A NEW DESIGN CONCEPT FOR CYLINDRICAL COMPOSITE SHELLS UNDER AXIAL COMPRESSION

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Abstract
Thin-walled cylindrical shells are prone to buckling. Imperfections which are defined as deviations from the perfect geometrical shape or homogeneous loading distribution can reduce the buckling load significantly compared to that of the perfect shell. For the design of unstiffened cylindrical shells mostly the NASA SP-8007 guideline is used, which recommends reducing the buckling load of the perfect shell using a knock-down factor. Existing knock-down factors are conservative and structural behavior of composite shells is not considered. A new proposal for an improved guideline is the advanced Single-Perturbation Load Approach (α–SPLA). The design loads of the new approach are, depending on the ply-layup, at least 40 % larger compared to the design loads of the NASA SP-8007 and the behavior of composite structures is taken into account. Based on test results the concept is validated.

1. Introduction

In the 1950s and 60s, a large number of buckling experiments of cylindrical shells have been carried out. **Figure 1** [1] summarizes the results of the test series. The knock-down factor ρ (the ratio of buckling load of imperfect and perfect shell) is shown for axially compressed cylindrical shells depending on their slenderness (ratio of radius and wall thickness). The results, presented as dots, show a large variance. In addition a decreasing of the knock-down factor with increasing slenderness is noticeable. The significant discrepancy between experiment and classical buckling theory motivated scientists to investigate this subject the past 50 years. KOITER was the first to develop a rational explanation for the large deviation of experiment and theory. He showed the extreme sensitivity of buckling loads of unstiffened cylindrical shells to initial geometric imperfections. An English translation of KOITER's thesis (dutch) was given by RIKS in [2]. Based on the test series shown in **Figure 1** the NASA SP-8007 guideline (1968) [6] provides knock-down factors for cylindrical shells which are conservative and in the case of cylindrical composite shells do not consider the structural behavior correctly.

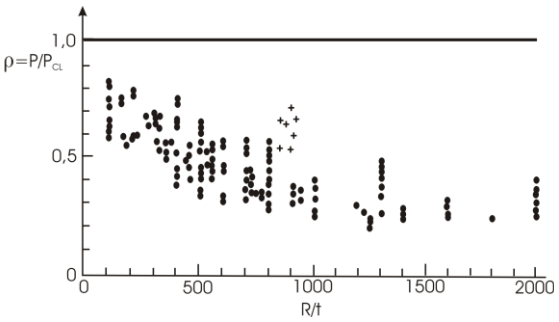

Figure 1. Results of large number of buckling experiments of cylindrical shells [1]

The recently developed Single-Perturbation Load approach (SPLA) is a new physical based design concept for unstiffened cylindrical composite shells and respects the influence of geometric imperfections on the buckling load [3]. This design concept leads to much less conservative design loads then the NASA SP-8007. However the buckling load depends not only on the influence of geometric imperfections. In this paper it will be shown that the influence of deviations from the perfect, homogeneous loading (generally known as load asymmetry, loading imperfections [4]) on the buckling load of the cylindrical shells is at least as critical as the influence of geometric imperfections. Therefore it is essential for a modern design concept to respect load asymmetry and geometric imperfections.

2. Single-Perturbation Load principle

In this chapter the basic idea of the SPLA is presented. Then the probable cause of load asymmetry and their influence on the buckling load of composite cylinders is described. Afterwards a proposal for an advanced Single-Perturbation Load approach (a-SPLA) is given.

2.1. Single-Perturbation Load approach (SPLA)

The SPLA is a design concept for thin-walled cylindrical composite shells. It is based on the assumption that geometrical imperfections are most critical for the structural behavior [3]. Details on magnitude or the amplitude of the imperfections, as needed for probabilistic approaches, are not required. If a defined perturbation load is applied to a cylindrical shell a single buckle is caused, as illustrated in **Figure 2** (left). The single buckle is a geometric imperfection and according to [3] one of the worst imperfections with respect to the stability behavior.

