MULTI-OBJECTIVE AND MULTI-CONSTRAINT OPTIMISATION OF VARIABLE STIFFNESS COMPOSITE LAMINATES

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Abstract

The fibre paths of variable stiffness laminates are described through the fibre angles at the nodes of a finite element (FE) representation of the structure. An algorithm is presented to optimise the fibre angles efficiently. To reduce the number of required FE analyses a multi-level approach is used: the exact solution is first approximated in laminate stiffness space. The second level approximation is a Gauss-Newton quadratic approximation in fibre-angle space. To ensure manufacturability, a steering constraint is introduced: the norm of the gradient of the fibre angle distribution is constrained. Two formulations are proposed: either the average steering is constrained; or the local element-wise steering is constrained. The resulting quadratically constrained quadratic optimisation problem is solved using an interior-point method. It is shown that the local steering constraint performs best, at the cost of increasing the size of the problem.

1. Introduction

Today, composite materials are frequently used in the aviation industry and the first composite-dominated planes like the B-787 or A400M are being built. Traditionally, the fibres within a layer have the same orientation, leading to the same mechanical properties everywhere, named constant stiffness composites. However, fibre placement machines have evolved and now it is possible to vary the fibre orientation inside a layer leading to varying mechanical properties. These composites will be referred to as variable stiffness laminates.

The following three-step approach is a proven method to optimise variable stiffness laminates. The first step is to find the optimal stiffness distribution in terms of the lamination parameters. This has been performed and is discussed in detail in [1, 2]. The second step is to find the optimal manufacturable fibre angle distribution, the focus of this paper. The third and final step is to retrieve manufacturable fibre paths from the fibre angle distribution [3]. This step will not be discussed. A schematic overview of this approach can be seen in Figure 1.

This paper is organised as follows: first the problem formulation is discussed in section 2, next the manufacturing constraints are discussed in section 3. The solution procedure is explained in

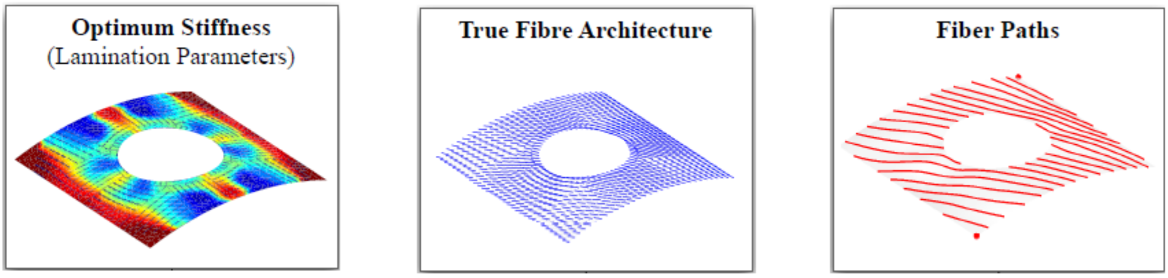


Figure 1: schematic overview of the three-step approach

section 4, followed by the results in section 5 and finally the conclusion is given in section 6.

2. Problem formulation

In structural optimisation, the minimisation of an objective response (e.g., weight or compliance) subject to performance constraints (e.g., on stresses or displacements) is studied. More generally, the worst case response, for example in the case of multiple load cases, may be optimised. Additional constraints not related to structural responses may be added to guarantee certain properties of the optimum, for example smoothness. Many of these additional constraints arise from manufacturing considerations. When optimising variable stiffness laminates, a steering constraint is posed, leading to the following problem formulation:

$$\begin{array}{ll} \min & \max(f_1, f_2, ..., f_n) \\ s.t. & f_{n+1}, ..., f_m \leq 0 \\ & \varsigma^2 - \varsigma_U^2 \leq 0 \end{array} \tag{1}$$

where f_1 up to f_n denote structural responses that are optimised and f_{n+1} up to f_m denote structural responses that are constrained; ς is the steering and ς_U is the maximum allowed steering.

