THEORY OF SINGLE AND MULTILAYERED MICROPOLAR ORTHOTROPIC PLATES

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Abstract

In this paper the above mentioned hypotheses are generalized for micropolar single and multilayered orthotropic plates. On the basis of these hypotheses general applied theories of single and multilayered micropolar elastic thin plates are constructed. With the help of the constructed applied theory of micropolar thin plates the bending of rectangular plate is studied, when it is hinged- supported and is loaded by evenly distributed external load. On the basis of numerical results effective properties of stiffness and rigidity of plate micropolar material are revealed.

1. Introduction

Mechanics of generalized continuas, particularly, micropolar (momental, asymmetric) theory of elasticity or Cossera's model [3] develops quickly, calling a big theoretical interest in structural mechanics and physics of solids. Cossera's model received also a substantial development for construction of new generalized models of thin plates and shells [1].
In papers [5] asymptotic behavior of the solution of boundary-value problem of three-dimensional micropolar theory of elasticity is studied in thin domain of the plate and shell. In papers these qualitative results of the solution are accepted as main hypotheses and on the basis of these hypotheses general applied theories of multilayered micropolar elastic thin plates are constructed.

2. Problem statement

Orthotropic plate of constant thickness $2h$ is considered as a three-dimensional micropolar elastic body. We'll start from the basic equations of spatial static problem of linear micropolar theory of elasticity for orthotropic material with free fields of displacements and rotations [4,2]:
Equilibrium equations:

$$\nabla \cdot \vec{\sigma} = 0, \quad \nabla \cdot \vec{\mu} + \vec{\sigma}_x = 0. \tag{1}$$

Geometrical relations:

$$\vec{\varepsilon} = \nabla \cdot \vec{U} + \vec{I} \times \vec{\omega}, \quad \vec{\chi} = \nabla \cdot \vec{\omega}. \tag{2}$$

Physical relations:

$$\vec{\gamma} = A \cdot \vec{\sigma}, \quad \vec{\chi} = B \cdot \vec{\mu}. \tag{3}$$

Boundary conditions:

$$\vec{n} \cdot \vec{\sigma} = \vec{t}^{\,0}, \quad \vec{n} \cdot \vec{\mu} = \vec{m}^{0} \quad \text{on} \quad S_f,$$
$$\vec{U} = \vec{U}^{0}, \quad \vec{\omega} = \vec{\omega}^{0} \quad \text{on} \quad S_U. \tag{4}$$

Here $\vec{\sigma}, \vec{\mu}$ are tensors of forces and moments; $\vec{\gamma}, \vec{\chi}$ are tensors of deformations and bending-torsions; $\vec{U}, \vec{\omega}$ vectors of displacement and free rotation of plate points; $\hat{A}, \hat{B}$ are matrices of elastic constants of micropolar orthotropic material [2]; ∇ is Hamilton's three-dimensional differential operator; $\vec{I}$ is unit matrix; $\vec{\sigma}_x$ is vector-invariant of force stresses; S is external surface of palte; S_f is part of the surface S, where force and moment stresses (t^0, m^0) are given; S_U is part of the surface S, where displacements and rotations $(\vec{U}^0, \vec{\omega}^0)$ are given; $\vec{n}$ is the external normal of the surface S.
Energy balance equation and general variation principle are constructed for the above introduced micropolar theory of elasticity [4].

3. Theory of micropolar orthotropic elastic thin plates with free fields of displacements and rotations

Here we'll present basic principle of reduction of three-dimensional equations of micropolar theory of elasticity for orthotropic material to general two-dimensional equations of thin plates. The principle is the following. On the basis of qualitative results of initial approximation of the asymptotic method of integration of the stated three-dimensional boundary-value problem (1)-(4) rather general hypotheses are formulated [5]. Three-dimensional problem is reduced to general applied model of micropolar orthotropic elastic thin plates with the help of the formulated hypotheses.
By their content the accepted hypotheses can be viewed as kynematic and static.
In accordance with the kinematic formulation assumptions of linear distribution of components of displacement and free rotation vectors are introduced along the coordinate x_3 $(i = 1,2)$:

$$U_i = x_3 \psi_i(x_1, x_2), \quad U_3 = w(x_1, x_2), \tag{5}$$
$$\omega_i = \Omega_i(x_1, x_2), \quad \omega_3 = x_3 \iota(x_1, x_2). \tag{6}$$

Kynematic hypothesis (5) for displacements is the known Timoshenko-Mindlin hypothesis in the classical theory of elastic plates. The kinematic hypothesis (5), (6), as in paper, is called Timoshenko-Mindlin generalized hypothesis in the micropolar theory of plates.
The static hypotheses are the followings:
1. In the generalized Hook's law (3) for γ_{11}, γ_{22} force stress σ_{33} can be neglected in relation to the force stresses σ_{11}, σ_{22}.

