HYSTERETIC BEHAVIOUR OF FIBRE-REINFORCED COMPOSITES

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Abstract
This papers deals with the development of a fractional model for a carbon fibre reinforced composite which includes both in-ply damage and hysteresis under shear loading. Continuum damage mechanics is used to describe in-ply failure behaviour and damage and irreversible strain evolution equations are derived to finally obtain material parameters from experimental data. A fractional model is used to deal with the hysteretic behaviour of the material under in-plane shear stress. The efficiency of this model has been analysed by comparison with experimental data. It has been shown that the model is able to reproduce the damage and hysteretic behaviour of the material with the use of only a few parameters.

1 Introduction
Nowadays composite materials are essential in applications in which significant stiffness combined with lightweight are needed. Knowledge of their behaviour until rupture is necessary in order to optimise the structures made of these materials.
Carbon fibre reinforced composite materials have been studied for many years. Their linear orthotropic elastic behaviour is well known and important work has been done on the understanding and modelling of damage by analysis at the mesoscale of the elementary ply [1-3]. Interest is centred here on the hysteretic behaviour of these materials together with its coupling with damage mechanics.
Fractional calculus has successfully been used to model hysteretic behaviour shown by the variation in electrical resistances with strain in conductive polymers [4] and polymer viscoelastic behaviour [5].

The main advantage of fractional models is the need of a very few number of parameters for the material characterisation and their ability to deal with phenomena which are history-dependent, such as hysteretic phenomena.

The aim of this analysis is to develop a model which includes both damage phenomena and inelastic strain, together with hysteretic behaviour, restricted to a quasi-static shear stress case.

2 Fibre reinforced composites model

The model is written at the mesoscale of the layer which establishes a good compromise between the scales of the constitutive materials and the structure. A plane-stress state is assumed and only small strains are taken into account. In what follows, subscripts 1 and 2 stand for the fibre direction and the transverse direction, respectively.

2.1 Constitutive laws

The model is applied to a fibre reinforced composite ply, which is modelled as a homogeneous orthotropic elastic or viscoelastic damaging material whose properties are degraded after loading mainly by matrix microcracking. Continuum damage mechanics theory is used to describe ply degradation by the use of internal damage parameters [1].

In order to study the behaviour of the material, shear tests with discharge were performed on unidirectional carbon fibre epoxy-matrix laminates with a $[0,90]_s$ stack sequence. In order to maintain the orientation, perpendicularly to carbon fibres, glass fibres were included in each ply. The inelastic strains and the loading-unloading hysteretic behaviour observed (fig. 1) may be due to the slide with friction which takes place between fibre and matrix as a result of the damage.

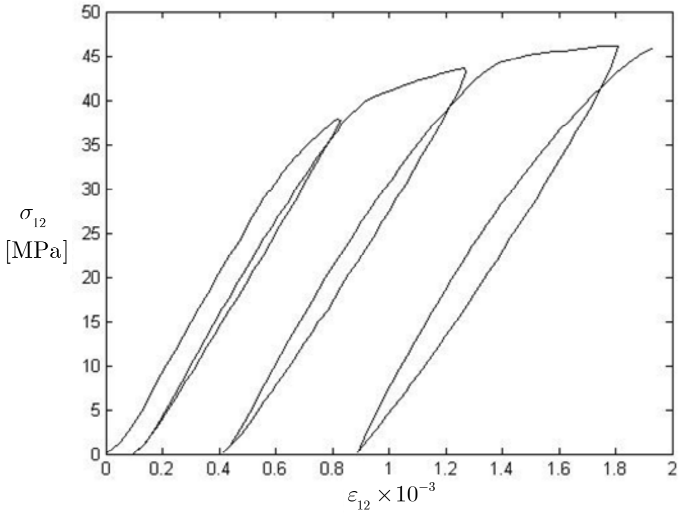

Figure 1. Stress-strain relation in shear

To deal with the reversible and irreversible effects, the following split of total strain ε_{12} is assumed:

$$\varepsilon_{12} = \varepsilon_{12}^{\mathrm{e}} + \varepsilon_{12}^{\mathrm{i}} \tag{1}$$