The structural responses, like buckling load, compliance and strength, are calculated in a finite element (FE) environment. However, since each FE analysis is computationally expensive, greater efficiency may be achieved by using structural approximations to reduce the number of required FE analyses. The exact FE solution f is first approximated in terms of the in-plane stiffness matrix $\boldsymbol{A}$ and out-of-plane stiffness matrix $\boldsymbol{D}$ and their reciprocals:

$$f \approx \sum_n \boldsymbol{\phi}_m : \boldsymbol{A}^{-1} + \boldsymbol{\phi}_b : \boldsymbol{D}^{-1} + \boldsymbol{\psi}_m : \boldsymbol{A} + \boldsymbol{\psi}_b : \boldsymbol{D} \tag{2}$$

where the : operator represents the Frobenius inner product, meaning $\boldsymbol{A} : \boldsymbol{B} = tr(\boldsymbol{A} \cdot \boldsymbol{B}^T)$; $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$ are calculated from a sensitivity analysis, m denotes the membrane, b the bending part and n runs over all the nodes. This function is computationally much cheaper to compute, but it is not convex in fibre angle space. Hence, a second approximation in terms of the change in fibre angles is made:

$$f \approx f^0 + \sum_n \boldsymbol{g} \cdot \Delta\boldsymbol{\theta} + \Delta\boldsymbol{\theta}^T \cdot \boldsymbol{H} \cdot \Delta\boldsymbol{\theta} \tag{3}$$

where f^0 denotes the value, $\boldsymbol{g}$ the gradient and $\boldsymbol{H}$ the Gauss-Newton part of the Hessian of the first approximation at the approximation point. This function is the only part that will be optimised. Based on the outcome, the stiffness matrices of the first approximation are changed and the gradient and Hessian are updated for the next optimisation. When the first approximation has converged, or a pre-set number of iterations is done, the FE analysis is done again, updating the sensitivities of the first approximation.

This multi-level approach to solve the optimisation problem works well, but there is no guarantee of global convergence: it is not a given the approximation functions are conservative at the next iterate, meaning the approximation is larger than the function approximated. To make sure every step is an improvement step, it is tried to make every approximation conservative. To achieve this, Svanberg proposed to add a function, called a damping function in the remainder [4]. This damping function consists of the shape of the function and a damping factor. The shape is chosen such that the function value and gradient at the approximation point are not changed. The damping factor is used to control the step size: if it is too large, the next iterate will be unconservative, if it is too small, a lot of iterations will be needed. This factor will be updated after each iteration.

After updating the damping factor, whether the next iterate is an improvement of the approximated function is checked. If it is not an improvement, the point is rejected and the optimisation is done again with the updated damping factor; if the next iterate is an improvement, the point is accepted and the function value, gradient, and Hessian are updated using the first approximation function.

3. Manufacturing constraints

To ensure the optimised laminate can be manufactured using fibre placement techniques, the rate of change in fibre angles should not be too high. This has two physical reasons. One, the fibre placement machine has to be able to follow the curvature of the fibre path without tow wrinkling. Two, the convergence or divergence of fibres should not be excessive to avoid too many gaps/overlaps. Hence, the norm of the gradient of the fibre angles, measuring the amount of *steering* is constrained. There are two ways this can be done: either the average steering is constrained, leading to one constraint per layer; or the local steering value is constrained, leading to one constraint per finite element per layer. The local approach allows more precise control of the steering at the cost of greatly increasing the number of constraints.

3.1. Global steering constraint

The steering ς is given by

$$\varsigma^2 = \nabla\theta \cdot \nabla\theta \tag{4}$$

The average steering can be found using

$$\bar{\varsigma}^2 = \frac{1}{\Omega} \int_{\Omega} \varsigma^2 d\Omega \tag{5}$$

where Ω is the total area of all elements. This is rewritten as

$$\varsigma^2 = \frac{2}{\Omega} \cdot \boldsymbol{\theta}^T \cdot \boldsymbol{L} \cdot \boldsymbol{\theta} \tag{6}$$

where $\boldsymbol{L}$ is the standard FEM discretisation of the Laplacian.

Using the global steering constraint has the advantage that only one constraint per layer is posed. However, while the average steering of each layer is constrained, the maximum steering is not taken into account. Hence, manufacturability is not guaranteed using the global steering constraint. To guarantee manufacturability local constraints are applied as shown in the next section.

3.2. Local steering constraint

When the local steering constraint is used, the steering of each element is constrained. It is assumed the steering in each element is constant, hence this time the average over an element is calculated. The steering of each element is calculated using

$$\varsigma^2 = \frac{2}{\Omega} \cdot \boldsymbol{\theta}^T \cdot \boldsymbol{L_e} \cdot \boldsymbol{\theta} \tag{7}$$

where the subscript e denotes the element. For triangular elements this equation gives the exact curvature, for 4-node elements, it is the average over the element.

4. Solution procedure

As seen previously in section 2, a multilevel approximation approach is adopted for the solution of the optimisation problem. The crucial numerical component is the solution of the optimisation problem (eq. 1) at the second level approximation (eq. 3). This is a quadratically constrained optimisation problem the solution of which is the subject of this section. The solution procedure will be explained for the local steering constraint. The same formulation will work for the global curvature constraints. A flowchart of the algorithm can be found in Figure 2.