2. During the determination of deformations, bending-torsions, force and moment stresses, first for the force stresses σ_{3i} and moment stress μ_{33} we'll take:

$$\sigma_{3i} = \overset{0}{\sigma}_{3i}(x_1, x_2) \quad (i = 1{,}2), \quad \mu_{33} = \overset{0}{\mu}_{33}(x_1, x_2)\,. \tag{7}$$

After determination of mentioned quantities, values of σ_{3i} and μ_{33} will be finally defined as the sum of the correspondent values (7) and the result of integration of the first two and the sixth equilibrium equations of (1), for which the condition will be required, that quantities, averaged along the plate thickness, are equal to zero.

3. In the generalized Hook's law (3) for χ_{i3} $(i = 1{,}2)$ moment stresses μ_{3i} can be neglected in relation to moment stresses μ_{i3} $(i = 1{,}2)$.

On the basis of the kynematic hypothesis (5), (6) we'll obtain following formulas for deformations and bending-torsions from formulas (2):

$$\begin{gathered}\gamma_{11} = x_3 K_{11}(x_1, x_2), \quad \gamma_{12} = x_3 K_{12}(x_1, x_2), \quad \gamma_{31} = \Gamma_{31}(x_1, x_2), \quad \gamma_{13} = \Gamma_{13}(x_1, x_2),\\ \gamma_{22} = x_3 K_{22}(x_1, x_2),\ \gamma_{21} = x_3 K_{21}(x_1, x_2),\ \gamma_{32} = \Gamma_{32}(x_1, x_2),\ \gamma_{23} = \Gamma_{23}(x_1, x_2),\ \gamma_{33} = 0\,.\end{gathered} \tag{8}$$

$$\begin{gathered}\chi_{11} = k_{11}(x_1, x_2), \quad \chi_{12} = k_{12}(x_1, x_2)\ , \quad \chi_{31} = 0\ , \quad \chi_{13} = x_3 l_{13}(x_1, x_2), \quad \chi_{33} = k_{33}(x_1, x_2),\\ \chi_{22} = k_{22}(x_1, x_2), \quad \chi_{21} = k_{21}(x_1, x_2), \quad \chi_{32} = 0, \quad \chi_{23} = x_3 l_{23}(x_1, x_2)\,.\end{gathered} \tag{9}$$

Here following notations are accepted:

$$\begin{gathered}K_{11} = \frac{\partial \psi_1}{\partial x_1}, \quad K_{22} = \frac{\partial \psi_2}{\partial x_2}, \quad K_{12} = \frac{\partial \psi_2}{\partial x_1} - \iota\,, \quad K_{21} = \frac{\partial \psi_1}{\partial x_2} + \iota\,,\\ \Gamma_{31} = \psi_1 - \Omega_2\,, \quad \Gamma_{32} = \psi_2 + \Omega_1\,, \quad \Gamma_{13} = \frac{\partial w}{\partial x_1} + \Omega_2\,, \quad \Gamma_{23} = \frac{\partial w}{\partial x_2} - \Omega_1\,,\end{gathered} \tag{10}$$

$$k_{11} = \frac{\partial \Omega_1}{\partial x_1}\,, \quad k_{22} = \frac{\partial \Omega_2}{\partial x_2}, \quad k_{12} = \frac{\partial \Omega_2}{\partial x_1}\,, \quad k_{21} = \frac{\partial \Omega_1}{\partial x_2}\,, \quad k_{33} = \iota\,, \quad l_{13} = \frac{\partial \iota}{\partial x_1}\,, \quad l_{23} = \frac{\partial \iota}{\partial x_2}\,. \tag{11}$$

