A DIRECT APPROACH FOR THE EXPLICIT DETERMINATION OF THE IN-SITU MODULUS OF MATRICES IN FIBRE REINFORCED PLASTICS

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Abstract
An analytical formulation is developed for the prediction of the in-situ modulus of polymer matrices in glass-fibre polymer matrix composites. An explicit formulation is derived to calculate the Young's modulus of the matrix resulting from the transverse strain-blocking effect induced by the embedded fibres. For the prediction of the in-situ modulus a phenomenologically based mixing rule for the Young's moduli parallel to the fibre direction as well as perpendicular to the fibre direction is utilised. This results in a better correlation between measured and calculated moduli of the composites. Further the in-situ modulus of the matrix can be utilized within mixing rules for strength properties to receive a more realistic analytical prediction for matrix dominated strength properties of unidirectional laminae. Compared to established methods for the determination of the in-situ modulus of the matrix deviations of up to 15 % can be found for glass-fibre reinforced polymers.

1 Introduction
Throughout material testing of fibre reinforced plastics (FRP) a significant scatter of the fibre volume content (FVC) can be observed. On one hand this results in a significant scatter of the mechanical properties to result from the tests. On the other hand the direct comparison of results from different testing campaigns or between FRP structures and coupon testing results are hindered.
Thus it is necessary to convert modulus and strength values of FRP testing results over the FVC. To enable the conversion of those highly FVC-dependent properties, reliable mixing rules such as from HASHIN and ROSEN as published in [1] are available. As well for the conversion of strength in fibre direction over FVC theories can be found [4]. The matrix dominated strength properties, such as intra laminar shear strength and fibre perpendicular direction tensional strength, are much more difficult to calculate based on mixing rules. Thus a conversion over FVC is currently not based on reliable theories.
The matrix dominated strength properties of FRP are significantly driven by the strain blocking effect of the matrix due to the much stiffer fibres. The in-situ modulus of the matrix is a good measure for this strain blocking effect. Based on this the calculation of the matrix dominated strength utilizing micro-mechanical models is possible.
Within this paper a theory will be presented which enables the FRP-designer to explicitly calculate the in-situ modulus of the matrix within FRP resulting from the strain blocking

effect induced by the much stiffer fibres incorporated in the matrix. Using analytically based mixing rules the potential of this theory will be shown in comparison with testing results.

2 Micromechanical calculation of elasticity properties

Within this section a short overview will be given on the current situation of the calculation of elasticity properties of unidirectional (UD) composites. The focus lies on the calculation based on micro-mechanical models. Though the mixing rules introduced by HASHIN and ROSEN [1] will be taken as a reference since they are widely accepted. In [8] they are proposed for the prediction of elastic properties of UD composites. The properties of a UD lamina are given in a right-handed Cartesian coordinate system were the 1-axis is parallel to the fibre direction. The 2-axis is perpendicular to the fibre direction in the lamina plane and the 3-axis is perpendicular to the lamina plane.

2.1 Elastic properties parallel to the fibre direction

The rule of mixtures for the Young's modulus in fibre direction is given by the following equation. Therein E_{FL} is the Young's modulus of the fibre in fibre direction, while E_M is the Young's modulus of the isotropic matrix and φ is the fibre volume content (FVC).

$$E_1 = E_L = E_{FL}\varphi + E_M(1-\varphi) \tag{1}$$

2.2 Elastic properties perpendicular to the fibre direction

Usually the derivation of micro-mechanically based mixing rules for elastic properties of UD composites is based on the following assumptions:

- Simple cross section of the fibre (square, rectangular or circle)
- Regular array of the fibres (square or hexagonal array)
- Two-phase composite consisting of fibre and matrix neglecting voids or interphases
- Parallel fibres
- Ideal connection between fibres and matrix

As a simple example a square array of fibres with a square cross-section is given by Beér [6]. The simplest approach is given by the so-called inverse rule of mixtures as shown within the following equation

$$\frac{1}{E_2} = \frac{1}{E_T} = \frac{\varphi}{E_{FT}} + \frac{1-\varphi}{E_M} \tag{2}$$