Where $\varepsilon_{12}^{\mathrm{e}}$ stands for the elastic shear strain and $\varepsilon_{12}^{\mathrm{i}}$ stands for the irreversible shear strain.
Taking into account Helmholtz's free energy ψ, under quasi-static conditions and only considering shear loading conditions, the following constitutive law for in-plane stress σ_{12} is obtained [1,2]:

$$\sigma_{12} = 2G_{12}^{0}\left(1 - d_{12}\right)\varepsilon_{12}^{\mathrm{e}} \tag{2}$$

where G_{12}^0 and d_{12} stand for the undamaged elastic shear modulus and damage variable associated with stiffness loss in shear due to matrix microcracking. In the general damage mechanics formulation [1], the thermodynamic force Y_{12}, associated to damage in shear, monitors damage development and is defined by:

$$Y_{12} = 2G_{12}^0(\varepsilon_{12}^{\mathrm{e}})^2 \tag{3}$$

In order to model the hysteretic behaviour observed in experimental tests (fig. 1), a fractional model is chosen. Taking into account the clockwise hysteresis loops observed in fig. 2, a fractional model based on fractional derivatives is used [4], leading to the following modified fractional constitutive law:

$$\sigma_{12}(t) = 2G_{12}^0\left(1 - d_{12}\right)\varepsilon_{12}^{\mathrm{e}}(t) + \sum_{k=1}^{N} 2G_k \mathrm{D}^{\alpha_k}\varepsilon_{12}^{\mathrm{e}}(t) \tag{4}$$

where G_k are material parameters and D^{α_k} is the a fractional derivative of order α_k, $0 \le \alpha_k \le 1$ and $k \in [1,\ldots,N]$. G_k and α_k are functions of damage d_{12} and irreversible strain $\varepsilon_{12}^{\mathrm{i}}$ levels.

Fractional calculus allows defining the derivative and integral of generalised order. The fractional derivative of a function $f(t)$ can be defined as [6, 7]:

$$\mathrm{D}^{\alpha} f(t) = \frac{\mathrm{d}^{\alpha} f(t)}{\mathrm{d}t^{\alpha}} = \frac{1}{\Gamma(1-\alpha)}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t (t-y)^{-\alpha} f(y)\,\mathrm{d}y \tag{5}$$

where Γ is the gamma function [7].

Depending on the definition used and on the order of derivatives, fractional derivatives and integrals at any time t_n can be calculated numerically by using several numerical algorithms [6,7]. Using the L1-algorithm, the expression obtained is:

$$\mathrm{D}^{\alpha} f(t) = \frac{(\Delta t)^{\alpha}}{\Gamma(2-\alpha)}\left[\left(\frac{1-\alpha}{n^{\alpha}} f_0 + \sum_{j=0}^{n-1}\left((j+1)^{1-\alpha} - j^{1-\alpha}\right)\left(f_{m-j} - f_{m-j-1}\right)\right]\right. \tag{6}$$

2.2 Evolution laws

The damage parameter d_{12} is defined by damage evolution function f_{12}. After experimental results on unidirectional fibre reinforced composites [1], assuming that damage during unloading remains constant until further positive loading is applied and causes further damage accumulation, the parameter $\bar{Y}_{12}$, based on the maximum value reached by the thermodynamic damage force along the previous loading history, is defined to describe the damage development. Thus:

$$\bar{Y}_{12} = \max_{\tau \le t}\left(\sqrt{Y_{12}(\tau)}\right) \tag{7}$$

A linear form for was found to be a good approach for shear behaviour at lower strains, thus the following damage evolution law is as follows:

$$d_{12} = f_{12}(\bar{Y}_{12}) = \frac{\left\langle \bar{Y}_{12} - \sqrt{Y_{12}^{0}} \right\rangle_{+}}{m_{12}} \text{ if } d_{12} < 1 \text{ and } \bar{Y}_{12} < Y_{12}^{c} \tag{8}$$

where $\langle \ \rangle_{+}$ is the Macaulay bracket, Y_{12}^{0} is the damage initiation thermodynamic force and m_{12} is the inverse of the slope of the curve. A non-local formulation for $\bar{Y}_{12}$ could be chosen in order to avoid theoretical or numerical drawbacks.