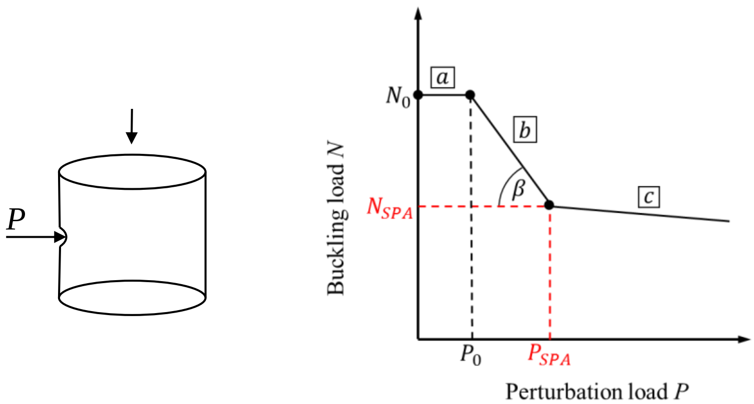

Figure 2. Illustration of the SPLA principle (left) - Buckling load versus perturbation load diagram (right)

Figure 2 (right) shows the buckling load plotted over the perturbation load. The figure can be divided into three different sections. In the first section a the buckling load is constant. With increased perturbation load the influence of the individual buckle is authoritative and a reduction of the buckling load N_0 is recognizable. The relation between perturbation load and buckling load in the second section b is linear. The β parameter describes the extent to which the buckling load decreases by increasing the perturbation load. In the third section c the buckling load is almost unchanged, although the perturbation load is increased further. The specific perturbation load P_{SPA} is the intersection between the straight section of b and c, and leads to the design load N_{SPA}. Single buckles with such large amplitudes are clearly visible and are assumed to be unrealistic. Therefore, the buckling load N_{SPA} at the intersection point is defined to be the lower limit of buckling loads of realistic geometric imperfect composite shells.

HÜHNE showed in [3] that in general the SPLA delivers larger and less conservative design loads compared to the design loads of the NASA SP-8007. He also stated that for some shells it was not possible to predict a safe lower limit of the buckling load. The reason for this is that the SPLA only covers geometric imperfections, which occur during the manufacturing process of an unstiffened composite cylinder, for example small dimples. If imperfections like load asymmetry occur it is not possible to determine a safe lower limit of the buckling load of an unstiffened composite cylinder using the SPLA.

2.2. Deviations of the perfect, homogeneous loading

In **Figure 3** (left) results of experiments conducted by HÜHNE are shown. It was noticed that the perturbation load reduced the buckling load depending on its position in the circumferential direction, illustrated in **Figure 3** (right).

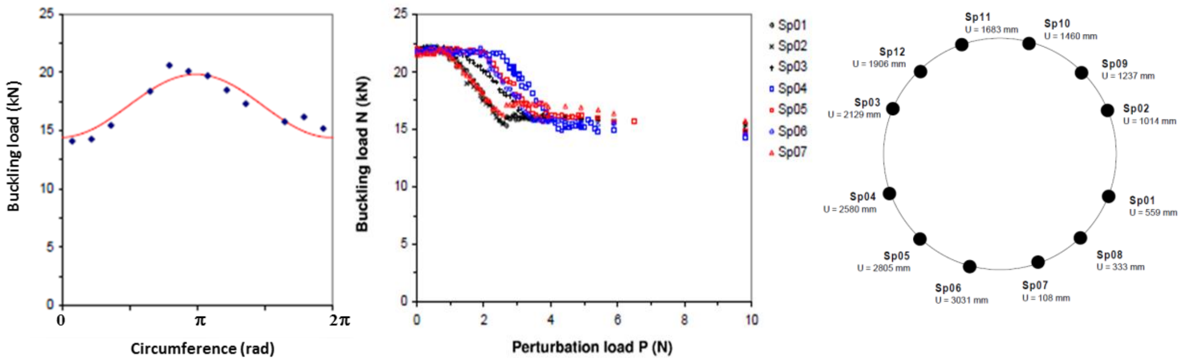

Figure 3. Buckling load of a cylindrical composite shell depending on the position of the perturbation load along the circumferential direction (left) – affiliated Buckling load versus perturbation load diagram (middle) – for the buckle experiments used equidistant pattern over the circumference (right) [3]

This behavior does not appear in the simulation with a perfect composite cylinder shell. The structural behavior for a perfect shell is independent from the perturbation load position. The scattering also appears for big perturbation loads (see **Figure 3**, middle) which means that geometric imperfections cannot be the cause of this behavior. This indicates that a small inclination was applied to the test shells caused by the test setup. In **Figure 4** the assumed inclination of the shells is illustrated.