Using the static hypotheses 1, 2 and substituting formulas (8), (9) into generalized Hook's law (1.3), we'll obtain following formulas for force and moment stresses:

$$\begin{gathered}\sigma_{11} = \frac{a_{22}}{a_{11}a_{22} - a_{12}^2} x_3 \left(K_{11} - \frac{a_{12}}{a_{22}} K_{22} \right)\ , \quad \sigma_{22} = \frac{a_{11}}{a_{11}a_{22} - a_{12}^2} x_3 \left(K_{22} - \frac{a_{12}}{a_{11}} K_{11} \right),\\ \sigma_{12} = \left(\frac{a_{88}}{a_{77}a_{88} - a_{78}^2} K_{12} - \frac{a_{78}}{a_{77}a_{88} - a_{78}^2} K_{21} \right) x_3, \ \sigma_{21} = \left(\frac{a_{77}}{a_{77}a_{88} - a_{78}^2} K_{21} - \frac{a_{78}}{a_{77}a_{88} - a_{78}^2} K_{12} \right) x_3\,,\\ \sigma_{13} = \frac{\tilde{a}_{55}}{\tilde{a}_{55}a_{66} - a_{56}^2} \Gamma_{13} - \frac{a_{56}}{\tilde{a}_{55}a_{66} - a_{56}^2} \Gamma_{31}, \quad \sigma_{23} = \frac{a_{55}}{a_{44}a_{55} - a_{45}^2} \Gamma_{23} - \frac{a_{45}}{a_{44}a_{55} - a_{45}^2} \Gamma_{32},\\ \overset{0}{\sigma}_{32} = \frac{a_{44}}{a_{44}a_{55} - a_{45}^2} \Gamma_{32} - \frac{a_{45}}{a_{44}a_{55} - a_{45}^2} \Gamma_{23}, \quad \overset{0}{\sigma}_{31} = \frac{a_{66}}{\tilde{a}_{55}a_{66} - a_{56}^2} \Gamma_{31} - \frac{a_{56}}{\tilde{a}_{55}a_{66} - a_{56}^2} \Gamma_{13},\\ \sigma_{31} = \overset{0}{\sigma}_{31}(x_1; x_2) + \left(\frac{h^2}{6} - \frac{x_3^2}{2} \right) \left(\frac{\partial \overset{1}{\sigma}_{11}}{\partial x_1} + \frac{\partial \overset{1}{\sigma}_{21}}{\partial x_2} \right),\end{gathered} \tag{12}$$

$$\sigma_{32} = \overset{0}{\sigma}_{32}(x_1;x_2) + \left(\frac{h^2}{6} - \frac{x_3^2}{2}\right)\left(\frac{\partial \overset{1}{\sigma}_{22}}{\partial x_2} + \frac{\partial \overset{1}{\sigma}_{12}}{\partial x_1}\right), \quad \sigma_{33} = -x_3\left(\frac{\partial \sigma_{13}}{\partial x_1} + \frac{\partial \sigma_{23}}{\partial x_2}\right).$$

$$\mu_{11} = \frac{\begin{vmatrix} b_{22} & b_{23} \\ b_{23} & b_{33} \end{vmatrix}}{\Delta_b} \mathrm{k}_{11} - \frac{\begin{vmatrix} b_{12} & b_{13} \\ b_{23} & b_{33} \end{vmatrix}}{\Delta_b} \mathrm{k}_{22} + \frac{\begin{vmatrix} b_{12} & b_{13} \\ b_{22} & b_{23} \end{vmatrix}}{\Delta_b} \mathrm{k}_{33},$$

$$\mu_{22} = \frac{\begin{vmatrix} b_{11} & b_{13} \\ b_{13} & b_{33} \end{vmatrix}}{\Delta_b} \mathrm{k}_{22} - \frac{\begin{vmatrix} b_{12} & b_{23} \\ b_{13} & b_{33} \end{vmatrix}}{\Delta_b} \mathrm{k}_{11} - \frac{\begin{vmatrix} b_{11} & b_{13} \\ b_{12} & b_{23} \end{vmatrix}}{\Delta_b} \mathrm{k}_{33}, \tag{13}$$

$$\overset{0}{\mu}_{33} = \frac{\begin{vmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{vmatrix}}{\Delta_b} \mathrm{k}_{33} + \frac{\begin{vmatrix} b_{12} & b_{22} \\ b_{13} & b_{23} \end{vmatrix}}{\Delta_b} \mathrm{k}_{11} - \frac{\begin{vmatrix} b_{11} & b_{12} \\ b_{13} & b_{23} \end{vmatrix}}{\Delta_b} \mathrm{k}_{22}, \qquad \Delta_b = \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{vmatrix},$$

$$\mu_{12} = \frac{b_{88}}{b_{77}b_{88} - b_{78}^2} k_{12} - \frac{b_{78}}{b_{77}b_{88} - b_{78}^2} k_{21}, \quad \mu_{21} = \frac{b_{77}}{b_{77}b_{88} - b_{78}^2} k_{21} - \frac{b_{78}}{b_{77}b_{88} - b_{78}^2} k_{12},$$