A more sophisticated phenomenological approach based on a cylindrical fibre model in hexagonal array (CFMh) is given by

$$E_{Th} = \frac{2E_M}{\sqrt{3}}\left[\frac{\sqrt{3}}{2} - \sqrt{2\sqrt{3}\frac{\varphi}{\pi}} - \frac{\pi}{2\left(1-\frac{E_M}{E_{FT}}\right)} + \frac{\frac{\pi}{2}+\arctan\left(\frac{\sqrt{2\sqrt{3}\frac{\varphi}{\pi}\left(1-\frac{E_M}{E_{FT}}\right)}}{\sqrt{1-2\sqrt{3}\frac{\varphi}{\pi}\left(1-\frac{E_M}{E_{FT}}\right)^2}}\right)}{\left(1-\frac{E_M}{E_{FT}}\right)\sqrt{1-2\sqrt{3}\frac{\varphi}{\pi}\left(1-\frac{E_M}{E_{FT}}\right)^2}}\right] \tag{3}$$

where φ is the FVC, E_{FT} is the Young's modulus of the fibre in the direction perpendicular to the fibre and E_{Th} is the Young's modulus of the UD composite in fibre-perpendicular direction. This mixing rule is preferred since it results in transversal isotropic behaviour.
On the left hand side of Figure 3 a comparison between the mixing rules of HASHIN and ROSEN and the mixing rule in Equation (3), shown by the dotted line, can be seen. The results of the mixing rule are supposed to lie close to the upper bound (ub) by HASHIN and ROSEN from [1]. Obviously the Young's modulus for the direction perpendicular to the fibre is estimated too low. This is due to the fact that the transverse strain-blocking of the matrix by the incorporated fibres is not taken into account.
In [2] CHAMIS and SENDECKYJ state that the differences between bulk and in-situ properties of the matrix should thoroughly be investigated.

3 In-situ modulus of the matrix

This effect has been well known up to now and therefore is incorporated in the estimation of mechanical properties of composites. The most common way will be shown within the following section.

3.1 Prediction of the in-situ modulus of the matrix based on 2D-stress-state

A well established way to take the strain-blocking effect of the matrix into account is based on the assumption of a two-dimensional state of stress [3]. Thus the in-situ modulus of the matrix is calculated by

$$E'_M = \frac{E_M}{1-\nu_M^2} \tag{4}$$

where ν_M is the Poisson's ratio of the matrix. The fact that the strain-blocking of the matrix is taken into account is indicated by an apostrophe. Within the mixing rule given in Equation (3) this results in an increase of the predicted Young's modulus as shown by the dashed line on the left side of Figure 3. Unfortunately the further result is that up to 10 % FVC the mixed Young's modulus is overestimated while at higher FVC values the resulting Young's modulus follows the lower bound of HASHIN's and ROSEN's mixing rules. Based on experience and as stated in [1] this still underestimates the measured Young's modulus of a UD composite in fibre-perpendicular direction.
Within the following section an approach will be presented that takes into account that the strain blocking effect of the matrix induced by the incorporated fibres is a 3D-effect.

3.2 Prediction of the in-situ modulus of the matrix based on 3D-stress-state

The matrix can be assumed to be restrained by an elastic material with the Young's modulus of the fibre as shown in Figure 1. It is easy to imagine that this effect is dependent on the FVC. Thus the FVC will be taken into account.

Converting Equation (1) results in

$$\frac{E_L}{1-\varphi} - E_M = \frac{\varphi}{1-\varphi} E_{FL} \tag{5}$$

The contribution of the Young's modulus of the fibre to that of the UD composite in the direction perpendicular to the fibre can be calculated by

$$E_T(\varphi) - E_M \tag{6}$$

when related to the volume content of the matrix. Herein $E_T(\varphi)$ can be any fibre perpendicular Young's modulus calculated neglecting the strain blocking effect of the matrix due to the fibres. The expressions from Equations (5) and (6) represent the elastic restraint of the matrix due to the incorporated fibres related to the volume content of the matrix. They can be combined and written in the form of a tension-compression quadrant of a 3D-stiffness-matrix.