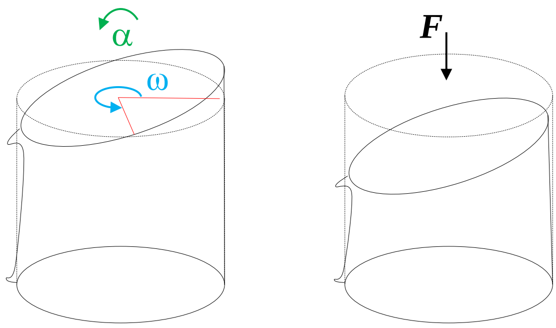

Figure 4. Illustration of the assumed inclination of the cylindrical shells

It can be seen that the shell is bent with the bending angle α about an axis, which is described by the circumferential angle ω. Therefore a bending moment was applied to the shells, which reduced the buckling load of the cylinders additionally. The inclination angles α have not been measured. KRIEGESMANN determined the angles indirectly in [4]. The angles α vary in a range from at least 0.0075 – 0.015°.

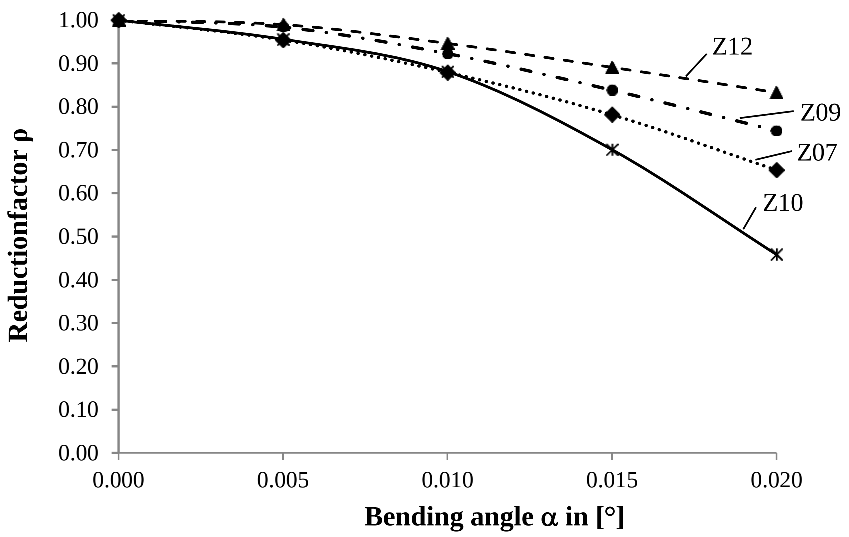

Figure 5. Knock-down factor versus bending angle for the cylindrical shells Z07-Z12

Figure 5 shows the knock-down factor (ratio of the buckling load of a perfect shell burdened with load asymmetry and a perfect shell) versus the bending angle α for the shells Z07-Z12 described by HÜHNE [3]. The shells Z07-Z12 all have the same geometry and material parameters, but the ply layup is different, for the finite element model of the shells the same model, solver and boundary conditions (clamped) as in [3] were used. It is noticeable that the reduction due to the load asymmetry depends on the layup of the composite structures. Z10 is classified as sensitive test shell according to [3] which implies that the corresponding buckling load responds sensitive to geometric imperfections. Z12 on the contrary is an insensitive shell. Knock-down factors shown in **Figure 5** reflect the same pattern of sensitivity against imperfections. Shell Z10 is extraordinary sensitive to load asymmetry; a bending angle of α = 0.02° reduces the buckling load by more than 50 %. In comparison the buckling load of shell Z12 reduces only by approximately 17 %. The other shells Z07 and Z09 are in between. Therefore it seems that shells which are sensitive to geometric imperfections are also sensitive to load asymmetry and vice versa.