$$\mu_{31} = -x_3\left(\frac{\partial \mu_{11}}{\partial x_1} + \frac{\partial \mu_{21}}{\partial x_2} + \sigma_{23} - \sigma_{32}\right), \quad \mu_{32} = -x_3\left(\frac{\partial \mu_{12}}{\partial x_1} + \frac{\partial \mu_{22}}{\partial x_2} + \sigma_{31} - \sigma_{13}\right),$$

$$\mu_{33} = \overset{0}{\mu}_{33}(x_1;x_2) + \left(\frac{h^2}{6} - \frac{x_3^2}{2}\right)\left(\frac{\partial \overset{1}{\mu}_{13}}{\partial x_1} + \frac{\partial \overset{1}{\mu}_{23}}{\partial x_2} + \overset{1}{\sigma}_{12} - \overset{1}{\sigma}_{21}\right), \ \mu_{13} = x_3 \frac{1}{b_{66}} l_{13}, \ \mu_{23} = x_3 \frac{1}{b_{44}} l_{23}.$$

In order to bring three-dimensional problem of micropolar theory of elasticity to two-dimensional one, which has been already done for deformations, bending-torsions, displacements, force and moment stresses, in micropolar theory of elastic thin plates instead of the components of the tensors of force and moment stresses statically equivalent to them integral characteristics- forces N_{i3}, N_{3i}, moments from force stresses M_{ii}, M_{ij}, moments from moment stresses L_{ii}, L_{33}, L_{ij} and hypermoments from moment stresses Λ_{i3} are introduced:

$$N_{i3} = \int_{-h}^{h} \sigma_{i3} dx_3, \ N_{3i} = \int_{-h}^{h} \sigma_{3i} dx_3, \ \mathrm{M}_{ii} = \int_{-h}^{h} x_3 \sigma_{ii} dx_3, \ \mathrm{M}_{ij} = \int_{-h}^{h} x_3 \sigma_{ij} dx_3,$$

$$\mathrm{L}_{ii} = \int_{-h}^{h} \mu_{ii} dx_3, \ \mathrm{L}_{33} = \int_{-h}^{h} \mu_{33} dx_3, \ \mathrm{L}_{ij} = \int_{-h}^{h} \mu_{ij} dx_3, \ \Lambda_{i3} = \int_{-h}^{h} x_3 \mu_{i3} dx_3. \ (i = 1,2) \tag{14}$$

The basic system of equations of the general theory of static bending deformation of micropolar orthotropic elastic thin plates with free fields of displacements and rotations, constructed on the basis of the accepted kynematic and static hypotheses, is the following:

Equilibrium equations:

$$\frac{\partial N_{13}}{\partial x_1}+\frac{\partial N_{23}}{\partial x_2}=-\tilde{p}_3,\qquad N_{3i}-\left(\frac{\partial M_{ii}}{\partial x_i}+\frac{\partial M_{ji}}{\partial x_j}\right)=h\tilde{p}_i\,,$$

$$\frac{\partial L_{ii}}{\partial x_i}+\frac{\partial L_{ji}}{\partial x_j}+(-1)^j\left(N_{j3}-N_{3j}\right)=-\tilde{m}_i\,,\quad L_{33}-\left[\frac{\partial \Lambda_{13}}{\partial x_1}+\frac{\partial \Lambda_{23}}{\partial x_2}+(M_{12}-M_{21})\right]=h\tilde{m}_3. \tag{15}$$

Physical relations:

$$N_{13}=\tilde{C}_{55}\Gamma_{13}+C_{56}\Gamma_{31}\,,\quad N_{31}=C_{66}\Gamma_{31}+C_{56}\Gamma_{13}\,,\quad N_{23}=C_{55}\Gamma_{23}+C_{45}\Gamma_{32}\,,\quad N_{32}=C_{44}\Gamma_{32}+C_{45}\Gamma_{23}\,,$$

$$M_{11}=D_{11}K_{11}+D_{12}K_{22}\,,\qquad M_{22}=D_{22}K_{22}+D_{12}K_{11}\,,$$

$$M_{12}=D_{88}K_{12}+D_{78}K_{21}\,,\qquad M_{21}=D_{77}K_{21}+D_{78}K_{12}\,, \tag{16}$$

$$L_{11}=d_{11}k_{11}+d_{12}k_{22}+d_{13}k_{33}\,,\quad L_{22}=d_{22}k_{22}+d_{21}k_{11}+d_{23}k_{33}\,,\quad L_{33}=d_{33}k_{33}+d_{31}k_{11}+d_{32}k_{22}\,,$$

$$L_{12}=d_{88}k_{12}+d_{78}k_{21}\,,\quad L_{21}=d_{77}k_{21}+d_{78}k_{12}\,,\quad \Lambda_{13}=\lambda_{66}l_{13}\,,\quad \Lambda_{23}=\lambda_{44}l_{23}\,.$$