First, the optimisation problem is rewritten:

$$\begin{array}{rc} \min & z \\ s.t. & f_i \cdot \boldsymbol{e} - z \leq 0 \\ & \varsigma^2 - \varsigma_U^2 \leq 0 \end{array} \tag{8}$$

where $\boldsymbol{e}$ is a vector consisting of only ones and zeros: one if the function is an objective, zero if it is a constraint. The Lagrangian of the problem is

$$\begin{aligned} \mathcal{L}(\lambda_o, \lambda_e, \theta, z, s_o, s_e) = z + \sum_o \lambda_o \cdot \left(f_o^0 + \boldsymbol{g_o} \cdot \Delta\boldsymbol{\theta} + \Delta\boldsymbol{\theta}^T \cdot \boldsymbol{H_o} \cdot \Delta\boldsymbol{\theta} + s_o\right) + \frac{1}{2} \cdot \Delta\boldsymbol{\theta} \cdot \left(\sum_e \lambda_e \cdot \boldsymbol{L}_e\right) \cdot \Delta\boldsymbol{\theta} \\ -\varsigma_U^2 \cdot \left(\frac{1}{2} \cdot \sum_e \lambda_e \cdot |\Omega_e|\right) + \frac{1}{2} \cdot \left(\sum_e \lambda_e \cdot s_e \cdot |\Omega_e|\right) - \mu \cdot \left(\sum_o \ln(s_o) \sum_e |\Omega_e| \cdot \ln(s_e)\right) \end{aligned} \tag{9}$$

where λ_o denotes the Lagrangian multipliers of the structural responses and λ_e denotes the Lagrangian multiplier of an element, which both have to be non-negative. The slack of the structural responses and of the constraints are given by s_o and s_e respectively. The homotopy factor is denoted by μ. Next, Newton's method is used to find the optimality criteria with respect to each variable. The next iterate is found by solving for these optimality criteria.