3 Parameters identification of the model

The identification procedure consists in determining, by means of experimental data, the values of the parameters appearing in the constitutive relations of the model.

3.1 Elastic properties, damage and irreversible strain

In this stage, initial elastic shear modulus G_{12}^{0} and actual elastic shear modulus in loop M G_{12}^{M} can been obtained from experimental data (fig. 1). This enables to define damage parameters d_{12}^{M}. From the same experimental data, the different $\varepsilon_{12}^{\mathrm{i}\ M}$ can also be obtained.

3.2 Damage evolution law

Under shear loading conditions, and having identified damage parameters, the shear damage evolution master curve can be determined (fig. 2). For the range studied, a linear form has been obtained.

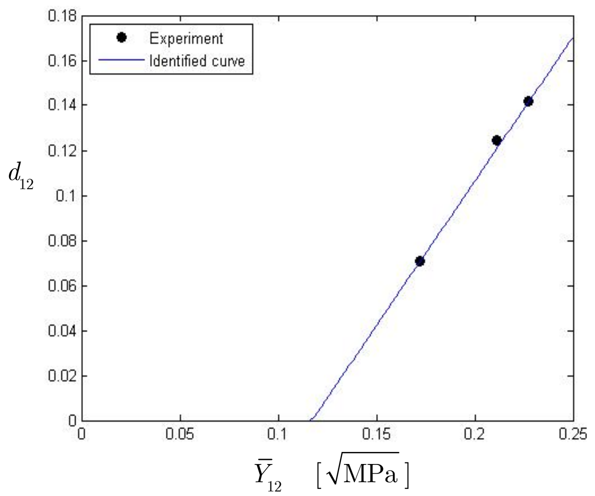

Figure 2. Shear damage evolution master curve

3.3 Irreversible strain evolution law

The irreversible strains can be obtained from experimental data (fig. 3). Their values correspond to those for which shear stress is zero. The irreversible strain master curve is presented in fig. 3. It can be observed that the evolution is no longer linear. A quadratic form seems to give good agreement with experimental data and permits to estimate the irreversible strain threshold Y_{12}^{i}.

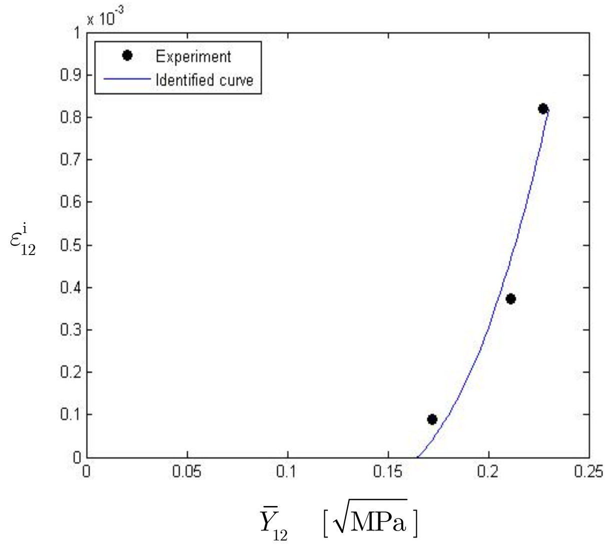

Figure 3. Irreversible strain evolution master curve

3.4 Fractional model parameters

In order to obtain the values for the parameters of the fractional model, an optimisation problem has been formulated. The basic idea consists in minimising the error function e, which can be calculated adding all the individual data point errors in the interval considered. The error function e^m at time t_m has been defined as:

$$e(t_m) = e^m = (\bar{\sigma}_{12}^m - \sigma_{12}^m)^2 \tag{9}$$

where $\bar{\sigma}_{12}^m$ is the value of the shear stress at $t = t_m$ obtained from experimental tests and σ_{12}^m is the shear stress at $t = t_m$ obtained by the mathematical model.

