PLASTIC DEFORMATION FLOW EQUATIONS OF TI_2ALN/TIAL COMPOSITES

D. L. Sun[a*], T. Sun[a], Q. Wang[a], D. Q. Wang[a], X. L. Han[a], J. Wang[b]， G. H. Wu[a]

[a]*School of Materials Science and Engineering, Harbin Institute of Technology, China)*
[b] *China Academy Launch Vehicle Technology, Beijing, China*
**sundl1958@hit.edu.cn*

Keywords: TiAl matrix composites, Ti_2AlN particles, plastic deformation flow equation

Abstract

Compression tests of different temperatures and strain rates were carried out on TiAl alloy and Ti_2AlN particle reinforced TiAl matrix composites of two different volume fractions by Gleeble-1500D thermal simulated test machine. According to the experimental data, $\dot{\varepsilon}\sim\sigma_p$ and $\sigma_p\sim 1/T$ curves of different materials were obtained. The thermal deformation activation energy of three different materials in series of temperature and strain rate was calculated through the slope of each curve. Then plastic deformation flow equation was established. The study result showed that the thermal deformation stress of composite was higher than that of matrix alloy. The establishment of plastic deformation flow equation provided theoretical references for machine-shaping and application of Ti2AlN/TiAl composites.

1. Introduction

To improve the high-temperature performance of TiAl matrix alloy, kinds of TiAl matrix composites have been developed[1]. Varieties of fibers，particles and whiskers are added into TiAl matrix alloy to prepare TiAl matrix composites. Recent research shows that Ti_2AlN particle reinforced TiAl matrix composites are expected to be used in the manufacture of high temperature components because of its great high temperature mechanical properties[2,3,4,5]. But deformation behavior and mechanism in high temperature of Ti2AlN particle reinforced TiAl matrix composites are not further realized. So discussing the deformation behavior and mechanismis at high temperature helps to guiding the process and expanding the practical application. In this paper, compression tests of different temperature and strain rate were carried out on TiAl alloy and Ti_2AlN particle reinforced TiAl matrix composites of two different volume fractions by Gleeble-1500D thermal simulated test machine. The thermal deformation activation energy of three different materials was calculated. Then the plastic deformation flow equations were established.

2. Experimental Methods

Based on Arrhenius equation, the relation between flow stress and strain rate was:

$$\dot{\varepsilon} = A[\sinh(\alpha\sigma)]^n \exp(-Q/RT) \tag{1}$$

Deformation activation energy was calculated through differential of Equation (1):

$$Q = R\left\{\frac{\partial \ln(\dot{\varepsilon})}{\partial \ln[\sinh(\alpha\sigma)]}\right\}_T \left\{\frac{\partial \ln[\sinh(\alpha\sigma)]}{\partial(1/T)}\right\}_{\dot{\varepsilon}} \tag{2}$$

If the hyperbolic sine was expanded through the function definition, equation (1) was:

$$\dot{\varepsilon} = A\left(\frac{\exp(\alpha\sigma) - \exp(-\alpha\sigma)}{2}\right)^n \exp(-Q/RT) \tag{3}$$

If $\alpha\sigma$ <0.8 (low stress level), $\frac{\exp(\alpha\sigma) - \exp(-\alpha\sigma)}{2} \approx \alpha\sigma$. And the equation (3) was:

$$\dot{\varepsilon} = A\alpha^n\sigma^n \exp(-Q/R\mathrm{T}) = A_1\sigma^{n_1} \exp(-Q/RT) \tag{4}$$

In equation (4), n was n_1. If A_1 was equal to $A\,\alpha^n$ and take logarithm on both ends of equation (4) at the same time, the equation was:

$$\ln\dot{\varepsilon} = \ln A_1 + n_1 \ln\sigma - Q/RT \tag{5}$$

$$n_1 = \left(\frac{\partial \ln\dot{\varepsilon}}{\partial \ln\sigma}\right)_T, B_1 = \frac{Q}{nR} = \left(\frac{\partial \ln\sigma}{\partial(1/T)}\right)_{\dot{\varepsilon}} \tag{6}$$