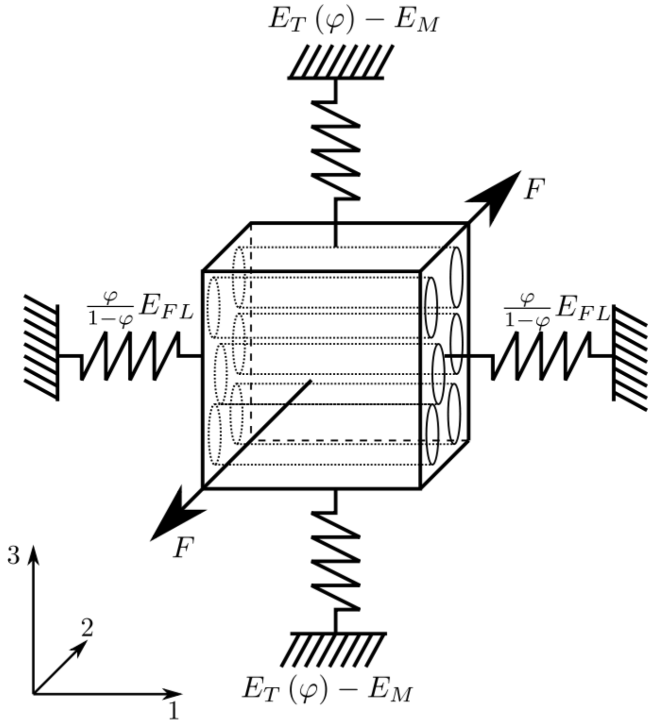

Figure 1. Matrix restrained by fibres

$$[A_{Fr}] = \begin{bmatrix} \frac{\varphi}{1-\varphi}E_{FL} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & E_T(\varphi) - E_M \end{bmatrix} \tag{7}$$

The tension-compression-quadrant of the stiffness-matrix of the polymeric matrix can be written in the form

$$[A_M] = E_M \begin{bmatrix} \frac{1-\nu_M}{1-\nu_M-2\nu_M^2} & \frac{\nu_M}{1-\nu_M-2\nu_M^2} & \frac{\nu_M}{1-\nu_M-2\nu_M^2} \\ \frac{\nu_M}{1-\nu_M-2\nu_M^2} & \frac{1-\nu_M}{1-\nu_M-2\nu_M^2} & \frac{\nu_M}{1-\nu_M-2\nu_M^2} \\ \frac{\nu_M}{1-\nu_M-2\nu_M^2} & \frac{\nu_M}{1-\nu_M-2\nu_M^2} & \frac{1-\nu_M}{1-\nu_M-2\nu_M^2} \end{bmatrix} \tag{8}$$

Superposition of the two expressions in Equations (7) and (8) results in

$$[A_{MFr}] = E_M \begin{bmatrix} \frac{\varphi E_{FL}}{(1-\varphi)E_M} + \frac{1-\nu_M}{1-\nu_M-2\nu_M^2} & \frac{\nu_M}{1-\nu_M-2\nu_M^2} & \frac{\nu_M}{1-\nu_M-2\nu_M^2} \\ \frac{\nu_M}{1-\nu_M-2\nu_M^2} & \frac{1-\nu_M}{1-\nu_M-2\nu_M^2} & \frac{\nu_M}{1-\nu_M-2\nu_M^2} \\ \frac{\nu_M}{1-\nu_M-2\nu_M^2} & \frac{\nu_M}{1-\nu_M-2\nu_M^2} & \frac{1-\nu_M}{1-\nu_M-2\nu_M^2} + \frac{E_T}{E_M} - 1 \end{bmatrix} \tag{9}$$

Inversion of the matrix expression in Equation (9) and formulating the reciprocal of the 22-entry of the matrix as well as rearranging results in

$$E'_{MT} = \frac{E_{FL}\left(\frac{E_T}{E_M}(1-\nu'^2_{MT})+\nu'^2_{MT}\right)\varphi + E_T(1-\varphi)}{\frac{E_{FL}}{E_M}\left(\frac{E_T}{E_M}(1-3\nu'^2_{MT}-2\nu'^3_{MT})+2(\nu'^2_{MT}+\nu'^3_{MT})\right)\varphi+\left(\frac{E_T}{E_M}(1-\nu'^2_{MT})+\nu'^2_{MT}\right)(1-\varphi)} \tag{10}$$

with $$\nu'_{MT} = \left(1-\frac{\sqrt{\nu_{FLT}\nu_{FTT}}E_M}{\nu_M\sqrt{E_{FL}E_{FT}}}\right)\nu_M \tag{11}$$