2.3. Advanced Single-Perturbation Load approach (α-SPLA)

Based on the conclusions of the previous chapter the shell is loaded with a bending moment before the Single-Perturbation Load is applied. An illustration of this procedure is shown in **Figure 6**.

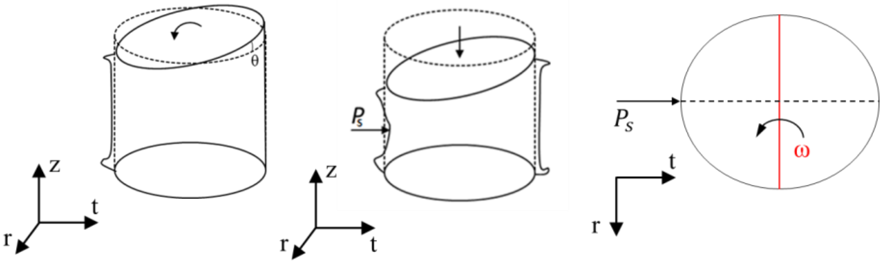

Figure 6. Illustration of the advanced Single-Perturbation Load principle

In this case it is assumed that geometrical imperfections and load asymmetry will always occur together and only their influence on the stability behavior of an unstiffened composite cylinder is relevant, because both cause a deviation from the assumed uniaxial stress state.

For a modern design with the α-SPLA it is desirable that the design process remains deterministic, which means that information on magnitude of geometric imperfections and load asymmetry is not necessary. Therefore the bending angle α has to be estimated for every shell. For that reason the bending angle α is now expressed in terms of an assumed displacement Δu, also shown in **Figure 7**.

$$\Delta u = R \cdot \sin(\alpha) \tag{1}$$

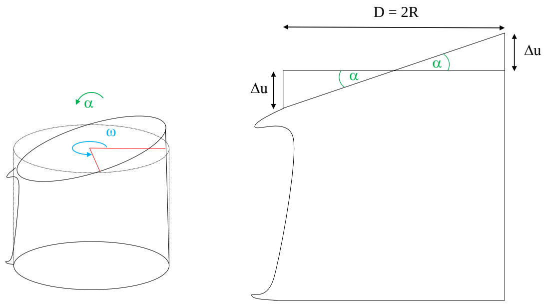

Figure 7. Expression of the bending angle α in terms of a displacement

In **Table 1** are the approximated bending angles α shown [4]. It should be mentioned that the angles α vary in a range from 0.0075° – 0.015° and are not fixed for every shell. In a worst case scenario every shell could be at least burdened with the maximum bending angle α = 0.015°. The listed bending angles serve only as an indication for the real magnitude of the bending angle α.

Shell	**Z07**	**Z09**	**Z10**	**Z12**
approximated bending angle α_{app} [°]	0.01	0.0075	0.0125	0.015

Table 1. Approximated bending angles α_{app} for the shells Z07-Z12 [4]

With the bending angles α of **Table 1** the corresponding displacement Δu (see **Table 2**) can be calculated using equation (1). For a better understanding of the magnitude of Δu the displacements u_{spla} of the SPLA (which correspond to the buckling load N_{spla}) are also listed in **Table 2**.

Shell	Z07	Z09	Z10	Z12
displacement Δu [mm]	0.043	0.032	0.054	0.065
displacement u_{spla} [mm]	0.255	0.212	0.203	0.285
ratio Δu / u_{spla}	0.168	0.151	0.266	0.229

