ESTIMATION OF THE PROBABILITY OF CRITICAL DAMAGE USING BAYESIAN THEOREM AT DELAMINATION IDENTIFICATION VIA THE EPCM

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Abstract

This study is about method for estimation of probability of occurrence of critical damage at damage identification problem by using Bayesian theorem. For the probable evaluation of the damage diagnostic result using regression analysis, in this study, regression coefficients for multiple regression model are estimated as not fixed value but the distribution parameter using MCMC. With the combination of the regression analysis with non fixed coefficients and the Bayesian theorem, probability distribution of true event, it means true damage probability, is derived. Moreover, residual strength distribution is calculated from evaluation event probability distributions. And estimation of probability of occurrence of critical damage by external force assumption is performed using limit state function method. In this paper, the proposed method is applied to the delamination crack identification of the CFRP structure using the electric potential change method.

1 Introduction

An inverse problem for the damage or fault identification is one of the optimization problems to minimize the estimation error. For the method, to solve the relation between sensor measurement and the damage properties, several optimization methods like a response surface, neural network, genetic algorithm and so on or statistical analysis like a multiple regression, discriminant analysis, spatial statistics and so on are used.

The estimated error is reduced by the advancement of the method. However, for the evaluation of the reliability of the structure from the diagnostic results, evaluation of the probability of underestimation of the severe damage, which cause the failure of the structure, is important. Hence, evaluation of the probability distribution of the occurrence of true damage size over the estimated result is required. Then in this research, the method of estimating the probability of occurrence of true damage properties over the arbitrary estimated results is proposed, using occurrence probability estimation of the cause events via the Bayesian theorem. In this paper, the residual strength distribution from the acquired distribution is estimated, and estimation of the probability of failure estimation is conducted. And for the validation of the method, proposed method is applied to the delamination identification problem of CFRP beam using the electric potential method.

2 Estimation of the probability of occurrence of each size damage using estimation of probability of occurrence of the cause event using Bayesian theorem

In general, for the damage identification problem, maximum or average of the estimation error is mainly discussed for the evaluation of each method. Hence, the distribution of the estimated value over the true value is important. However, when evaluating the reliability from the diagnostic result, the distribution of the true value over the estimated value of damage is important. When the probability distribution of occurrence of the every damage size is uniform, and the distribution of the regression error for each damage size is equivalent, the distribution of the true value over the estimated value and that of the estimated value over the true value become almost equivalent. However for the damage identification problem, since the large damage is caused by the accumulation or the growth of small damage, the probability of occurrence of large damage is small compared with small damage, and this assumption is not correct. Moreover, the range where the higher accuracy is demanded at damage evaluation is the range where the failure of detection of occurrence of the damage is critical. The accuracy of estimation over sufficiently small damage is comparatively unimportant, and the identification accuracy does not need to be uniform at all the ranges of damage size. Then in this research, estimation of the distribution of the probability of occurrence of the true damage size over the estimated result from the distribution of the regression error using the Bayesian theorem are performed.

2.1 Estimation of the distribution of probability of occurrence of true damage size over the diagnostic result using Bayesian theorem

Bayesian theorem is a simple mathematical theorem used for calculating conditional probabilities. In this study, by the theorem, the probability of occurrence of true damage is estimated as the conditional probabilities under the estimated damage size. The ordinary form of Bayesian theorem is shown as follows.

$$P(E_i \mid F) = \frac{P(E_i)P(F \mid E_i)}{\sum_j P(E_j)P(F \mid E_j)} \qquad (i, j = 1, 2, \cdots) \qquad (1)$$

where $P(E_i)$ is the probability of occurrence of the event E_i and $P(F \mid E_i)$ is the conditional probability which event F causes under event E_i. $P(E_i \mid F)$ of the left side is conditional probability which event E_i causes under event F, and is called posterior probability. For the identification problem of the damage sizes, the event of the cause of generating the arbitrary estimated results is occurrence of the damage, and the $P(a_i)$ is the probability of occurrence of true damage size a_i. The probability that occurrence of the damage a_i causes the arbitrary estimated results $EstA_k$ is $P(EstA_k|a_i)$. In this case, the formula (1) modified as follows.

$$P(a_i \mid EstA_k) = \frac{P(a_i)P(EstA_k \mid a_i)}{\sum_j P(a_j)P(EstA_k \mid a_j)} \qquad (2)$$

where, the left side is posterior probability. Hence, $P(a_i|EstA_k)$ is the probability of occurrence of $EstA_k$ because of the true damage size a_i. By estimating occurrence probability about all the a_i, the occurrence probability distribution of the true damage parameter to the estimated value is deduced.

