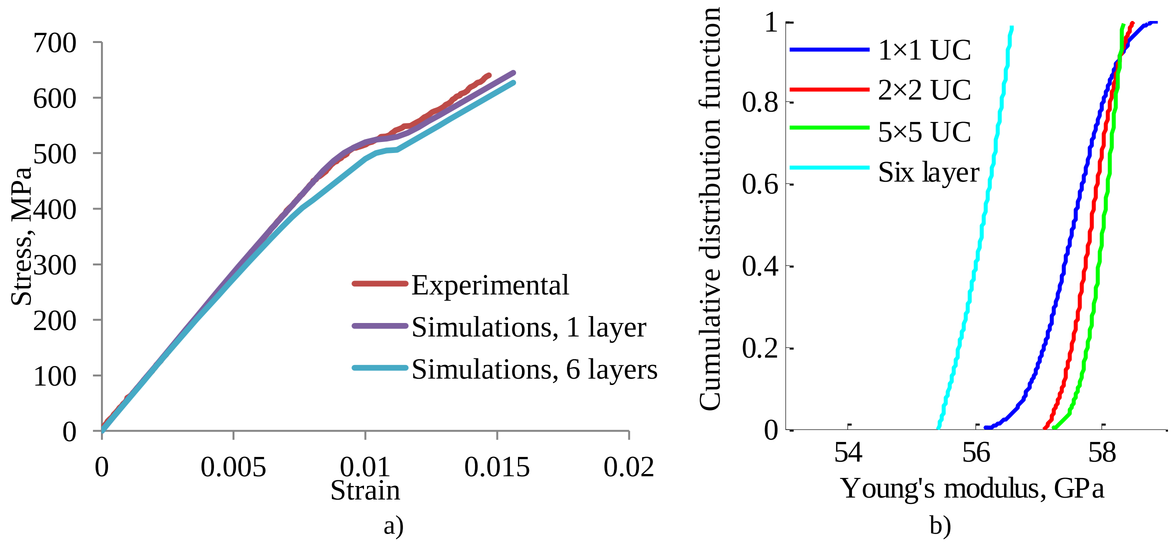

Figure 7. a) Predicted and experimental stress-strain curves for regular nesting; b) Cumulative distribution functions of the Young's moduli

The multi-layer models of the composites with layer shift were generated using the geometry measured from μCT samples. Figure 8 shows results of numerical modelling and experimental results which are in good agreement.

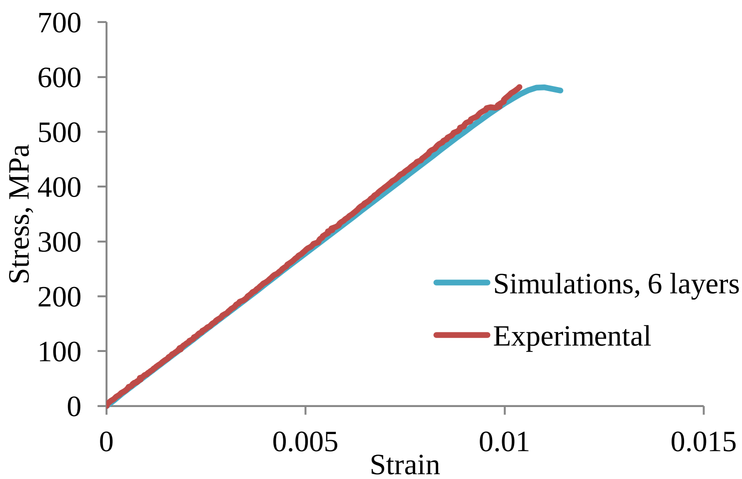

Figure 8. Predicted and experimental stress-strain curves for composites with random nesting

4. Conclusions

The structure of the twill weave textile reinforcement was analysed experimentally at meso- and macro-scales by means of image analysis. It was found that structural variability at the meso-scale is of the same order as the resolution of image acquisition (15μm). Variability at the macro-scale was found to be more significant e.g. yarn path deviations were up to 0.7 mm from a nominal straight line. Yarn path variability was modeled using an Ornstein-Uhlenbeck random field which was fitted to actual macro-scale data and its parameters, standard deviation and inverse correlation lengths, were determined. TexGen software was used to create FE models of the composites. Results of non-linear numerical analysis were in a good qualitative agreement with experimental results. The models for both laminates with no layer shift and random layer shift were able to capture kinks in stress-strain curves. Distributions of the Young's modulus were predicted for the laminate with no layer shift using models of various sizes. The standard deviation of the model was underestimated compared to experimental data for all the cases.

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