Incorporating an adapted Poisson's ratio ν'_{MT} for the matrix delivers a non-restrained matrix in the case of equal materials for fibre and matrix. It is remarkable that due to this approach the matrix has to be regarded as an orthotropic material. A similar approach for a restrained Young's modulus of the matrix parallel to the fibre direction results in

$$E'_{ML} = \frac{E_T(1-\nu'_{ML})+E_M\nu'_{ML}}{\frac{E_T}{E_M}(1-\nu'_{ML})+\nu'_{ML}+2\nu'^2_{ML}\left(1-\frac{E_T}{E_M}\right)} \tag{12}$$

with $$\nu'_{ML} = \left(1-\frac{\nu_{FTL}E_M}{\nu_M E_{FT}}\right)\nu_M \tag{13}$$

The influence on the Young's modulus of a UD composite perpendicular to the fibre direction can be seen on the right side in Figure 3. Including the 3D-strain-blocking in the mixing rule in equation (3) results in a prediction for the fibre-perpendicular Young's modulus that is much closer to the upper bound of the HASHIN and ROSEN formulation.

This approach can as well be used for the shear modulus since due to local 3D-stresses in the matrix an influence of the strain blocking effect is expected. Assuming that the Poisson's ratio of the matrix is constant the shear modulus can be calculated by

$$G'_{MTL} = \frac{E'_{MT}+E'_{ML}}{4(1+\nu_M)} \text{ and } G'_{MTT} = \frac{E'_{MT}}{2(1+\nu_{Meff})} \text{ with } \nu_{Meff} = \nu_M\frac{1+\nu_M-\nu_{TL}\frac{E_M}{E_L}}{1-\nu_M^2+\nu_M\nu_{TL}\frac{E_M}{E_L}} \tag{14}$$

The third equation in (14) was formulated by Foye in [7]. The in-situ Young's and shear moduli of the matrix within UD composites can be understood as a representation of the complex 3D-stress-state in the matrix resulting from the external loading of the composite.

4 Incorporation into mixing rules for matrix dominated strength

From the engineers perspective it is usually preferred to measure strength values for composites. Unfortunately it is practically impossible to produce coupons for testing with exactly the same FVC as within the composite part to be designed and manufactured. Therefore the testing results have to be scaled over the FVC. In fibre direction mixing rules for strength values of UD composites are available and show realistic behaviour. The following equation can be found in [5].

$$R_L^{(+)} = R_{FL}^{(+)}\left(\varphi+\frac{(1-\varphi)E_M}{E_{FL}}\right) \tag{16}$$

Herein $R_L^{(+)}$ is the tensional strength of the UD in fibre direction while $R_{FL}^{(+)}$ is the tensional strength of the fibre in its longitudinal direction. For matrix dominated tensional strength a mixing rule will be derived within the following section.

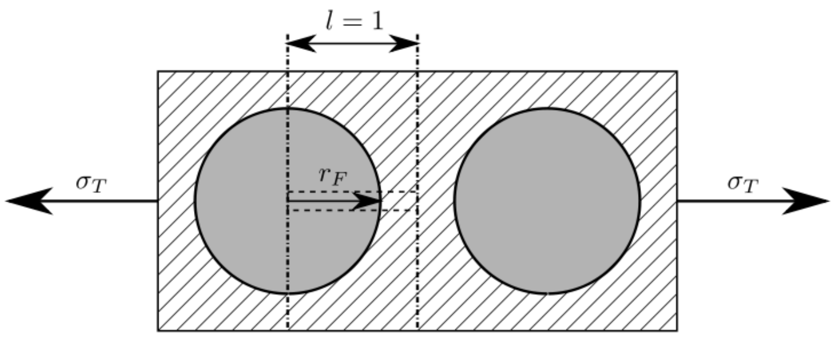

Figure 2. Cross-section of two fibres embedded in matrix

The strain of a strip consisting of fibre and matrix distributes according to the Young's moduli of the constituents.

$$\varepsilon_T = \frac{\sigma_T}{E_T} = \frac{\sigma r_F}{E_{FT}} + \frac{\sigma(1-r_F)}{EM} \text{ with } r_F = \sqrt{2\sqrt{3}\frac{\varphi}{\pi}} \text{ for hexagonal array} \tag{17}$$