The error e considering all the data points is the interval $[t_0, t_n] = [0, t_n]$ is the defined as follows:

$$e = \sum_{m=0}^{n} e^m = \sum_{m=0}^{n} (\bar{\sigma}_{12}^m - \sigma_{12}^m)^2 \tag{10}$$

The unknown model parameters can be then obtained by minimising the error.

To do so, based on (4), the following behaviour law is proposed:

$$\sigma_{12}(t) = 2G_0\varepsilon^{\mathrm{e}}_{12}(t) + \sum_{k=1}^{N} 2G_k \mathrm{D}^{\alpha_k}\varepsilon^{\mathrm{e}}_{12}(t) \tag{11}$$

This will enable to validate the optimisation procedure by comparing the term $G^0_{12}\left(1-d_{12}\right)$ in (4) with the term G_0 in (11). Thus, after the optimisation procedure, the material parameters G_0 and G_k as well as the fractional derivative order α_k will be obtained.

4 Results

From fig. 1, $G^0_{12} = 27.92\,\mathrm{GPa}$ has been obtained for the initial shear modulus. From the damage evolution master curve (fig. 2), the damage initiation thermodynamic force $Y^0_{12} = 1.37\times10^{-2}\,\mathrm{MPa}$ and the inverse of the slope $m_{12} = 0.774\,(\mathrm{MPa})^{1/2}$ have been obtained. From the shear irreversible strain evolution master curve (fig. 3) the irreversible strain threshold thermodynamic force $Y^{\mathrm{i}}_{12} = 2.68\times10^{-2}\,\mathrm{MPa}$ has been obtained. In this case a quadratic expression has been retained for its evolution.

For the fractional model, in this preliminary work only one derivative order has been retained, that is, $N = 1$. Therefore, after the minimisation procedure only G_0, G_1 and α_1 have been taken into account. Nevertheless, in order to analyse their evolution, their respective values for the M-th loop, G^M_0, G^M_1 and α^M_1, have been calculated.

After error minimisation, the values of the parameters have been obtained. The comparison between the experimental results and the ones provided by the model are shown in fig. 4