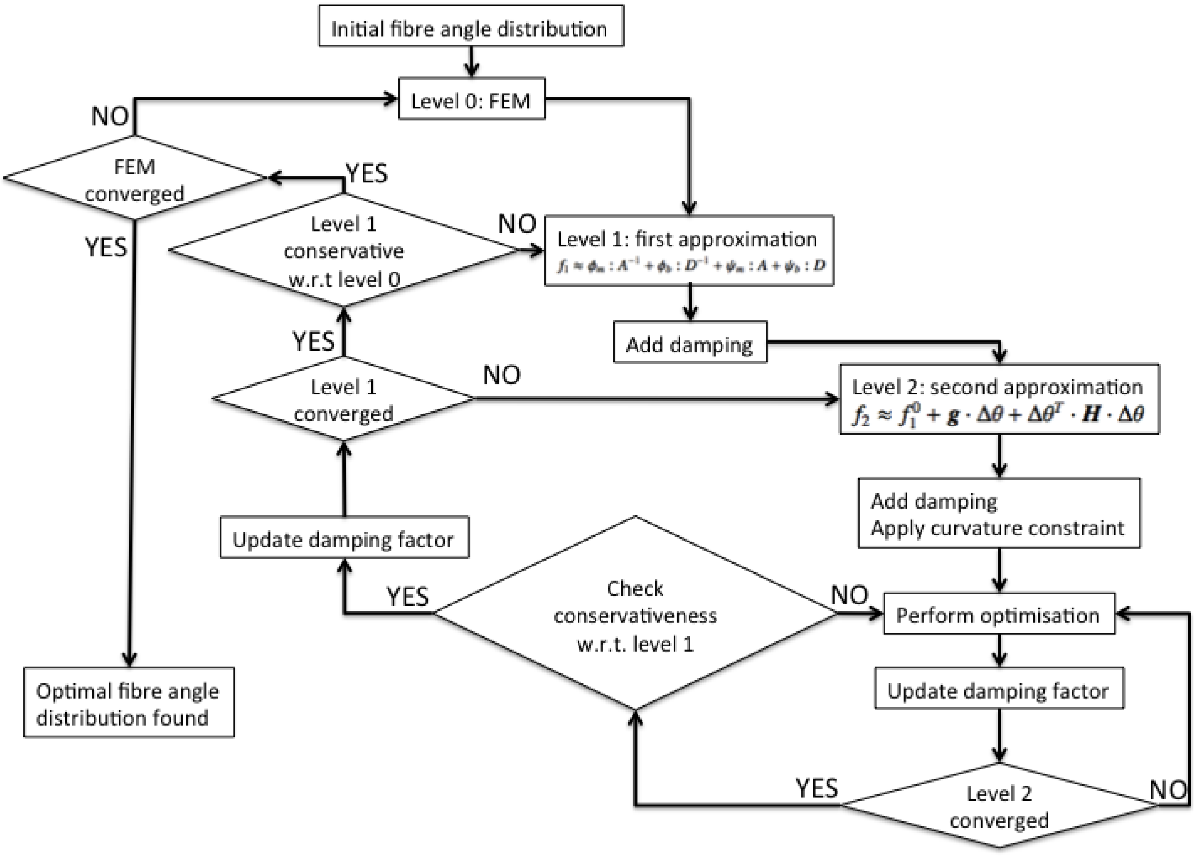


Figure 2: flowchart of the optimisation

5. results

To demonstrate the proposed optimisation algorithm, consider the cylindrical panel with a hole shown in Figure 3. Both length L and width W of the panel are 0.5 m, the radius of curvature R is 0.75 m and the hole has a radius of 0.12 m. The panel is optimised for maximum buckling load. To account for possible modal interactions the first two modes are considered. The straight edges are simply supported, the curved edges are clamped. The panel is subjected to uni-axial compression at the curved edges. The material used has an E1-modulus of 154 GPa, an E2-modulus of 10.8 GPa, a shear modulus of 4.02 GPa, and a Poisson ratio of 0.317.

The laminate to be optimised consists of 16 layers in total, leading to a total thickness of 3.6 mm, by choosing the laminate to be balanced and symmetric only 4 layers need to be taken into account during the optimisation. The results shown are normalised using the buckling load of a quasi-isotropic (QI) laminate.

First the optimisation is performed in lamination parameter space. This shows that compared to a QI laminate, the best constant stiffness laminates offers an increase of 20.1% in buckling load; while using a variable stiffness laminate the buckling load can be increased by 98.3%. Thus, optimally, steering may cause an up to 65% increase in performance compared to the best constant stiffness laminate.

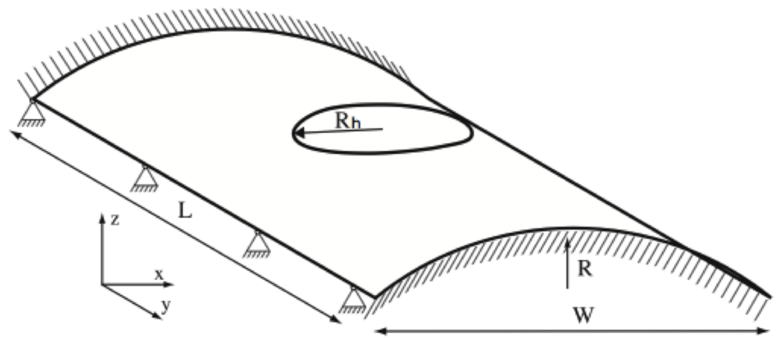


Figure 3: graphical representation of the structure, loading and boundary condition

maximum global steering	buckling load 1	buckling load 2	maximum local steering layer 1	maximum local steering layer 2	maximum local steering layer 3	maximum local steering layer 4	number of FE analyses
1	1.2558	1.2958	3.7961	2.5628	2.5377	2.5081	5
2	1.4641	1.5103	6.9759	5.2010	4.8976	5.0588	6
3	1.6177	1.7141	9.3337	8.0402	7.6818	8.0790	6
4	1.7481	1.8740	16.9370	11.1651	10.5510	11.6299	6
5	1.8130	1.9069	36.1727	27.5095	15.7851	14.5229	6

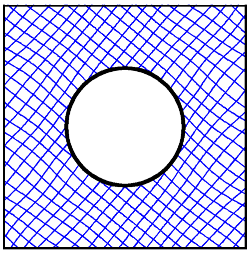
Table 1: Overview of the results using the global steering constraint

To optimise the fibre angles, first, the global steering constraint is used. In Table 1, the first column gives the maximal average steering, the next two columns give the buckling load normalised with respect to the QI buckling load, the next four columns give the maximal steering observed in each layer. The final column gives the number of FE analyses that were needed. The results are as expected: the higher the allowed steering, the higher the buckling load. Furthermore, the first two buckling modes are close but not identical; for the optimum in lamination parameter space they were equal. The number of FE analyses is low compared to the number of times the first approximation is evaluated, which is around 40 times for all cases. The most important result, however, is that the maximum local steering in the different layers is much higher than the allowed global steering, and no relationship between them can be found. These high maximum local steering values may cause manufacturing problems.