So deformation activation energy was known through equation (7).

$$Q = n_1 R B_1 = R\left(\frac{\partial \ln\dot{\varepsilon}}{\partial \ln\sigma}\right)_T \left(\frac{\partial \ln\sigma}{\partial(1/T)}\right)_{\dot{\varepsilon}} \tag{7}$$

If $\alpha\sigma$ >1.2(high strain level), $\frac{\exp(\alpha\sigma) - \exp(-\alpha\sigma)}{2} \approx \frac{\exp(\alpha\sigma)}{2}$. So equation (4) was:

$$\dot{\varepsilon} = A(\frac{\exp(\alpha\sigma)}{2})^n \exp(-Q/RT) = \frac{A}{2^n}\exp(n\alpha\sigma)\exp(-Q/RT) \tag{8}$$

If $A_2 = \frac{A}{2^n}, \beta = n\alpha$, the equation was taken as exponential as:

$$\dot{\varepsilon} = A_2 \exp(\beta\sigma)\exp(-Q/RT) \tag{9}$$

Take logarithm on both ends of equation (9) at the same time, the equation was:

$$\ln\dot{\varepsilon} = \ln A_2 + \beta\sigma - Q/RT \tag{10}$$

$$\beta = \left(\frac{\partial \ln \dot{\varepsilon}}{\partial \sigma}\right)_T, B_2 = \frac{Q}{\beta R} = \left(\frac{\partial \sigma}{\partial (1/T)}\right)_{\dot{\varepsilon}} \tag{11}$$

So deformation activation energy was known through equation (12).

$$Q = \beta R B_2 = R\left(\frac{\partial \ln \dot{\varepsilon}}{\partial \sigma}\right)_T \left(\frac{\partial \sigma}{\partial (1/T)}\right)_{\dot{\varepsilon}} \tag{12}$$

If 0.8< $\alpha\sigma$ <1.2, the logarithm of Equation (1) which was the original expression of plastic deformation was:

$$\ln \dot{\varepsilon} = \ln A + n \ln[\sinh(\alpha\sigma)] - Q / RT \tag{13}$$

$$n = \left(\frac{\partial \ln \dot{\varepsilon}}{\partial \ln[\sinh(\alpha\sigma)]}\right)_T, B = \frac{Q}{nR} = \left(\frac{\partial \ln[\sinh(\alpha\sigma)]}{\partial (1/T)}\right)_{\dot{\varepsilon}} \tag{14}$$

So deformation activation energy was:

$$Q = nRB = R\left(\frac{\partial \ln \dot{\varepsilon}}{\partial \ln[\sinh(\alpha\sigma)]}\right)_T \left(\frac{\partial \ln[\sinh(\alpha\sigma)]}{\partial (1/T)}\right)_{\dot{\varepsilon}} \tag{15}$$

3. Results and Analysis.

The compression strength (peak stress σ_p) of TiAl alloy and Ti_2AlN/TiAl composites under the condition of test was shown in Table 1.

material	temperature [℃]	strain rate [s^{-1}]		
		0.001	0.01	0.1
TiAl	800	785.2	975.2	1045.2
	900	510.9	683.1	869.0
	1000	270.0	424.9	621.0
	1100	157.2	271.3	411.6
20Ti_2AlN/TiAl	800	861.5	958.6	1021.7
	900	564.8	748.8	900.3
	1000	325.3	515.1	742.6
	1100	152.5	293.8	618.4
50Ti_2AlN/TiAl	800	851.0	926.7	964.0
	900	674.0	799.1	797.4
	1000	382.9	539.0	707.4
	1100	161.3	374.2	577.6

Table 1. Compression strength of three kinds of materials at different conditions

Table 1 showed that strain rate reduced as deformation temperature increased. The peak flow stress of composites was higher than that of TiAl alloy expect the condition of low temperature (800℃) and high strain rate(0.1s^{-1}). The peak flow stress of the composite which had lower volume fraction of reinforcement was higher than the other one in high strain rate. As strain rate reduced, the stress of the material was high whose volume fraction of reinforcement was low in 800℃ while the stress of the material was high whose volume fraction of reinforcement was high in high temperature (900-1100℃).