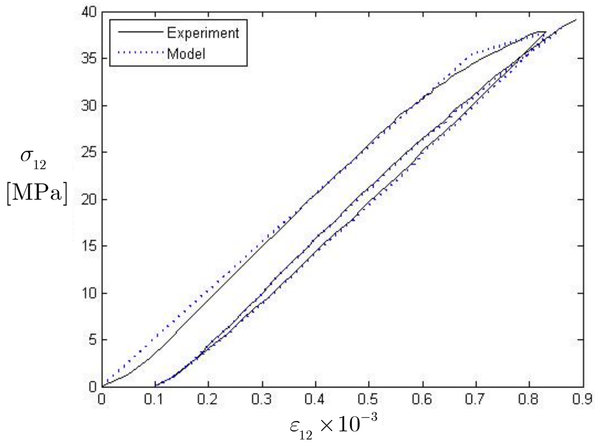

Figure 4a. Stress-strain comparison for the first loop

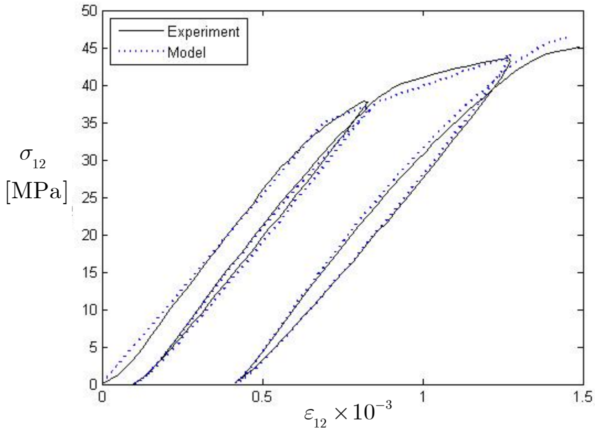

Figure 4b. Stress-strain comparison for the second loop

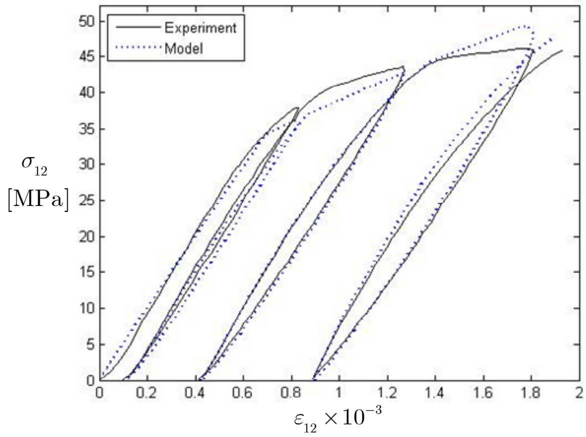

Figure 4c. Stress-strain comparison for the third loop

Table 1 shows the unknown parameters for the model obtained from error minimisation.

Loop	G_0 [GPa]	G_1 [GPa]	α_1
1	51060	136	0.093
2	49737	1136	0.138
3	47360	2965	0.162

Table 1. Parameters of the fractional model

After comparing the values for G_0 and G_{12}^0, a good correlation has been found, as the values obtained for the damage parameters are consistent with the ones obtained experimentally.

In order to identify the relation between the rest of the fractional model parameters and damage, their evolutions have been plotted (fig. 5). It can be observed (fig. 5a) that, in the damage range studied, the fractional derivative order depends linearly on the damage parameter.

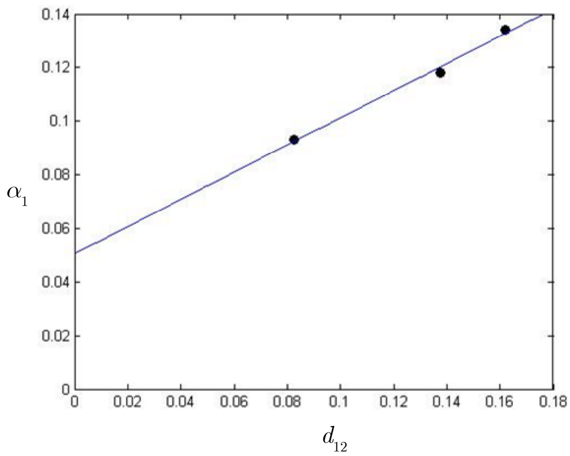

Figure 5a. α_1 evolution with damage

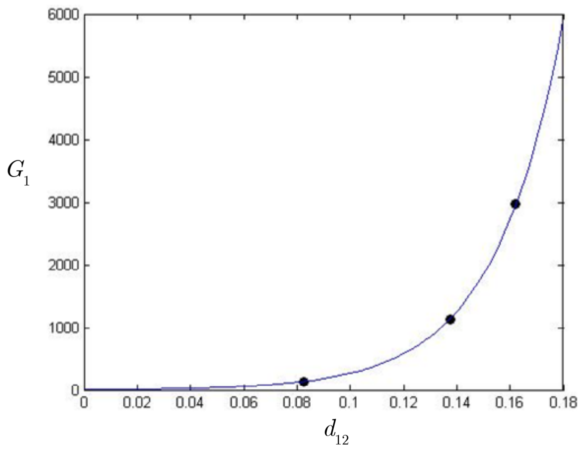

Figure 5b. G_1 evolution with damage

As far as G_1 is concerned, there seem to be an exponential evolution (fig. 5b). Nevertheless, more experimental data is needed to confirm this statement.

From fig. 4, the model's ability to reproduce the clockwise hysteresis loops is shown. However, for the loading stage after each of the loops, the model gives higher loading slopes, especially in the case presented in fig. 4c, than the ones obtained from experimental data. Ongoing work is concerned with improving this behaviour.

4 Conclusion

A model based on fractional calculus has been developed for a carbon fibre reinforced composite which includes both intra-ply damage and hysteresis under shear loadings. The construction of the model is based on the thermodynamics framework of irreversible processes, continuum damage mechanics and the use of fractional calculus in order to reproduce the hysteretic behaviour. An important feature of the model is that only a very few number of parameters are needed for its characterisation.

It has been shown that the model is able to reproduce the damage and hysteretic behaviour of the material. Further work is in progress concerning model and its implementation in finite element codes.

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