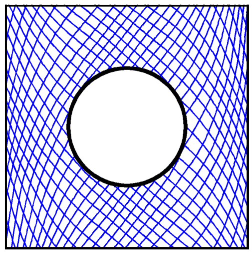
To compare the local and global steering constraints, the maximum local steering found for the global steering is set as local steering constraint. From a manufacturing point of view this is more realistic: if this steering can be laid down locally, it is possible at all places so there is no need to constrain steering at other places to be lower. In Table 2, the global steering constraint is shown in the first column, the next two columns give the optimal buckling loads found normalised with respect to the quasi-isotropic design, the fourth column gives the maximal local steering found, the next two columns give the optimal buckling load found using the local steering constraint; finally the last two columns compare the buckling loads and the number of FE analyses necessary. The local steering constraint always leads to a better result, although the

maximum global steering	buckling load 1	buckling load 2	maximum local steering	buckling load 1	buckling load 2	difference local Vs global	difference in FE analyses
1	1.2558	1.2958	3.7961	1.5603	1.6370	+ 24.2%	0
2	1.4641	1.5103	6.9759	1.7939	1.8751	+ 22.5%	0
3	1.6177	1.7141	9.3337	1.8584	1.9424	+ 14.9%	0
4	1.7481	1.8740	16.9370	1.9428	2.0317	+ 11.1%	+1
5	1.8130	1.9069	36.1727	1.9635	2.0430	+ 8.3%	+1

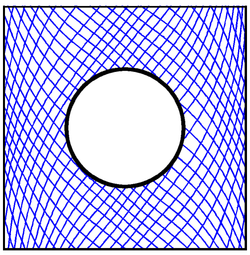
Table 2: Overview of the results using the local steering constraint of the optimum obtained using the global constraint



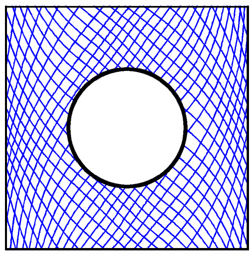
(a) fibre paths of layer 1 and 2 (outer layers)



(b) fibre paths of layer 3 and 4



(c) fibre paths of layer 5 and 6



(d) fibre paths of layer 7 and 8 (at symmetry plane)

Figure 4: fibre paths optimised using local steering constraints

difference gets smaller for larger global steering. This is due to the limited improvement that can be made at a certain time, one also notices this when the global steering constraint is used: the difference between a maximum steering of 4 and 5 is small, while the difference between 1 and 2 is considerably larger. Using the maximum local steering constraint of 36.1727, the steering constraint was never active; the maximum steering was just below 30. Furthermore, the theoretical optimum in lamination parameter space is 1.983, hence the buckling load found in lamination parameter space is only 1% higher than the buckling load of the laminate found. The small difference in buckling load is due to the constraint that only 16 layers are used, not due to the steering constraint.

For the case where the maximum local steering is set to 3.7961, the top view of the fibre paths are shown in Figure 4. In this figure, both the layer and its balanced counterpart are shown.

6. conclusion

A method has been developed to optimise the stacking sequences of variable stiffness laminates. The stacking sequences are defined at the finite element nodes using a vector of fibre angles. This direct parametrisation in terms of fibre angle results directly in the information needed to determine the fibre paths and hence to manufacture the laminates. Additional steering constraints are imposed on the norm of the gradient to assure the smoothness, and, hence, the manufacturability, of the fibre angle distributions. The method offers true stacking sequence optimisation.

To reduce computational time a multi-level approach was used: the result of the FE analysis was used to build an approximation function in terms of the in-plane and bending stiffness matrices, which was consecutively used to build a second approximation in terms of the change in fibre angles. To ensure each step is an improvement step and let the approximation be conservative, a damping function was added to the approximations. This approach has been proven to work well: a limited number of FE analyses was needed to find the optimum fibre angle distribution while the first, computationally cheaper, approximation was frequently evaluated.

The initial results indicate the effectiveness of the proposed method. When manufacturing constraints are inactive, the obtained optimum matches the performance found in lamination parameter space. When the manufacturing constraints are active, the use of local steering constraints resulted in improved optima compared to the use of average, per layer, steering constraints.

The current algorithm does not allow thickness change: the number of layers used in the optimisation is pre-specified by the user. The capability to change the laminate thickness and steering will be addressed in future work.

7. acknowledgements

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