AN ENERGY BASED FAILURE CRITERION FOR MATRIX CRACK INDUCED DELAMINATION IN LAMINATED COMPOSITE STRUCTURES

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Abstract
A new failure criterion to predict matrix crack induced delamination in laminated composites is presented. The failure criterion is developed in the field of Fracture Mechanics, comparing the energy released at ply level due to the presence of a matrix crack and the fracture toughness of the interface between plies. The predicted failure loads obtained with the model are in agreement with available experimental data.

1 Introduction
The use of composite material parts has increased in the last decades. In composite structures, different damage mechanisms can promote structural failure and they must be considered in the design process. These damage mechanisms could be of two different types: interlaminar or intralaminar damage. The intralaminar damage takes place within the ply and can be as matrix cracking, fibre breakage, fibre kinking and fibre matrix debonding. Interlaminar damage takes place at the interface between two adjacent plies and can be as delamination that it is often the major failure mode in composite structures.

Delamination can be caused by different phenomena, being the three most common: induced by an external impact, caused by the interlaminar stresses that appear at the free edges or triggered by matrix cracks. Different approaches are used to predict delamination failure in composite materials. One powerful approach is using Cohesive Zone Models [1], that are able to predict delamination onset and propagation in a precise manner, but they are very expensive in time and computing capabilities. In consequence, these formulations are not affordable in many cases and faster criteria are needed. There exist two types of criteria: stress/strain based or energy based. On the stress/strain based ones, threshold values or mathematical expressions containing mainly out of plane properties are used [2]. In contrast, energy based criteria are settled on Fracture Mechanics [3,4], where the balance between the stored energy and the energy necessary to expand the crack are compared to define the initiation threshold for delamination.

In this work, an energy-based criterion for matrix crack induced delamination is proposed. This criterion can be used to locate the most critical zones for delamination failure in a
composite part and to obtain a first approach of the failure load. With that purpose, the proposed criterion has been implemented in a UVARM user post processing subroutine for ABAQUS. Finally, proposed criterion has been validated with experimental data that is available in bibliography and with finite element method using cohesive zone elements.

2 Fracture Mechanics based failure criterion for matrix cracking induced delamination

A new failure criterion to predict matrix cracking induced delamination is presented in this section. The failure criterion is obtained by comparing the elastic energy released in a laminate due to the presence of a matrix crack and the fracture toughness of the material.

The critical stress for delamination growth in a laminate containing a matrix crack and loaded in the direction perpendicular to the crack plane is developed in this section. The crack growth will undergo under pure mode II loading.

The specific elastic energy stored at a ply loaded in the direction perpendicular to the fibres and assuming plane stress conditions [4,5] is:

\[
\psi_2 = \frac{E_{22}}{2(1 - v_{12}v_{21})} \epsilon_2^2
\]  

(1)

If a matrix crack appears at this ply, the elastic energy released due to the presence of a matrix crack reads [4]:

\[
G_2 = \psi_2 \cdot \frac{1}{\kappa_2}
\]

(2)

where \(\kappa_2\) is a mechanical parameter that depends on the relative stiffness of the cracked and uncracked laminate [4], and \(l\) is a geometric factor that depends on the location of the cracked ply within the laminate:

\[
l = \begin{cases} 
  t_{\text{ply}} & \text{outer ply} \\
  \frac{t_{\text{ply}}}{2} & \text{inner ply}
\end{cases} 
\]

(3)

where \(t_{\text{ply}}\) is the ply thickness.

If the elastic energy released is equal to the interlaminar fracture toughness \((G_2 = G_{\text{ILC}})\), the matrix crack induces delamination growth at the interface with neighbouring plies.

On the other side, if a laminate containing a matrix crack parallel to the fibres of the cracked ply and loaded perpendicular to the crack plane in tension, as shown in Figure 1, induces a delamination at the interface with adjacent plies, the available strain energy release rate \(G\) reads:

\[
G = \frac{1}{2} (\epsilon_{\text{LAM}})^2 \cdot \frac{1}{a_{\text{LAM}}} \cdot \left[\frac{a_{\text{LD}}}{a_{\text{LAM}}} - 1\right]
\]

(4)
where $a_{LAM}$ is obtained from the laminate constitutive equation. For example, if the laminate is loaded as shown in Figure 1, the laminate constitutive equation reads:

$$
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{xy}
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} & a_{16} \\
a_{21} & a_{22} & a_{26} \\
a_{61} & a_{62} & a_{66}
\end{bmatrix}
\begin{bmatrix}
N_{xx} \\
0 \\
0
\end{bmatrix}
$$

(5)

and $a_{LAM} = a_{11}$. On the other side, $a_{LD}$ is the same compliance term like for $a_{LAM}$ but for a laminate without the cracked ply.