Geometrical relations have the form (10), (11). Here

$$\tilde{C}_{55}=2h\frac{\tilde{a}_{55}}{\tilde{a}_{55}a_{66}-a_{56}^2}\,,\quad C_{66}=2h\frac{a_{66}}{\tilde{a}_{55}a_{66}-a_{56}^2}\,,\quad C_{56}=-2h\frac{a_{56}}{\tilde{a}_{55}a_{66}-a_{56}^2}\,,$$

$$C_{55}=2h\frac{a_{55}}{a_{44}a_{55}-a_{45}^2}\,,\quad C_{44}=2h\frac{a_{44}}{a_{44}a_{55}-a_{45}^2}\,,\quad C_{45}=-2h\frac{a_{45}}{a_{44}a_{55}-a_{45}^2}\,,$$

$$D_{11}=\frac{2h^3}{3}\frac{a_{22}}{a_{11}a_{22}-a_{12}^2}\,,\quad D_{22}=\frac{2h^3}{3}\frac{a_{11}}{a_{11}a_{22}-a_{12}^2}\,,\quad D_{12}=-\frac{2h^3}{3}\frac{a_{12}}{a_{11}a_{22}-a_{12}^2}\,,$$

$$D_{88}=\frac{2h^3}{3}\frac{a_{88}}{a_{77}a_{88}-a_{78}^2}\,,\quad D_{77}=\frac{2h^3}{3}\frac{a_{77}}{a_{77}a_{88}-a_{78}^2}\,,\quad D_{78}=-\frac{2h^3}{3}\frac{a_{78}}{a_{77}a_{88}-a_{78}^2}\,,$$

$$d_{11}=2h\frac{\begin{vmatrix} b_{22} & b_{23}\\ b_{23} & b_{33}\end{vmatrix}}{\Delta_b}\,,\quad d_{12}=-2h\frac{\begin{vmatrix} b_{12} & b_{13}\\ b_{23} & b_{33}\end{vmatrix}}{\Delta_b}\,,\quad d_{13}=2h\frac{\begin{vmatrix} b_{12} & b_{13}\\ b_{22} & b_{23}\end{vmatrix}}{\Delta_b}\,,$$

$$d_{22}=2h\frac{\begin{vmatrix} b_{11} & b_{13}\\ b_{13} & b_{33}\end{vmatrix}}{\Delta_b}\,,\quad d_{21}=-2h\frac{\begin{vmatrix} b_{12} & b_{23}\\ b_{13} & b_{33}\end{vmatrix}}{\Delta_b}\,,\quad d_{23}=-2h\frac{\begin{vmatrix} b_{11} & b_{13}\\ b_{12} & b_{23}\end{vmatrix}}{\Delta_b}\,,$$

$$d_{33}=2h\frac{\begin{vmatrix} b_{11} & b_{12}\\ b_{12} & b_{22}\end{vmatrix}}{\Delta_b}\,,\quad d_{31}=2h\frac{\begin{vmatrix} b_{12} & b_{22}\\ b_{13} & b_{23}\end{vmatrix}}{\Delta}\,,\quad d_{32}=-2h\frac{\begin{vmatrix} b_{11} & b_{12}\\ b_{13} & b_{23}\end{vmatrix}}{\Delta_b}\,,$$

$$d_{88}=2h\frac{b_{88}}{b_{77}b_{88}-b_{78}^2}\,,\quad d_{77}=2h\frac{b_{77}}{b_{77}b_{88}-b_{78}^2}\,,\quad d_{78}=-2h\frac{b_{78}}{b_{77}b_{88}-b_{78}^2}\,,$$

$$\lambda_{66}=\frac{2h^3}{3}\frac{1}{b_{66}}\,,\ \lambda_{44}=\frac{2h^3}{3}\frac{1}{b_{44}}\,.$$

"Softened" boundary conditions on the boundary contour Γ of the plate middle plane should be added to the system of equations (15), (16), (10), (11) of model with free fields of displacements and rotations of micropolar elastic thin plates (on $x_1 = const$):

$$
\begin{aligned}
&M_{11} = M_{11}^* \text{ or } K_{11} = K_{11}^*,\ M_{12} = M_{12}^* \text{ or } K_{12} = K_{12}^*,\ N_{13} = N_{13}^* \text{ or } w = w^*,\\
&L_{11} = L_{11}^* \text{ or } k_{11} = k_{11}^*,\ L_{12} = L_{12}^* \text{ or } k_{12} = k_{12}^*,\ \Lambda_{13} = \Lambda_{13}^* \text{ or } l_{13} = l_{13}^*.
\end{aligned}
\tag{17}
$$

The system of equations (15), (16), (10), (11) and boundary conditions (17) present the mathematical model with free fields of displacements and rotations of micropolar orthotropic elastic thin plates, when transverse shears and related deformations are completely taken into account. It is a system of differential equations of 12th order with 6 boundary conditions on each side of the plate middle plane.

