A BI-PHASIC APPROACH TO MODEL PROGRESSIVE MATRIX DAMAGE IN COMPOSITES: DEVELOPMENT AND APPLICATION

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Abstract
A constitutive law for progressive matrix damage in unidirectional and fabric plies of long fibre reinforced composites is presented. The material model is based on the decomposition of properties into two idealized fibre and matrix phases. A novel algorithm for the decomposition is presented and tested considering several types of composite plies. A thermodynamically consistent damage model is applied to the matrix phase to model the evolution of progressive damage in different stress states by using a single damage variable. Application to carbon reinforced composites shows that the approach can correctly capture the non-linear response and the evolution of Poisson's coefficients in angle-ply laminates made of unidirectional and fabric plies.

1 Introduction
The initiation and multiplication of different damage modes is a peculiar characteristic of inelastic processes in carbon-fibre reinforced composites. From the engineering point of view, attention can be focused on the consequences of damage onset and evolution on composite structural parts, and an important distinction can be introduced between damage processes that progressively evolve and degrade the laminate and other damage mechanisms, such as fibre failures, which represent a more immediate treat for the integrity of the composite part. Progressive damage accumulation is a characteristic of some intra-ply damage processes such as transverse matrix cracking and matrix-fibre interfacial debonding, which are known to represent the first damage mechanisms activated in laminates with multiple directions of fibre reinforcement in monotonic or cyclic loading conditions [1-5].
Numerical modelling of damage accumulation in the matrix can be performed by applying continuum damage mechanics formulations to models developed at the scale of the homogenized ply. Though complex models, based on tensorial damage variables, have been proposed [6,7], composite inelastic mechanisms can be represented by means of several different scalar damage variables, because the different damage mechanisms tend to selectively affect the stiffness properties in different directions [7]. The approaches presented in [8, 9] model the progressive damage that evolves in matrix and in matrix/resin interface by means of two scalar damage variables that degrade the transverse elastic modulus and the shear modulus of unidirectional and fabric plies.
The use of damage variables that separately degrades the elastic constants can be applied to distinguish between progressive damage accumulation in the matrix and brittle fibre failures,

but an alternative approach is represented by a bi-phasic model, which allows introducing a neat separation between matrix-dominated and fibre-dominated inelastic responses. In a bi-phasic approach the contribute to composite stiffness due to the presence of a continuous reinforcement is attributed to an idealized fibre phase, whereas a generalized matrix phase provides the remaining stiffness [10-12]. The principal objective of such bi-phasic decomposition is the possibility to model the response by using a limited number of damage variables and damage evolution laws. The approach has been typically adopted in impact analyses [11-14] but it presents some appealing aspects for a general purpose ply model.

Indeed the attribution of matrix-dominated responses to a separate material phase can be exploited to couple the effects of different stress components that originate matrix damage accumulation. Moreover, the bi-phasic standpoint could be very useful for modelling composite visco-elastic behaviour and introducing the effects of environmental conditions. In fact, the effects of loading rate, temperature and humidity absorption are known to differently affect matrix- and fibre-dominated responses. Despite these possible advantages, a weak point of the bi-phasic approach is represented by the need of a preliminary decomposition of the composite properties into the two idealized material phases.

The paper presents a bi-phasic model that can be adopted for unidirectional and fabric composite plies. A novel decomposition algorithm is presented and applied to a set of materials. A thermodynamically consistent damage model is developed for the matrix phase. A single damage parameter is used and the interaction between the stress components is inherently taken into account. Finally, the model is applied to an unidirectional and to a fabric carbon/epoxy ply. Numerical-experimental correlations show that the non-linear tensile responses in angle-ply laminates can be correctly captured by the bi-phasic approach, including the evolution of Poisson’s coefficients during the tests.