![Figure 1](image)

*Figure 1* Laminate with a matrix crack loaded in tension. The grey areas represent the unloaded areas due to the presence of the matrix crack and the delamination growth at the interface.

Therefore, comparing equations (2) and (5), and assuming strain compatibility before ply cracking, the mechanical parameter $\kappa_2$ reads:

$$
\kappa_2 = \frac{E_{22} \cdot l}{1 - \nu_{12} \cdot \nu_{21}} \left[ \frac{a_{LAM}^2}{a_{LD} - a_{LAM}} \right]
$$

(6)

Using equation (1) and (6) in (2), the ply strain $\varepsilon_{22}^{MCID}$ at which the presence of a matrix crack will induce delamination reads:

$$
\varepsilon_{22}^{MCID} = \sqrt{2G_{IIc}} \frac{a_{LAM}^2}{a_{LD} - a_{LAM}}
$$

(7)

or, alternatively, assuming plane stress conditions, the transverse ply stress $\sigma_{22}^{MCID}$ at which the presence of a matrix crack will induce delamination reads:

$$
\sigma_{22}^{MCID} = \frac{E_{22}}{1 - \nu_{12} \cdot \nu_{21}} \sqrt{2G_{IIc}} \frac{a_{LAM}^2}{a_{LD} - a_{LAM}}
$$

(8)

If the laminate is loaded under pure shear, then a matrix crack may induce delamination growth under mode III loading. The specific elastic energy under pure shear loading is:

$$
\Psi_s = \frac{1}{2} \cdot G_{12} \cdot \varepsilon_{12}^2
$$

(9)

If a matrix crack appears at this ply, the elastic energy released due to the presence of a matrix crack reads [4]:
where, following the same development presented in previous section, the mechanical parameter \( \kappa_6 \) reads:

\[
\kappa_6 = G_{12} \cdot l \left( \frac{a_{6,\text{LAM}}^2}{a_{6,\text{LD}} - a_{6,\text{LAM}}} \right) \tag{11}
\]

where \( a_{6,\text{LAM}} \) and \( a_{6,\text{LD}} \) are the term \( a_{66} \) in equation (5) of the whole laminate and the laminate without the cracked ply, respectively.

The ply shear strain \( \varepsilon_{12}^{\text{MCID}} \) at which the presence of a matrix crack will induce delamination reads:

\[
\varepsilon_{12}^{\text{MCID}} = \sqrt{2G_{11\text{c}} a_{6,\text{LAM}}^2 / a_{6,\text{LD}} - a_{6,\text{LAM}}} \tag{12}
\]

or, alternatively, the in-plane shear stress \( \sigma_{12}^{\text{MCID}} \) at which the presence of a matrix crack will induce delamination reads

\[
\sigma_{12}^{\text{MCID}} = G_{12} \sqrt{2G_{11\text{c}} a_{6,\text{LAM}}^2 / a_{6,\text{LD}} - a_{6,\text{LAM}}} \tag{13}
\]

where \( G_{11\text{c}} \) is the mode III fracture toughness of the material.

If the laminate is loaded by a combination of normal and shear load perpendicular to the crack plane, delamination growth will be induced when:

\[
G_2 + G_6 = G_c \tag{14}
\]

where \( G_c \) is the mixed-mode Fracture Toughness and depends on the pure mode II and pure mode III fracture toughness. Like there does not exist many experimental data it is assumed here that \( G_c = G_{11\text{c}} \). Then, using equations (2)-(8) and (10)-(13) in equation (15) the failure criterion under mixed-mode loading reads:

\[
\frac{(\sigma_{22}^{\text{MCID}})^2}{w_2 \cdot G_{11\text{c}}} + \frac{(\sigma_{12}^{\text{MCID}})^2}{w_6 \cdot G_{11\text{c}}} = 1 \tag{15}
\]

where:

\[
w_2 = 2 \frac{a_{6,\text{LAM}}}{a_{6,\text{LD}} - a_{6,\text{LAM}}} \left( \frac{E_{22}}{1 - \nu_{12} \cdot \nu_{21}} \right)^2 \tag{16}
\]
3 Validation of the failure criterion
The predictions made with the proposed formulation were validated with experimental data available in the bibliography [6]. Also a Cohesive Zone Model (CZM) was used to validate the proposed [1,7].