3.1 Hot compression deformation activation energy and plastic deformation flow equation of TiAl alloy

Table 2 showed the result of univariate regressions about TiAl alloy. The linearly dependent coefficients were approximate. The linearly dependent coefficient of power exponent equation was slightly higher while β was approximate and the effect of temperature on stress index (n) was less. So power-exponent equation at high stress level was more suitable to describe the relation between $\dot{\varepsilon}$ and σ_p of high temperature compression deformation.

function	deformation temperature[℃]	regression equation	correlation coefficient	significance
exponential function	800	$\ln\dot{\varepsilon}$=14.79 $\ln\sigma_p$-105.7	0.9585	*
	900	$\ln\dot{\varepsilon}$=8.645 $\ln\sigma_p$-60.88	0.9985	***
	1000	$\ln\dot{\varepsilon}$=6.02 $\ln\sigma_p$-41.03	1	***
	1100	$\ln\dot{\varepsilon}$=4.756 $\ln\sigma_p$-31.05	0.9970	***
power exponential function	800	$\ln\dot{\varepsilon}$=0.01654 σ_p-20.07	0.9662	*
	900	$\ln\dot{\varepsilon}$=0.01285 σ_p-13.44	0.9998	***
	1000	$\ln\dot{\varepsilon}$=0.01372 σ_p-10.71	0.9990	***
	1100	$\ln\dot{\varepsilon}$=0.01804 σ_p-9.656	0.9997	***
hyperbolic sine function	800	$\ln\dot{\varepsilon}$=11.34 $\ln(\sinh(\alpha\sigma_p))$-6.8	0.9621	*
	900	$\ln\dot{\varepsilon}$=6.542 $\ln(\sinh(\alpha\sigma_p))$-5.82	0.9997	***
	1000	$\ln\dot{\varepsilon}$=4.543 $\ln(\sinh(\alpha\sigma_p))$-5.319	0.9946	***
	1100	$\ln\dot{\varepsilon}$=3.552 $\ln(\sinh(\alpha\sigma_p))$-5.125	0.9982	***

Table 2. Regression result of strain rate ($\dot{\varepsilon}$) and peak stress (σ_p) of TiAl
comment：highly significant on 0.01 level (***),significant on 0.05 level (*)

The fitting line about σ_p and $1/T$ of TiAl alloy was plotted through equation (11). The calculated linearly dependent coefficients were all more than 0.99 and significant on 0.01 level. As the slopes were all approximate, strain rate had almost no effect on B_2. Table 3 was the original calculation results. The peak deformation flow equation of TiAl alloy was:

$$\dot{\varepsilon} = \exp(27.3)\exp(0.01522\sigma_p) - 398.6 / RT \text{，} \quad 0.001s^{-1} \le \dot{\varepsilon} \le 0.1s^{-1} \tag{16}$$

T, ℃	$\dot{\varepsilon}$ [s^{-1}] B_2 B	0.001 3.12E+06 Q	 $\ln A_2$	0.01 3.51E+06 Q	 $\ln A_2$	0.1 3.14E+06 Q	 $\ln A_2$
800	0.01654	428.8	28	472.4	34	431.6	28.3
900	0.01258	326.2	20	356.9	24.2	328.3	20.2
1000	0.01372	355.7	22.9	390.2	27.1	358.0	23.1
1100	0.01804	467.7	31.3	516.2	36.5	470.7	31.6
—	0.01522*	394.6*	25.6*	403.9*	30.5*	397.2*	25.8*

Table 3. Deformation calculation parameter values of TiAl alloy at 800~1100℃

comment：* stood for statistical average of every column; the unit of deformation activation energy was kJ/mol