2 Bi-phasic models and decomposition procedures

The approach presented in this paper moves from the bi-phasic model introduced in [10-12] for unidirectional plies. The stress acting in the composite plies, which is an average stress in the representative volume element, is presented as the sum of the contributions due to two phases of the material, namely a fibre and a matrix phase. Hence, the bi-phasic model practically involves the decomposition of the composite stiffness tensor, C^C, into two parts, C^f and C^m:

$$\begin{aligned} \sigma &= \sigma^f(\bar{\varepsilon}) + \sigma^m(\bar{\varepsilon}) \\ \sigma &= C^C\varepsilon = \left(C^f + C^m\right)\varepsilon \end{aligned} \tag{1}$$

The bi-phasic model is developed considering that the contribution of the idealized fibre phase represents only the stress component in the reinforcement direction, σ_{11}^F, which is due to the effective stiffness of the fibres [10-12]. This is a very simplified interpretation of the role played by the fibres. The contribution due to the idealised matrix phase can be merely defined as the difference between the overall stress tensor carried by the composite and the contribution attributed to the fibre phase. Accordingly, the components of C^f and C^m are expressed as:

$$\begin{aligned} C_{ij}^f &= \begin{cases} c_i^F E_i^{F-bare}\delta_{ij} & \text{if } i \text{ is a reinforcement direction} \\ 0 & \text{otherwise} \end{cases} \\ C^m &= C^C - C^f \end{aligned} \tag{2}$$

where c_i^F is the fibre volumetric content in the *i*-th direction, E_i^{F-bare} is the bare fibre stiffness and δ_{ij} is the Kronecker symbol. Equation 2 defines a decomposition scheme that is based on a simple rule of mixture and relies on the knowledge of the bare fiber stiffness. This approach is suggested in [12] for an unidirectional ply model and could be considered adequate for the scopes of the bi-phasic model, which is not intended as a true micro-mechanical approach. Nevertheless, several problems may arise when the procedure is applied to real materials. In particular, it can be observed that a negative C_{11}^m is obtained in most cases by applying the E_i^{F-bare} values available in literature and that, in other cases, the limitations imposed to the Poisson's coefficient for the physical admissibility of the materials are violated.
A novel and more reliable method for the decomposition of composite stiffness may be derived from a different point of view. Indeed, the distinctive properties of the ply in the reinforcement direction derive from the continuity of fibers. Hence, the fiber phase could be defined as the contribution due to the continuity of reinforcement and such contribution could be expressed by means of a fibre modulus , E_i^{F-cont}:

$$C_{ij}^f = c_i^F E_i^{F-cont} \delta_{ij} \tag{3}$$

Considering that the stiffness matrices introduced in Eq. 1 are degraded in a continuum damage mechanics approach, the idealised matrix phase represents the characteristic of the composite when the idealised fibre phase is completely degraded. In the proposed interpretation of the bi-phasic approach, the degradation of C^f should not model the elimination of fibre material, but should rather be related to the interruption of fibre continuity. Hence, to find E_i^{F-cont}, attention can be focused on another remarkable effect of reinforcement continuity, namely the difference between the Poisson's coefficients when the ply is subjected to a uniaxial load along and transversely to the reinforcement direction. Generally speaking, the deformation in the fiber direction due to a stress applied transversely to the reinforcement is very low in long fiber reinforced composites. In fact, the ν_{21}^C term, for unidirectional plies, is much lower than ν_{12}^C, that is typically in the range ν_0 = 0.25÷0.35 [15]. A new decomposition procedure can be devised considering that, if fiber continuity is removed, the remaining material could be roughly considered as a composite reinforced by discontinuous inclusions and the Poisson's coefficient ν_{21}^m of such a material should be within the range that has been previously denoted by ν_0. A similar reasoning can be carried out for fabric plies in both the reinforcement directions. In a generic ply model, the Poisson's coefficients of the idealized matrix phase can be expressed as a function of the terms of the stiffness matrices:

$$\begin{aligned} \nu_{21}^m &= \frac{C_{12}^m}{C_{11}^m} = \frac{C_{12}^C}{C_{11}^C - C_{11}^f} = \frac{C_{12}^C}{C_{11}^C}\frac{C_{11}^C}{C_{11}^C - C_{11}^f} = \nu_{21}^C \frac{C_{11}^C}{C_{11}^C - C_{11}^f} \\ \nu_{12}^m &= \frac{C_{21}^m}{C_{22}^m} = \frac{C_{21}^C}{C_{22}^C - C_{22}^f} = \frac{C_{21}^C}{C_{22}^C}\frac{C_{22}^C}{C_{22}^C - C_{22}^f} = \nu_{12}^C \frac{C_{22}^C}{C_{22}^C - C_{22}^f} \end{aligned} \tag{4}$$