Table 2. Comparison of the displacement u_{spla} and the displacement Δu

Based on the results shown in **Table 2** it can be seen that the displacement Δu is at least 15 % of the magnitude of the displacement u_{spla}. The displacement Δu is the required variable to determine the bending angle α and is not known at the beginning of the design process for an unstiffened composite cylinder. The displacement Δu is now estimated by assuming that a linear relationship between Δu and u_{spla} can be expressed in the following form:

$$\Delta u = u_{spla} \cdot a \tag{2}$$

The term a in equation (2) is a reduction factor to adjust the displacement u_{spla}. The next task is to choose a reduction factor, based on experience it is known that the half of the knock-down factor ρ_{spla} in **Table 3** (the ratio of the buckling load N_{spla} and the buckling load N_{per} of a perfect shell) delivers promising results. Equation (2) is adjusted to:

$$\Delta u \approx u_{spla} \cdot \frac{\rho_{spla}}{2} \tag{3}$$

Using equation (3) the bending angle α can now be approximated:

$$\alpha \approx \sin^{-1}\left(\frac{u_{spla} \cdot \rho_{spla}}{2 \cdot R}\right) \tag{4}$$

This approach is entirely based on an empiric assumption and at this moment it cannot be proofed that this non-physical approach always works. Using this approach the insensitive shells are loaded with a large bending angle, sensitive shells are loaded with a less large bending angle which is at least equal to the measured angles of **Table 1**. In the **Table 3** are the bending angles α which were calculated using equation (4) and are compared to the maximum bending angle $\alpha_{app\text{-}max}$ = 0.015° of **Table 1**.

Shell	Z07	Z09	Z10	Z12
knock-down factor ρ_{spa}	0.57	0.90	0.64	0.94
bending angle α [°]	0.0167	0.0217	0.0147	0.031
ratio $\alpha/\alpha_{app\text{-}max}$	1.113	1.446	0.980	2.066

Table 3. Approximated bending angles α for the shells Z07-Z12 using equation (4) and the ratios ρ_{spa} and $\alpha/\alpha_{app\text{-}max}$

It can be seen in **Table 3** that it is possible to approximate the angles α with equation (4) so that they are nearly the same magnitude as the bending angles given in **Table 1**.

2.4. Advanced Single-Perturbation Load approach (α-SPLA) results

In this chapter the results of the α-SPLA are given below, they are compared with experimental results and the results of the different simulation types. In **Figure 8** the results of the different simulations and tests of two different set of shells (HÜHNE [3], PRIYADARSINI et al. [5]) are compared. For the two shell sets ply-layup and the ratio radius/wall thickness (R/t) are given in **Table 4**.

Shell	Ply-Layup	R/t
Z07, Z08	$[\pm 24, \pm 41]$	500
Z09	$[\pm 41, \pm 24]$	500
Z10, Z11	$[+24, +41, -41, -24]$	500
Z12	$[\pm 45, 0, -79]$	500
SP2-SP4	$[0, +45, -45, 0]_s$	300

Table 4. Ply-Layup and Ratio R/t for the Composite Cylinders of Figure 8

Figure 8 shows the knock-down factor ρ (the ratio of the buckling load N of the SPLA; experiment; NASA SP-8007; α-SPLA compared to the buckling load N_{per} of a perfect shell) for the investigated shells.

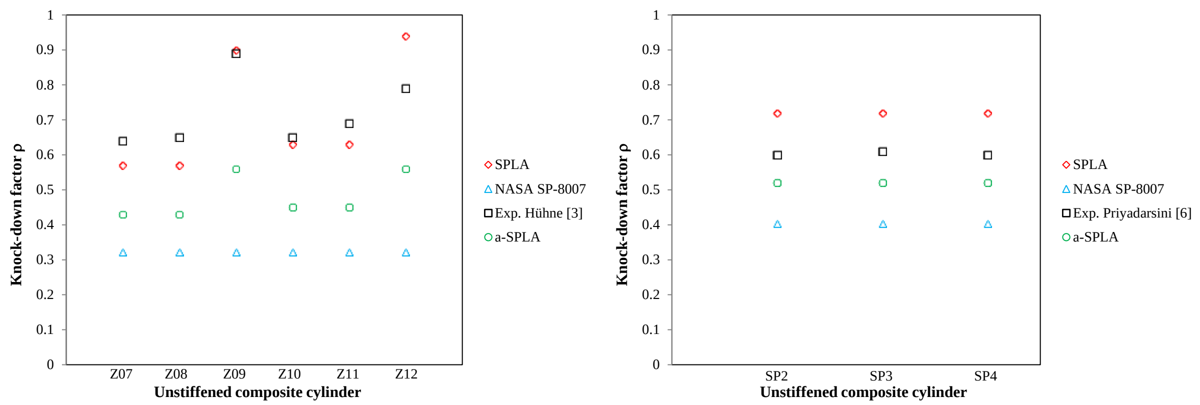

Figure 8. Results of test and simulation compared with NASA SP-8007 – Shells HÜHNE [3] (left) –Shells PRIYADARSINI et al. [5] (right)

The results illustrated in **Table 5** show that SPLA cannot provide a safe lower limit of the buckling load for all investigated shells.