3.2 Hot compression deformation activation energy and plastic deformation flow equation of $20Ti_2AlN/TiAl$ composites

Table 4 showed the result of univariate regression about $20Ti_2AlN/TiAl$ composites. The linearly dependent coefficients were approximate and significant on 0.01 level. So three functions were all available. But the linearly dependent coefficient of power-exponent

function	deformation temperature[℃]	regression equation	correlation coefficient	significance
exponential function	800	$\ln\dot{\varepsilon}$=26.44 $\ln\sigma_p$-185.8	0.9895	*
	900	$\ln\dot{\varepsilon}$=9.734 $\ln\sigma_p$-68.71	0.9928	***
	1000	$\ln\dot{\varepsilon}$=5.555 $\ln\sigma_p$-39.12	0.9978	***
	1100	$\ln\dot{\varepsilon}$=3.108 $\ln\sigma_p$-22.27	1	***
power exponential function	800	$\ln\dot{\varepsilon}$=0.02832 σ_p-31.43	0.9925	*
	900	$\ln\dot{\varepsilon}$=0.01368 σ_p-14.7	0.9984	***
	1000	$\ln\dot{\varepsilon}$=0.01101 σ_p-10.41	0.9986	***
	1100	$\ln\dot{\varepsilon}$=0.009241 σ_p-7.848	0.9992	***
hyperbolic sine function	800	$\ln\dot{\varepsilon}$=3.59 $\ln(\sinh(\alpha\sigma_p))$-5.289	0.9997	*
	900	$\ln\dot{\varepsilon}$=19.89 $\ln(\sinh(\alpha\sigma_p))$-8.867	0.991	***
	1000	$\ln\dot{\varepsilon}$=7.413 $\ln(\sinh(\alpha\sigma_p))$-6.025	0.9956	***
	1100	$\ln\dot{\varepsilon}$=4.187 $\ln(\sinh(\alpha\sigma_p))$-5.415	0.9999	***

Table 4. Regression results of strain rate ($\dot{\varepsilon}$) and peak stress (σ_p) of $20Ti_2AlN/TiAl$ composite

comment：highly significant on 0.01 level (***),significant on 0.05 level (*)

equation was slightly higher while the slope β was approximate and the effect of temperature on stress index (n) was less. So power-exponent equation at high stress level was more suitable to describe the relation between $\dot{\varepsilon}$ and σ_p of high temperature (800-1100℃) compression deformation on $20Ti_2AlN/TiAl$ composites.

The fitting line about σ_p and $1/T$ of $20Ti_2AlN/TiAl$ composites was known through equation (11). The linearly dependent coefficients were all more than 0.99 and significant on 0.01 level. As the slopes of two fitting lines which corresponded 0.001 s^{-1} and 0.01s^{-1} were approximate, strain rate had almost no effect on B_2. Table 5 showed the original calculation results. The peak deformation flow equation of $20Ti_2AlN/TiAl$ composites was:

$$\begin{cases} \dot{\varepsilon} = \exp(41.2)\exp(0.02\sigma_p) - 587.6/RT & \dot{\varepsilon} \le 0.01s^{-1} \\ \dot{\varepsilon} = \exp(28.4)\exp(0.02\sigma_p) - 533.7/RT & 0.01s^{-1} < \dot{\varepsilon} < 0.1s^{-1} \\ \dot{\varepsilon} = \exp(17.8)\exp(0.02\sigma_p) - 335.3/RT & \dot{\varepsilon} \ge 0.1s^{-1} \end{cases} \tag{17}$$

T, ℃	$\dot{\varepsilon}$ [s^{-1}]	0.001		0.01		0.1	
	B_2	3.55E+06		3.27E+06		2.01E+06	
	B	Q	lnA_2	Q	lnA_2	Q	lnA_2
800	0.02832	835.5	62.3	769.6	54.9	473.0	21.6
900	0.01368	403.6	26.7	371.7	23.4	228.5	8.74
1000	0.02877	848.7	69.8	781.8	63.5	480.5	35.0
1100	0.009241	272.6	16.0	251.1	14.2	154.4	5.68
—	0.020003	587.6*	43.7*	533.7*	39.0*	335.3*	17.8*