By applying Eq. 4, the terms of the stiffness tensor attributed to the fibre phase, C_{11}^f and C_{22}^f, can be immediately evaluated once the Poisson's coefficient of the material are set to a given value ν_0. Then, the properties of the idealized matrix phase can be evaluated:

$$C_{11}^{f}=c_{1}^{F}E_{1}^{F-cont}=C_{11}^{C}-\frac{\nu_{21}^{C}}{\nu_{0}}C_{11}^{C}$$
$$C_{22}^{f}=c_{2}^{F}E_{2}^{F-cont}=C_{22}^{C}-\frac{\nu_{12}^{C}}{\nu_{0}}C_{22}^{C} \quad (5)$$
$$C^{m}=C^{C}-C^{f}$$

Equation 5 defines a new decomposition algorithm that is valid for unidirectional ($c_2^F=0$) as well as for fabric plies ($c_1^F\neq 0;c_2^F\neq 0$). The engineering constants of the material equivalent to the idealized matrix phase, $E_{11}^m,E_{22}^m,G_{12}^m$, can be computed from the terms of C^m matrix. Table 1 summarizes the application of the procedure to a set of unidirectional and fabric plies, whose properties are reported in [15]. The results obtained by assuming ν_0 =0.25 are reported. It can be observed that the values E_i^{F-cont} turn out to be lower, with a single exception, but quite close to the values corresponding to the bare fibre moduli. Moreover, the obtained idealised matrix material is always a physically admissible material with stiffness properties similar to a plastic with a discontinuous reinforcement phase. The application of the decomposition algorithm with ν_0 = 0.35 leads to similar considerations.

Material	c_{11}^F, c_{22}^F	E_{11}^C (GPa)	E_{22}^C (GPa)	ν_{12}^C (-)	E^{F-cont} (Gpa)	$\frac{E^{F-cont}}{E^{F-bare}}$	E_{11}^m (GPa)	E_{22}^m (GPa)
T-500 12k967	0.6,0.0	151	9.0	0.30	235.0	0.896	10.00	8.33
T-300 15k 976	0.6,0.0	135	9.2	0.2	207.1	0.910	10.89	8.56
HITEX33-6k E7K8	0.6,0.0	125	8.6	0.31	192.6	0.846	9.92	8.00
AS4-12K E7K8	0.6,0.0	133	8.5	0.32	205.0	0.875	10.05	7.85
Ceilon-12k E7K8	0.6,0.0	137	8.3	0.29	214.1	0.913	9.42	8.24
AS4-12K 938	0.6,0.0	154	8.9	0.30	240.9	1.027	9.92	8.27
Ceilon-12k 938	0.6,0.0	136	9.3	0.32	208.1	0.887	10.94	8.63
AS4-12K 3502	0.6,0.0	133	9.3	0.30	207.9	0.886	10.40	8.66
IM-6 12k APC-2	0.60,0.0	149	8.8	0.34	229.7	0.833	11.12	8.12
Ceilon 3000 E7K8	0.28,0.28	67	66	0.06	179.1	0.759	14.29	14.29
HITEX33-6k E7K8	0.28,0.28	60	60	0.05	171.1	0.752	10.31	10.31
AS4 6k PR500	0.28,0.28	66	66	0.05	185.1	0.790	12.36	12.36

Table 1. Application of the new decomposition algorithm.

3 Thermodynamically consistent damage law for the idealised matrix phase

In the suggested bi-phasic approach, the effect of fibre failure can be modelled by degrading the contribution due the reinforcement continuity, whereas other types of damages can be represented by properly formulating the constitutive response of the matrix phase.

Following a classical approach in continuum damage mechanics, a thermodynamically consistent damage law can be formulated by considering the Gibbs potential of the matrix phase, which can be expressed by using the compliance matrix of the idealized phase:

$$\phi^m(\underline{\sigma},\underline{d}) = \frac{1}{2}\begin{Bmatrix}\sigma_{11}^m\\ \sigma_{22}^m\\ \sigma_{12}^m\end{Bmatrix}^T \begin{bmatrix} \dfrac{S_{11}^m}{(1-d_{11})} & S_{12} & 0\\ S_{21} & \dfrac{S_{22}^m}{(1-d_{22})} & 0\\ 0 & 0 & \dfrac{S_{33}^m}{(1-d_{33})}\end{bmatrix}\begin{Bmatrix}\sigma_{11}^m\\ \sigma_{22}^m\\ \sigma_{12}^m\end{Bmatrix} \tag{6}$$