Shell	Z07	Z08	Z09	Z10	Z11	Z12	SP2	SP3	SP4
$\rho_{\text{NASA SP-8007}}$	0.32	0.32	0.32	0.32	0.32	0.32	0.40	0.40	0.40
ρ_{SPLA}	0.57	0.57	0.90	0.63	0.63	0.94	0.72	0.72	0.72
$\rho_{\alpha\text{-SPLA}}$	0.43	0.43	0.56	0.45	0.45	0.56	0.52	0.52	0.52
$\rho_{\text{Experiment}}$	0.64	0.65	0.89	0.65	0.69	0.79	0.60	0.61	0.60

Table 5. Knock-down factors ρ (SPLA; EXPERIMENT; NASA SP-8007; α-SPLA) for the investigated shells

The SPLA delivers for Z12 and SP2-SP3 knock- down factors which are about 20 % larger than the associated experimental knock-down factors. For Z09 and Z10 the knock-down factors of SPLA are risky but and good indication on the amplitude of the buckling load. The knock down factors for Z07, Z08 and Z11 predict a safe design load for the experiments. It can be seen that the knock-down factors of α-SPLA are well below the experiments but compared to the results of the NASA SP-8007, a large improvement. An average increase of the buckling load of about 40 % compared to the knock-down factors of the NASA SP-8007 is apparent. The maximum increase of the buckling load is about 75 %.

3. Conclusion

In this Paper, the advanced Single-Perturbation Load approach (α-SPLA) for the design of thin-walled unstiffened composite cylindrical shells is proposed. The α-SPLA assumes that geometrical imperfections and load asymmetry will always occur together and only their influence on stability behavior of unstiffened composite cylinders is relevant, because both cause a deviation from the assumed uniaxial stress state. The α-SPLA does not need any information on imperfections, the amplitude or magnitude of the bending angle as it is the case with probabilistic approaches. The basic idea of the principle is presented in the second part of the paper. It is demonstrated that for all investigated shells the α-SPLA provides a safe lower limit of the buckling load. This lower limit is less than the lower limit of the standard SPLA but average about 40 % greater than the results of the NASA SP-8007. The maximum increase of the buckling load in comparison to NASA SP-8007 is about 75 %. However further investigations regarding the choice of the reduction factor in equation (3) are required. Because this approach is entirely based on an empiric assumption and at this moment it cannot be proofed that this non-physical approach always works. The range of application of the (α-SPLA) is still under investigation.

References

[1] V. I. Weingarten, E .J. Morgen and P. Seide. Elastic stability of thin-walled cylindrical and conical shells under axial compression. *AIAA Journal 3(3)*:500-505, 1965

[2] W. T. Koiter, On the Stability of Elastic Equilibrium. NASA-TT-F-10833, 1967

[3] C. Hühne, Robuster Entwurf beulgefährdeter, unversteifter Kreiszylinderschalen aus Faserverbundwerkstoff, PhD thesis, published as Mitteilungen des Instituts für Statik und Dynamik der Leibniz Universität Hannover, ISSN 1862-4650, 04/2006

[4] B. Kriegesmann, Probabilistic Design of Thin-Walled Fiber Composite Structures, PhD thesis, published as Mitteilungen des Instituts für Statik und Dynamik der Leibniz Universität Hannover, ISSN 1862-4650, 15/2012

[5] R. S. Priyadarsini, V. Kalyanaraman, and S. M. Srinivasan, International Journal of Structural Stability and Dynamics, Volume 12, Issue 4, 07/2012

[6] Buckling of Thin-Walled Circular Cylinders, NASA SP-8007, NASA, 1968