Table 5. Deformation parameter values of $20Ti_2AlN/TiAl$ composites at 800~1100℃

comment：* stood for statistical average of every column; the unit of deformation activation energy was kJ/mol

3.3 Hot compression deformation activation energy and plastic deformation flow equation of $50Ti_2AlN/TiAl$ composites

Table 6 showed the results of univariate regression about $50Ti_2AlN/TiAl$ composites. The linearly dependent coefficients were all low in 900℃ while they were all more than 0.97 in 800℃ which were significant on 0.05 level. The linearly dependent coefficients were nearby in 1000℃ or 1100℃ and significant on 0.01 level. So they all could be used. But the linearly dependent coefficient of power exponent equation was slightly higher while β was approximate at different temperatures and the effect of temperature on stress index (n) was less. So power-exponent equation at high stress level was more suitable to describe the relation between $\dot{\varepsilon}$ and σ_p of high temperature (800-1100℃) compression deformation on $50Ti_2AlN/TiAl$ composites.

The fitting line about σ_p and $1/T$ of $50Ti_2AlN/TiAl$ composites was received through equation (11). The linearly dependent coefficients were all more than 0.99 and significant on 0.01 level.

The slopes of two fitting lines were approximate in the range from $0.001s^{-1}$ to $0.01s^{-1}$. Strain rate had almost no effect on B_2. When strain rate increased to $0.1s^{-1}$, the slope of the fitting line was lower than that in the range from $0.001s^{-1}$ to $0.01s^{-1}$. So the plastic flow equation and deformation activation energy were related to strain rate. Table 7 was the original calculation results. The peak deformation flow equation of $50Ti_2AlN/TiAl$ composites was:

function	deformation temperature[℃]	regression equation	correlation coefficient	significance
exponential function	800	$\ln\dot{\varepsilon}=35.35\ \ln\sigma_p-245.6$	0.9783	*
	900	$\ln\dot{\varepsilon}=20.28\ \ln\sigma_p-139$	0.8605	
	1000	$\ln\dot{\varepsilon}=7.47\ \ln\sigma_p-51.41$	0.9978	***
	1100	$\ln\dot{\varepsilon}=3.491\ \ln\sigma_p-24.82$	0.9834	***
power exponential function	800	$\ln\dot{\varepsilon}=0.03924\ \sigma_p-40.47$	0.9813	*
	900	$\ln\dot{\varepsilon}=0.0276\ \sigma_p-25.5$	0.8600	
	1000	$\ln\dot{\varepsilon}=0.01418\ \sigma_p-12.31$	0.9997	***
	1100	$\ln\dot{\varepsilon}=0.01106\ \sigma_p-8.709$	0.9999	***
hyperbolic sine function	800	$\ln\dot{\varepsilon}=27.03\ \ln(\sinh(\alpha\sigma_p))-9.131$	0.9796	*
	900	$\ln\dot{\varepsilon}=15.25\ \ln(\sinh(\alpha\sigma_p))-8.207$	0.8604	
	1000	$\ln\dot{\varepsilon}=5.668\ \ln(\sinh(\alpha\sigma_p))-5.629$	0.9996	***
	1100	$\ln\dot{\varepsilon}=2.609\ \ln(\sinh(\alpha\sigma_p))-5.407$	0.995	***

Table 6. Regression results of strain rate ($\dot{\varepsilon}$) and peak stress (σ_p) of $50Ti_2AlN/TiAl$

Comment：0.01 level represented highly significant (***), 0.05 level represented significant (*)

$$\begin{cases} \dot{\varepsilon} = \exp(39.2)\exp(0.023\sigma_p) - 636.6/RT & \dot{\varepsilon} \le 0.01s^{-1} \\ \dot{\varepsilon} = \exp(24.3)\exp(0.023\sigma_p) - 537.3/RT & 0.01s^{-1} < \dot{\varepsilon} < 0.1s^{-1} \\ \dot{\varepsilon} = \exp(14.7)\exp(0.023\sigma_p) - 352.0/RT & \dot{\varepsilon} \ge 0.1s^{-1} \end{cases} \quad (18)$$