The energy balance equation for the model (15), (16), (10), (11), (17) of micropolar orthotropic elastic thin plates is the following:

$$
\begin{aligned}
\iint_{(s)} \tilde{W} ds = \frac{1}{2} \iint_{(s)} \left[\left(p_1 h \psi_1 + p_2 h \psi_2 + p_3 w \right) + \left(m_1 \Omega_1 + m_2 \Omega_2 + m_3 h \iota \right) \right] ds + \\
+ \int_l \left[\left(M_{11}^0 \psi_1 + M_{12}^0 \psi_2 + N_{13}^0 w \right) + \left(L_{11}^0 \Omega_1 + L_{12}^0 \Omega_2 + \Lambda_{13}^0 \iota \right) \right] dl,
\end{aligned}
\tag{18}
$$

where the density $\tilde{W}$ of deformation potential energy is expressed as follows:

$$
\begin{aligned}
\tilde{W} = \frac{1}{2} \Big[& D_{11} K_{11}^2 + D_{22} K_{22}^2 + 2 D_{12} K_{11} K_{22} + D_{88} K_{21}^2 + 2 D_{78} K_{12} K_{21} + D_{77} K_{12}^2 + \\
& + C_{66} \Gamma_{31}^2 + 2 C_{56} \Gamma_{31} \Gamma_{13} + \tilde{C}_{55} \Gamma_{13}^2 + C_{44} \Gamma_{32}^2 + 2 C_{45} \Gamma_{32} \Gamma_{23} + C_{55} \Gamma_{23}^2 + \\
& + d_{11} k_{11}^2 + d_{22} k_{22}^2 + d_{33} k_{33}^2 + 2 d_{12} k_{11} k_{22} + 2 d_{13} k_{11} k_{33} + 2 d_{23} k_{22} k_{33} + d_{77} k_{21}^2 + \\
& + 2 d_{78} k_{12} k_{21} + d_{88} k_{12}^2 + \lambda_{66} l_{13}^2 + \lambda_{44} l_{23}^2 \Big].
\end{aligned}
\tag{19}
$$

General variation functional for this model can be introduced in the following form:

$$
\begin{aligned}
I = \iint_s \Bigg\langle \tilde{W} - \Bigg\{ & M_{11} \left(K_{11} - \frac{\partial \psi_1}{\partial x_1} \right) + M_{22} \left(K_{22} - \frac{\partial \psi_2}{\partial x_2} \right) + + M_{12} \left[K_{12} - \left(\frac{\partial \psi_2}{\partial x_1} - \iota \right) \right] + \\
& + M_{21} \left[K_{21} - \left(\frac{\partial \psi_1}{\partial x_2} + \iota \right) \right] + N_{13} \left[\Gamma_{13} - \left(\frac{\partial w}{\partial x_1} + \Omega_2 \right) \right] + N_{23} \left[\Gamma_{23} - \left(\frac{\partial w}{\partial x_2} - \Omega_1 \right) \right] + \\
& + L_{11} \left(k_{11} - \frac{\partial \Omega_1}{\partial x_1} \right) + L_{22} \left(k_{22} - \frac{\partial \Omega_2}{\partial x_2} \right) + \\
& + L_{33} \left(k_{33} - \iota \right) + L_{12} \left(k_{12} - \frac{\partial \Omega_2}{\partial x_1} \right) + L_{21} \left(k_{21} - \frac{\partial \Omega_1}{\partial x_2} \right) + \Lambda_{13} \left(l_{13} - \frac{\partial \iota}{\partial x_1} \right) + \Lambda_{23} \left(l_{23} - \frac{\partial \iota}{\partial x_1} \right) \Bigg\} \Bigg\rangle ds - \\
& - \iint_s \left[\left(p_1 h \psi_1 + p_2 h \psi_2 + p_3 w \right) + \left(m_1 \Omega_1 + m_2 \Omega_2 + m_3 h \iota \right) \right] ds - \\
& - \int_l \left[\left(M_{11}^0 \psi_1 + M_{12}^0 \psi_2 + N_{13}^0 w \right) + \left(L_{11}^0 \Omega_1 + L_{12}^0 \Omega_2 + \Lambda_{13}^0 \iota \right) \right] dl.
\end{aligned}
\tag{20}
$$