In this work only the diagonal terms of compliance are damaged; such a choice leads to apply damage only to the engineering moduli $E_{11}^m, E_{22}^m, G_{12}^m$. In this general formulation, the expressions of the variables associated to damage process are:

$$\begin{aligned} \overline{Y}_{ii} &= -\frac{\partial\phi^m}{\partial d_{ii}^m} = \frac{1}{2}\left(\frac{1}{1-d_{ii}^m}\right)^2 S_{ii}\sigma_{ii}^{m2} \quad i=1,2\\ \overline{Y}_{33} &= -\frac{\partial\phi^m}{\partial d_{33}^m} = \frac{1}{2}\left(\frac{1}{1-d_{33}^m}\right)^2 S_{33}\sigma_{12}^{m2} \end{aligned} \tag{7}$$

In a thermodynamically consistent formulation, damage must be a function of these variables, which are known as thermodynamic forces. One of the main objective of the bi-phasic approach is the limitation of damage variables. Accordingly, a single scalar damage is used for the matrix and the evolution law of the damage variables included in Eq. 7 is identical:

$$d^m = d_{11}^m = d_{22}^m = d_{33}^m = d^m\left(\overline{Y}_{11}, \overline{Y}_{22}, \overline{Y}_{33})\right) \tag{8}$$

Though a single damage variable is used, the orthotropy of the composite material is taken into account in the definition of damage evolution. In the proposed model damage evolves according to the values of a function shaped as a classic Tsai-Wu's criterion in effective stresses space:

$$\begin{aligned} & f(\tilde{\sigma}_{11}, \tilde{\sigma}_{22}, \overline{\sigma}_{12}) = F_{11}\tilde{\sigma}_{11}^{m2} + F_{22}\tilde{\sigma}_{22}^{m2} + F_{33}\tilde{\sigma}_{12}^{m2} + F_1\tilde{\sigma}_{11}^m + F_2\tilde{\sigma}_{22}^m\\ & \tilde{\sigma}_{11}^m = \frac{\sigma_{11}^m}{1-d^m};\ \tilde{\sigma}_{22}^m = \frac{\sigma_{22}^m}{1-d^m};\ \tilde{\sigma}_{12}^m = \frac{\sigma_{12}^m}{1-d^m} \end{aligned} \tag{9}$$

By exploiting Eq. 7, f is expressed as a function of the thermodynamic forces and the matrix damage evolves as a function of *f:*

$$\begin{aligned} & f(\overline{Y}_{11}, \overline{Y}_{22}, \overline{Y}_{33}) = F_{11}\frac{2\overline{Y}_{11}}{S_{11}} + F_{22}\frac{2\overline{Y}_{22}}{S_{22}} + F_{33}\frac{2\overline{Y}_{33}}{S_{33}} + F_1\sqrt{\frac{2\overline{Y}_{11}}{S_{11}}} + F_2\sqrt{\frac{2\overline{Y}_{22}}{S_{22}}}\\ & d^m = d^m(g(f(\overline{Y}_{11}, \overline{Y}_{22}, \overline{Y}_{33}))) \end{aligned} \tag{10}$$

It can be observed that the proposed approach inherently models the coupling between the different stress components acting in the idealized matrix phase. The evolution law is chosen in order to represent a progressive saturation of damage after an initial threshold, determined by the value $f=1$. Moreover, a critical damage state D_u is defined at $f=f_u$. Beyond such level the evolution law can be shaped to obtain a strain-softening response in the matrix phase:

$$d^m = \int_t \dot{d}^m dt = \begin{cases} 0, \ f(Y) < 1 \\ c_1 + a_1 f(Y) + \dfrac{b_1}{f(Y)}, 1 \le f(Y) < f_1 \\ c_2 + a_2 f(Y) + \dfrac{b_2}{f(Y)}, \ f_1 \le f(Y) < f_u \\ 1 - (1 - D_u)\left(\dfrac{f_u}{f(Y)}\right)^p, \ f(Y) > f_u \end{cases} \tag{11}$$

By properly defining the F_{ij} and F_i coefficients in the Tsai-Wu function, damage onset and evolution are modelled depending on the stress state acting in the matrix. Moreover, the proposed damage law takes into account that damage differently affects the stiffness in compressive stress states, due the natural effect of closure of micro-cracks [16]. Accordingly, a crack closure parameter *h*, with $0 < h < 1$, is introduced. When axial stress components σ_{11}^m or σ_{22}^m are compressive, the effective damage used in the constitutive response is $h \cdot d^m$.