Multiplying with E_T and substituting σ_T by $R_T^{(+)}$ which is the tensional strength perpendicular to the fibre direction and σ by $R_M^{(+)}$, which is the tensional strength of the isotropic matrix, results in a mixing rule for this strength.

$$R_T^{(+)} = R_M^{(+)} \left(\frac{\sqrt{2\sqrt{3}\frac{\varphi}{\pi}}}{E_{FT}} + \frac{\left(1-\sqrt{2\sqrt{3}\frac{\varphi}{\pi}}\right)}{E_M} \right) \tag{18}$$

The resulting strength values for different in-situ Young's moduli for the matrix are shown on the right hand side of Figure 3.

5 Comparison of theory with testing results

For verification the results from coupon tests were evaluated. Therefore a UD non-crimped fabric (NCF) has been vacuum infused and tested according to DIN EN ISO 527-5B [9]. Further the mechanical properties of the composite have been estimated utilising the classical laminate theory (CLT). The testing results and the prediction for the fibre perpendicular modulus as well as strength as functions of the FVC of the composite are shown in comparison within Figure 4. The NCF tested has an area-mass of 1200 gsm. The total mass of the UD-fabric divides into 97 % in 0°-direction and 3 % in 90°-direction.

For the calculation of properties the values shown in Table 1 have been used. The roving used in the fabric is a PPG HYBON® 2001/2002 [10] and [11] while the polymer matrix is the epoxy resin MOMENTIVE™ RIM135 [12]. The comparison between measured data and estimated Young's modulus as well as strength is shown within Figure 4. Besides the fact that the strength data shows a significant scattering, the correlation in terms of the average values, as well for the Young's moduli as for the strength data, appears to be very good.

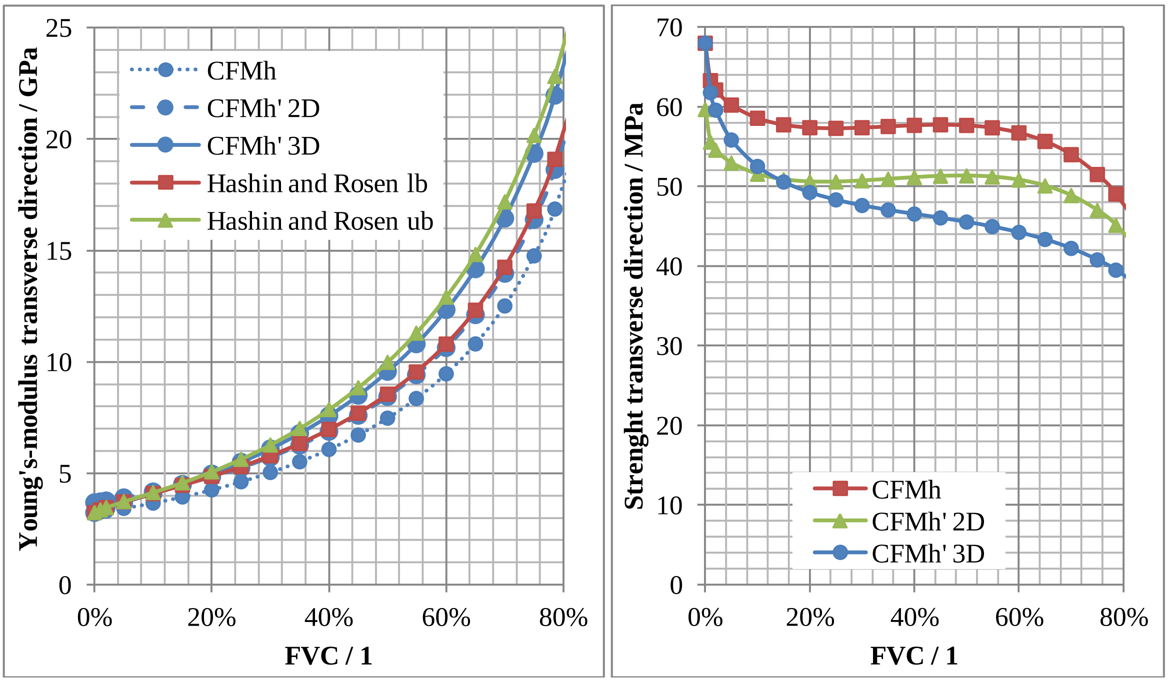

Figure 3. Comparison of mixing rules implying different in-situ Young's moduli for the matrix