3. Mathematical model of micropolar orthotropic elastic multilayered plates

Multilayered thin plates of n layers is considered. Material of plate each layer obeys to Hook's generalized law for micropolar orthotropic elastic material. It is assumed that plate layers are connected and work together without sliding.
Middle plane of some layer k is accepted as initial layer and is brought to Cartesian coordinate x_1, x_2. Transfer coordinate z is directed along the external normal of initial plane. We'll make following notations: z_i is the coordinate of upper limit of layer i; z_0, z_n are coordinates of lower and upper boundary surfaces.
We'll proceed from basic equations (1)-(3) of spatial static problem of linear micropolar theory of elasticity with free fields of displacements and rotations for the plate each layer.
It is assumed that the plate is thin, i.e. $2h << a$, where $2h$ is the whole thickness of the plate, a is the characteristic size of the plate.
Hypotheses are used during the construction of the theory of multilayered micropolar orthotropic elastic thin plates. It is assumed that generalized for micropolar case hypothesis (5), (6) and static hypotheses 1-3 are true for the whole and each package of the plate. Main system of applied two-dimensional theory of micropolar orthotropic elastic multilayered plates is obtained on the basis of these hypotheses:

Equilibrium equations:

$$\begin{gathered}
\frac{\partial T_{11}}{\partial x_1}+\frac{\partial S_{21}}{\partial x_2}=-\left(q_1^+ + q_1^-\right), \qquad \frac{\partial T_{22}}{\partial x_2}+\frac{\partial S_{12}}{\partial x_1}=-\left(q_2^+ + q_2^-\right), \\
\frac{\partial N_{13}}{\partial x_1}+\frac{\partial N_{23}}{\partial x_2}=-\left(q_3^+ + q_3^-\right), \quad N_{31}-\frac{\partial M_{11}}{\partial x_1}=z_n q_1^+ + z_0 q_1^-, \quad N_{32}-\frac{\partial M_{22}}{\partial x_2}=z_n q_2^+ + z_0 q_2^-, \\
\frac{\partial L_{11}}{\partial x_1}+\frac{\partial L_{21}}{\partial x_2}+N_{23}-N_{32}=-\left(m_1^+ + m_1^-\right), \qquad \frac{\partial L_{22}}{\partial x_2}+\frac{\partial L_{12}}{\partial x_1}+N_{31}-N_{13}=-\left(m_2^+ + m_2^-\right), \\
\frac{\partial L_{13}}{\partial x_1}+\frac{\partial L_{23}}{\partial x_2}+S_{12}-S_{21}=-\left(m_3^+ + m_3^-\right), \quad L_{33}-\frac{\partial \Lambda_{13}}{\partial x_1}-\frac{\partial \Lambda_{23}}{\partial x_2}=z_n m_3^+ + z_0 m_3^-.
\end{gathered} \tag{21}$$

Physical relations:

$$\begin{gathered}
T_{11}=C_{11}\Gamma_{11}+C_{12}\Gamma_{22}+R_{11}K_{11}+R_{12}K_{22}, \qquad T_{22}=C_{12}\Gamma_{11}+C_{22}\Gamma_{22}+R_{12}K_{11}+R_{22}K_{22}, \\
S_{12}=C_{88}\Gamma_{12}+C_{78}\Gamma_{21}+R_{88}K_{12}+R_{78}K_{21}, \qquad S_{21}=C_{77}\Gamma_{21}+C_{78}\Gamma_{12}+R_{77}K_{21}+R_{78}K_{12}, \\
N_{13}=\tilde{C}_{55}\Gamma_{13}+C_{56}\Gamma_{31}, \quad N_{31}=C_{66}\Gamma_{31}+C_{56}\Gamma_{13}, \quad N_{32}=C_{44}\Gamma_{32}+C_{45}\Gamma_{23}, \quad N_{23}=C_{55}\Gamma_{23}+C_{45}\Gamma_{32}, \\
M_{11}=D_{11}K_{11}+D_{12}K_{22}+R_{11}\Gamma_{11}+R_{12}\Gamma_{22}, \quad M_{22}=D_{12}K_{11}+D_{22}K_{22}+R_{12}\Gamma_{11}+R_{22}\Gamma_{22}, \\
H_{12}=D_{88}K_{12}+D_{78}K_{21}+R_{88}\Gamma_{12}+R_{78}\Gamma_{21}, \quad H_{21}=D_{77}K_{21}+D_{78}K_{12}+R_{77}\Gamma_{21}+R_{78}\Gamma_{12}, \\
L_{11}=d_{11}k_{11}+d_{12}k_{22}+d_{13}\iota, \quad L_{22}=d_{12}k_{11}+d_{22}k_{22}+d_{23}\iota, \quad L_{33}=d_{13}k_{11}+d_{23}k_{22}+d_{33}\iota, \\
L_{12}=d_{88}k_{12}+d_{78}k_{21}, \quad L_{21}=d_{78}k_{12}+d_{77}k_{21}, \quad L_{13}=d_{66}k_{13}+\eta_{66}l_{13}, \quad L_{23}=d_{44}k_{23}+\eta_{44}l_{23}, \\
\Lambda_{13}=\lambda_{66}l_{13}+\eta_{66}k_{13}, \qquad \Lambda_{23}=\lambda_{44}l_{23}+\eta_{44}k_{23},
\end{gathered} \tag{22}$$