The bi-phasic constitutive law can be completed by a brittle damage law, which is attributed to the idealized fibre phase. The law has been implemented in the Abaqus/Explicit code and applied to a set of available results referred to three types of material.

4 Application to progressive damage in three different types of composite

The presented approach has been applied to laminates made of unidirectional and fabric plies reinforced by T800 fibres with an X01 Hexcel epoxy resin. A series of tensile tests have been performed on $[\pm\vartheta]_S$ specimens with ϑ=0, 7.5°, 20°, 30°, 35°, 40° ,45° and 90° (the last one only for UD material). Three tests have been performed for each type of specimen. The parameters of the material model have been found by applying the decomposition algorithm formalized in Eq. 5, with ν_0 =0.25. Then the damage model has been calibrated by using the results of $[\pm 30°]_S$ and $[\pm 45°]_S$ tests. For the unidirectional material the linearity limit for $[90°]_S$ in compression has been assumed considering data referred to similar materials. Such limit has been used in the calibration process. The crack closure parameter *h* has been set to 0.25 for both the types of plies.

The material model has been assessed by considering single laminated shell elements and FE models of the specimens, as the one shown in Fig. 1-A, which is referred to the contour of matrix damage in the model of a $[\pm 45°]_S$ unidirectional specimen at failure. Figure 1-B and 1-C report the numerical-experimental correlation for the unidirectional and fabric materials, respectively. Results show that the proposed model can acceptably capture the onset and the evolution of non-linearities induced by progressive accumulation of damage in the matrix, though the matrix stress states in the considered cases are considerably different. It should be observed that in most of the experimental specimens failure has been influenced by edge effects. Consequently, the correlation with the stress levels at failure is not meaningful, though it is possible that a separate criterion should be introduced for the identification of critical damage levels and of the stress levels in failures due to matrix damage.

The evolution of Poisson’s coefficients in the experimental tests and in the numerical models is correlated in Figure 2. In the numerical model, the progressive degradation of the stiffness in the idealised matrix phase involves a variation of the Poisson’s coefficient of the composite ply. The curves reported in Fig. 2 point out that the resulting trends of the Poisson’s coefficients are in appreciable agreement with the experimental results.