Property	Carbon Fibre	Glass Fibre	Matrix
E_L in MPa	230150	81460	3240
E_T in MPa	14437	81460	3240
ν_{TL} in -	0.35	0.24	0.35
G_{TL} in MPa	65185	32913	1200
$R^{(+)}$ in MPa	4100	2290	68

Table 1. Material properties for calculation

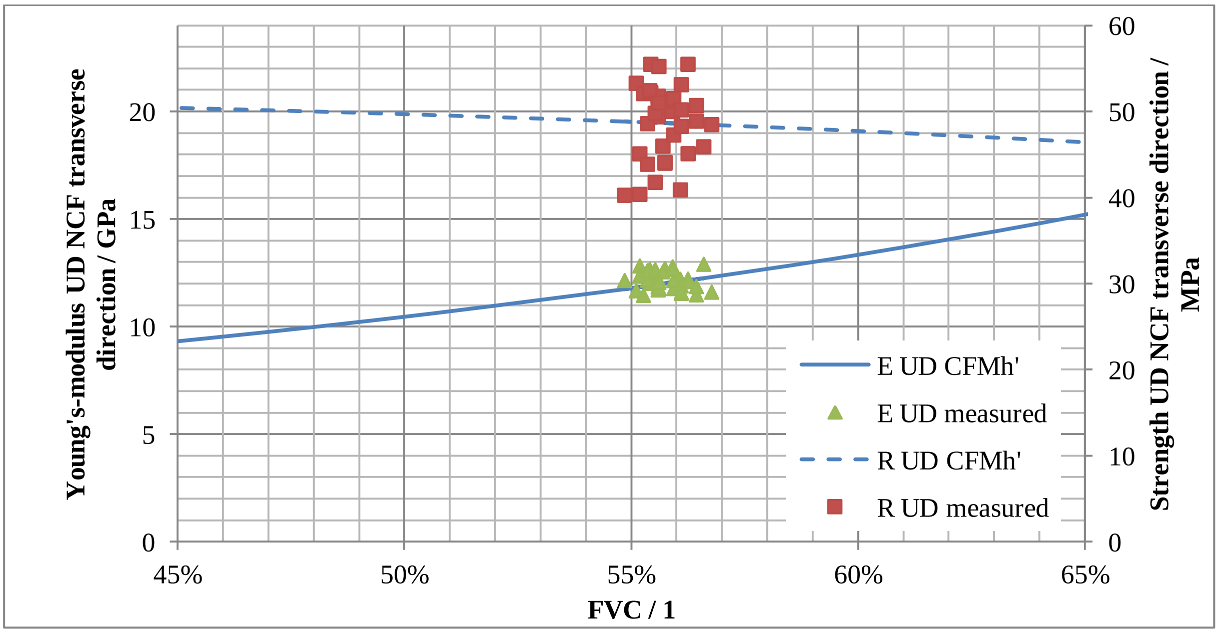

Figure 4. Comparison of testing results on UD NCF with calculated Young's modulus and strength over FVC

6 Results

Within this manuscript an approach for the explicid determination of the in-situ Young's modulus of the matrix within UD composites has been derived. For glass-fibre reinforced polymers the Young's modulus in fibre perpendicular direction incorporating the in-situ Young's modulus of the matrix has been estimated utilising a phenomenologically based mixing rule. The comparison with results based on [1] shows very good correlation with the upper bound which is assumed to be realistic. To show as well the capability of estimating mechanical properties of carbon-fibre reinforced polymers calculated values for 60 % FVC are shown within Table 2.

Mixing rule	E_T CFRP	E_T GFRP
Hashin and Rosen ub	7840 MPa	12911
Hashin and Rosen lb	7368 MPa	10804
CFMh' 3D	7847 MPa	12346

Table 2. Comparison of mixing rules for an FVC of 60 %

Further the strength of UD composites has been estimated based on micromechanics and here as well the correlation between measured data and estimated results is very good. Though the coupons only cover a range of round about 55 % to 57 % FVC. Therefore further validation is necessary for other FVC ranges. Additionally the comparison with measured data from CFRP composites has to be carried out in the future.

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