where

$$C_{11}=\sum_{i=1}^{n}\frac{a_{22}^i}{a_{11}^i a_{22}^i-\left(a_{12}^i\right)^2}\left(z_i-z_{i-1}\right), \qquad D_{11}=\sum_{i=1}^{n}\frac{a_{22}^i}{a_{11}^i a_{22}^i-\left(a_{12}^i\right)^2}\frac{1}{3}\left(z_i^3-z_{i-1}^3\right).. \tag{23}$$

It should be noted that when plate is composed of odd number of layers and the middle plane of the middle layer is accepted as initial plane, the system of equations (10), (11), (21), (22) and boundary conditions (17) will be separated into two different systems and boundary conditions for plane stress state problem and bending problem.
Geometric relations are (10), (11). In case of bending boundary conditions have the form (17). Energy balance equation and variation equation have the forms (18), (19), (20), where physical relations (22) must be taken into account for the whole package of the plate.

4. Problem of bending of hinged-supported micropolar isotropic elastic rectangular plate.

The equilibrium of micropolar isotropic, elastic rectangular plate is considered, when it is hinged-supported in four edges and is bending under normal, uniformly distributed load. On the basis of system of equations (2.11),(2.12),(2.6),(2.7), we'll obtain system of equations for unknown functions $w(x_1,x_2)$, $\psi_1(x_1,x_2)$, $\psi_2(x_1,x_2)$, $\Omega_1(x_1,x_2)$, $\Omega_2(x_1,x_2)$, $\iota(x_1,x_2)$:
The solution of the problem (4.1),(4.2), which satisfies boundary conditions of hinged support, is the following:

$$w = A_1 \sin\frac{k\pi x_1}{a}\sin\frac{k\pi x_2}{b}, \quad \Psi_1 = A_4 \cos\frac{k\pi x_1}{a}\sin\frac{k\pi x_2}{b}, \quad \Omega_1 = A_2 \sin\frac{k\pi x_1}{a}\cos\frac{k\pi x_2}{b},$$

$$\Psi_2 = A_5 \sin\frac{k\pi x_1}{a}\cos\frac{k\pi x_2}{b}, \quad \Omega_2 = A_3 \cos\frac{k\pi x_1}{a}\sin\frac{k\pi x_2}{b}, \quad \iota = A_6 \cos\frac{k\pi x_1}{a}\cos\frac{k\pi x_2}{b}. \qquad (24)$$

Algebraic linear inhomogeneous system of equations will be obtained for coefficients $A_1, A_2, A_3, A_4, A_5, A_6$ ($q_3 = const = q_0$).
Numerical results are introduced below: elastic constants: $\mu = 4\cdot 10^9\ N/m^2$, $\lambda = 2.8\cdot 10^{10}\ N/m^2$, $\alpha = 4\cdot 10^6\ N/m^2$, $\gamma = 1.936\cdot 10^8\ N$, $\varepsilon = 3.046\cdot 10^9\ N$, $\beta = 1\cdot 10^8\ N$, $q = 500 N/m^2$, $a = 0.1m$, $h = 0.001m$. Plate maximal bending is $w_{\max} = 0.8\cdot 10^{-5} m$, classical value is $w_{\max} = 2\cdot 10^{-5} m$. Thus, we can say that even when $\frac{\alpha}{\mu} = \frac{1}{1000}$, the micropolarity of plate material increases the plate rigidity. The same result is obtained also in case of the determination of maximal stresses.

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