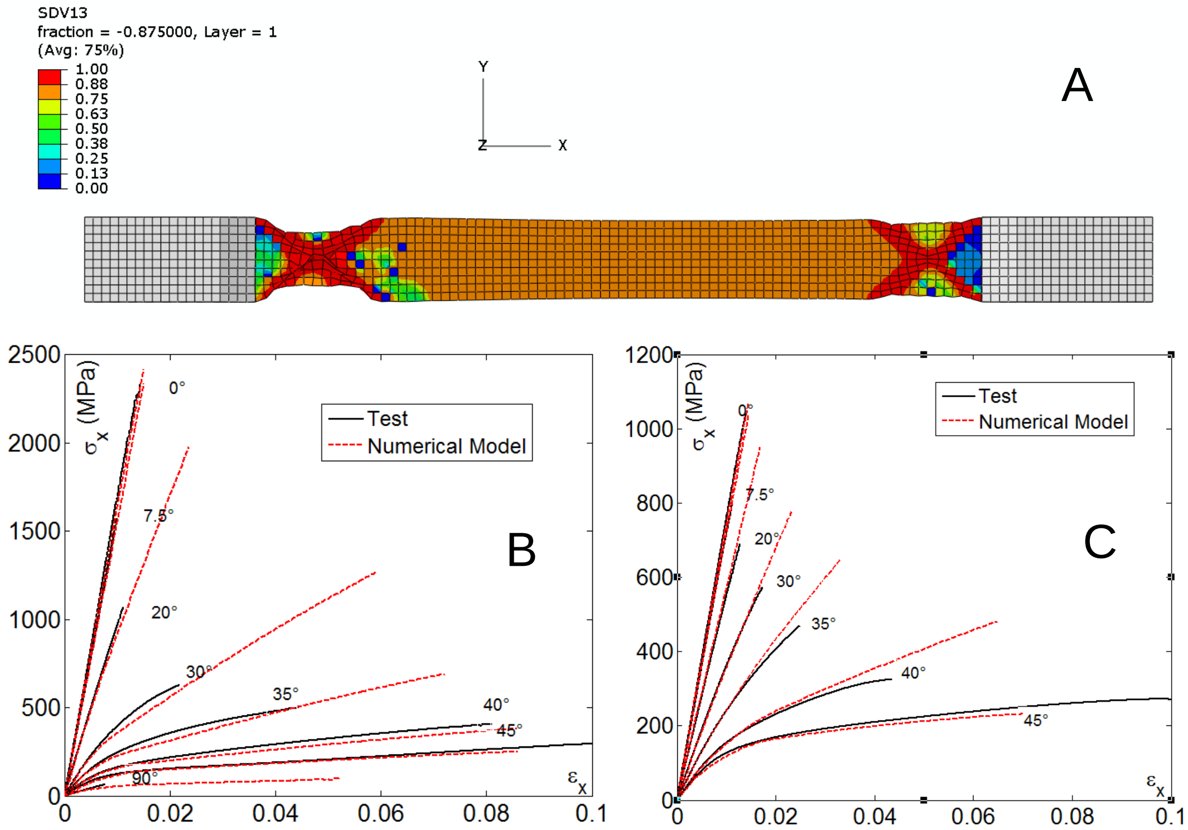

Figure 1. FE model of a [±45°]$_S$ unidirectional specimen (A), numerical-experimental correlation for unidirectional (B) and fabric (C) angle-ply tensile specimens [±ϑ]$_S$.

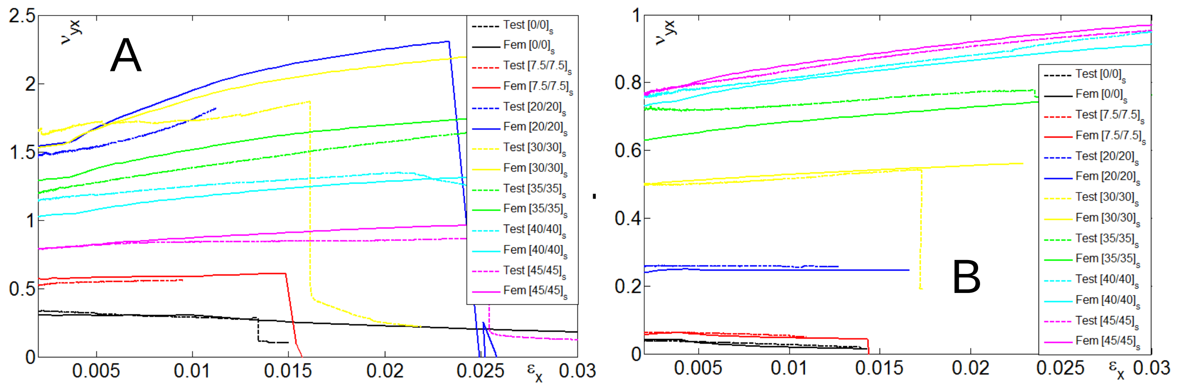

Figure 2. Numerical-experimental correlation for the evolution of Poisson's coefficients in unidirectional (A) and fabric (B) angle-ply tensile specimens [±ϑ]$_S$.

5 Concluding remarks

A bi-phasic approach has been developed and applied for modelling the accumulation of intra-ply damage in the matrix phase of multi-directional composite laminates. A novel algorithm for the decomposition of composite properties into two idealized phases has been devised. The decomposition procedure is based on the identification of the effects of reinforcement continuity and is applicable to both unidirectional and fabric plies. A thermodynamically consistent damage law has been developed for the progressive damage of the matrix. The law uses a single scalar damage parameter that evolves taking into account the mutual interaction of stress components and the different effects of damage in tension and compression. The application of the approach to a set of angle-ply laminates made of carbon reinforced unidirectional and fabric plies shows that the damage law can succesfully predict the onset and the evolution of the non-linearities that can be attributed to matrix damage accumulation. Moreover, the variation of the Poisson's coefficients during the tests, which is influenced by the progressive damage of the matrix in the plies, is correctly predicted. The separation between the constitutive laws attributed to fibre- and matrix-dominate properties can be effectively exploited for the introduction of the effects of loading rate, temperature and other enviromental conditions in the composite ply model.

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