The experimental campaign performed by Crossman and Wang had the objective of determining the different failure modes of a composite laminate depending on the thickness of 90° ply. The studied lay-up was [25/-25/90_n], where n could have the values of [0.5, 1, 2, 3, 4, 6 and 8]Figure 2. The test coupons were tested under tensile load under displacement control.

![Figure 2 Composite lay-up of the validation model.](image)

The results obtained with the experimental campaign determined that two different failure mechanisms provoked the failure of the coupons. These failure modes were directly influenced by the thickness of the 90° ply. For the cases where n was lower than 3 edge delamination was the origin of the failure; for values where n was higher than 4 the matrix cracking induced delamination was the failure origin; and for n values of 3 and 4 the transition between both failure modes was found.

Additionally CZM models were computed. The CZM model was made with one layer of elements for each unitary prepeg lamina that had the specimen physically. Because of that the thickness of the model varied from three to ten layers of elements. Only one half of the laminate thickness was modelled and a symmetry boundary condition was applied in the z direction. A 3D cohesive element layer was introduced between the -25 and 90 layers; the thickness of this layer was null and the model was the one proposed by [1]. In addition, the FEM model had a precrack in the middle of the length that expanded on the whole width and in z direction on the whole thickness of the 90 lamina.

In order to have a better scope of the edge delamination phenomenon a non-uniform mesh was proposed. In the thickness of the specimen an element in height for each ply (0.125mm), in the length elements of 0.1mm were located. Finally for the width two different partitions were made. First 10 elements of 1.25mm were located along the width, because the edge delamination process in the corner, the element located in the corner element was divided in additional ten more pieces having a 0.125mm width elements.

Finally, the T300-934 material properties of the model are summarized in Table 1.
<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$E_{11}$</td>
<td>134 GPa</td>
</tr>
<tr>
<td>$E_{22}$</td>
<td>10.2 GPa</td>
</tr>
<tr>
<td>$v_{12}$</td>
<td>0.3</td>
</tr>
<tr>
<td>$G_{12}$</td>
<td>5.52 GPa</td>
</tr>
<tr>
<td>$G_{IIc}$</td>
<td>0.216 kJ/m²</td>
</tr>
</tbody>
</table>

**Table 1** T300-934 Material properties

The results obtained with the CZM determined that for $n$ values that are smaller than 3 the failure mode is edge delamination Figure 3 and for $n$ values greater than 4 matrix cracked induced delamination Figure 4. These results are in agreement with the experimental data.

**Edge delamination initiation**

**Crack delamination state when the edge is delaminating**

**Figure 3** Edge delamination in [25/-25/90°s], where it is the main initiation failure because of the thin thickness of the 90° lamina.

**Crack delamination development – no edge delamination**

**Figure 4** Matrix Crack Induced Delamination I [25/-25/90°s], where it is the main initiation failure because of the thick thickness of the 90° lamina.
In the CZM models two different failure modes were obtained for each coupon-model, edge delamination and matrix crack induced delamination. Both fracture or decohesion points were obtained in nearly all models Figure 5.

The proposed failure criterion, eq. (16) has been implemented as a UVARM subroutine, a post processing tool that is computed at each integration point in every iteration. Additionally, the results obtained by applying the formulation proposed in [3] and [4] have been computed. The four preceding models; O’Brien, Maini and coworkers, CZM which is coincident with experimental data [6] and the proposed criteria have been compared Figure 6.

The results obtained with the subroutine are in agreement with the experimental data. Considering that the model is valid for n values that are higher than 3 the highest deviation of the model is less than the 10% when compared to the experimental data. Furthermore, the difference between O’Brien and the proposed formulation is negligible (less than 0.33% of difference).
4 Conclusions
A failure criterion for matrix cracking induced delamination has been presented in this work. The model has been validated with experimental data available in the literature and numerical predictions using cohesive elements. A good agreement between the failure values predicted by the failure criterion and experimental results are obtained for the range where the failure criterion is applicable. The failure criterion proposed can complement other failure criteria such as LaRC or Puck criteria that do not account for matrix crack induced delamination failure.

References