T, ℃	$\dot{\varepsilon}$, s^{-1}	0.001		0.01		0.1	
	B	3.45E+06		3.56E+06		2.32E+06	
	B	Q	$\ln A_2$	Q	$\ln A_2$	Q	$\ln A_2$
800	0.03924	1100.0	85.7	913.0	61.9	600.0	26.8
900	0.0276	766.3	55.7	642.2	40.4	422.0	17.8
1000	0.01418	381.5	26.1	329.9	18.9	216.8	8.2
1100	0.01106	292.1	19.1	257.3	13.8	169.1	6.1
—	0.02302*	636.6*	46.6*	537.3*	33.8*	352.0*	14.7*

Table 7. Deformation parameter values of $50Ti_2AlN/TiAl$ composites at 800~1100℃

comment： * stood for statistical average of every column; the unit of deformation activation energy was kJ/mol

4. Conclusions

(1) Ti_2AlN particles increased peak flow stress of TiAl matrix composites, especially significantly in high temperature. The composites were not obviously reinforced as the volume fraction of Ti_2AlN particles increased from 20% to 50%.

(2) The deformation flow equation of TiAl alloy in the temperature range from 800℃ to 1000℃ was:

$$\dot{\varepsilon} = \exp(27.3)\exp(0.01522\sigma_p) - 398.6 / RT \text{，} \quad 0.001s^{-1} \le \dot{\varepsilon} \le 0.1s^{-1}$$

(3) The deformation flow equation of $Ti_2AlN/TiAl$ composites in the temperature range from 800℃ to 1100℃ was related to strain rate. The deformation flow equation of $20Ti_2AlN/TiAl$ composites was：

$$\begin{cases} \dot{\varepsilon} = \exp(41.2)\exp(0.02\sigma_p) - 587.6 / RT & \dot{\varepsilon} \le 0.01s^{-1} \\ \dot{\varepsilon} = \exp(28.4)\exp(0.02\sigma_p) - 533.7 / RT & 0.01s^{-1} < \dot{\varepsilon} < 0.1s^{-1} \\ \dot{\varepsilon} = \exp(17.8)\exp(0.02\sigma_p) - 335.3 / RT & \dot{\varepsilon} \ge 0.1s^{-1} \end{cases}$$

The deformation flow equation of $50Ti_2AlN/TiAl$ composites was：

$$\begin{cases} \dot{\varepsilon} = \exp(39.2)\exp(0.023\sigma_p) - 636.6 / RT & \dot{\varepsilon} \le 0.01s^{-1} \\ \dot{\varepsilon} = \exp(24.3)\exp(0.023\sigma_p) - 537.3 / RT & 0.01s^{-1} < \dot{\varepsilon} < 0.1s^{-1} \\ \dot{\varepsilon} = \exp(14.7)\exp(0.023\sigma_p) - 352.0 / RT & \dot{\varepsilon} \ge 0.1s^{-1} \end{cases}$$

REFERENCES

[1] Y. M. Li. *Study on reactive synthesis and high temperature performance of Tif/TiAl3 composites*. Harbin Institute of Technology, Harbin, 2010.

[2] M. Yan, B. C. Mei, J. Q. Zhu, C. G. Tian, P. Wang. Study on preparation of Ti2AlN polycrystalline ceramics. *Journal of Wuhan University of Technology*, 29(3):20-22, 2007.

[3] M. Yan. *Synthesis, microstructures and properties of Ti2AlN and Ti2AN/TiN composite*. Wuhan University of Technology, Wuhan, 2007.

[4] J. F. Zhu, J. Q. Gao, F. Wang. Synthesis of TiA1-Ti2A1N composites by in-situ crystallization. *Trans. Nonferrous Met. Soc. China*, 16: 1928-1931, 2006

[5] B. H. Li, Y.Y. Chen, F.T. Kong. Isothermal deformation simulation and Package set casting of Ti-45Al-5Nb-0.3Y alloy. *Journal of Aeronautical Materials*, 27(3):42-46, 2007.