2.2 Procedure of estimation of the probability of failure over the estimated result

Estimation of the probability of failure is performed using the limit state function method. The probability of failure(PoF) is estimated by the following formula by the limit state function method.

$$Pof = P[g(R - S) < 0] \quad (3)$$

where, R is strength, S is applied force and g is the limit state function. The procedure of probability of failure estimation over the estimated result is shown in Fig. 1. First, the occurrence probability distribution of the true damage size over the estimated size is deduced by the procedure of the paragraph 2.1. Residual strength is the function of the damage properties. In this paper, the buckling failure because of the delamination is assumed and the distribution of buckling strength is calculated.

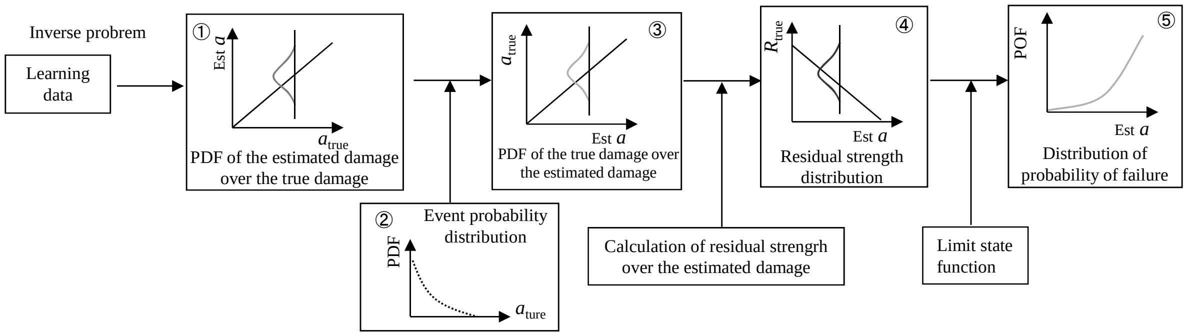


Figure 1. Procedure of the estimation of probability of failure

3 Application of the method to the damage identification using electric potential change method

3.1 Electric potential change method

As mentioned previously, this method for estimation of the probability of failure is applied to the identification of delamination in a CFRP Beam using the electric potential method[1-10] via the regression analysis. FEM analyses are employed for investigations in present study. A detailed description of FEM analysis is provided in our previous studies [2, 3]. Specimen configuration is shown in Fig.2. The specimen is a CFRP Beam with a thickness of 2mm and a stacking sequence of $[0_2/90_2]_s$. In order to measure the change in electric potential caused by a delamination crack, seven electrodes are mounted on the one surface of specimen. The lengths of the electrodes are 10mm. FEM analyses were performed using the commercially available FEM tool(ANSYS 9.0). In the present study, four-node-rectangular elements are adopted for analysis; each element is approximately 0.125 mm by 0.125 mm. A delamination crack is modeled by the release of a nodal point of the element. The electric conductance ratio is obtained from an experimental result regarding a CFRP laminate whose volume fraction is 62% as follows.

$\sigma_{90}/\sigma_0=3.7*10^{-2}$

$\sigma_t/\sigma_0=3.8*10^{-2}$

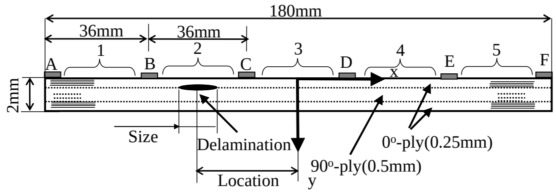


Figure 2. Model of Specimen

3.2 Identification of the delamination crack

Identification of delamination crack is conducted from the analytical result of change of electric potential caused by the delamination crack. The electric potential change of the each region is named as v_i (i=1 to 5). For diagnosis, following 6 parameters, a vector length Z and the standardized potential changes V_i are used for parameters.

$$Z = \sqrt{\sum_{i=1}^{5} v_i^{\ 2}}, \quad V_i = {v_i}/{Z} \ (i = 1 \cdots 5) \tag{4}$$

The delamination size identification is conducted by using the linear polynomial as follows.

$$y = \beta_0 + \sum_1^5 \beta_i V_i + \beta_6 Z \tag{5}$$

where, y is the predictor variable, in this case, the damage size. β_i are the regression coefficients. Number of the data for the regression is 74. The mean and distribution of the identification error to the true value are shown in Figure3 (a) and 3 (b). Abscissa shows the true size of the damage and vertical of (a) shows the mean and (b) shows the standard deviation. In this case, regression accuracies differ largely at each size of damage.

(a) Mean (b) Standard deviation

Figure 3. Mean and standard deviation of the estimation

3.3 Estimation of the distribution of true damage size over the estimated result using Bayesian theorem

By assuming the prior distribution of occurence of the true damage size, estimation of the true damage size over the estimated result by the Bayesian theorem was performed. The exponential distribution shown in the figure 4 was used for prior distribution. Example of the result is shown in Figure 5(a) and 5 (b). The abscissa of the figures shows the true value of the delamination size, and the vertical shows PDF. Figure 5(a) and 5(b) shows that the probability of occurrence of the damage size around the estimated size is the highest. Because of the prior distribution of the probability of occurrence, it is asymmetrical.

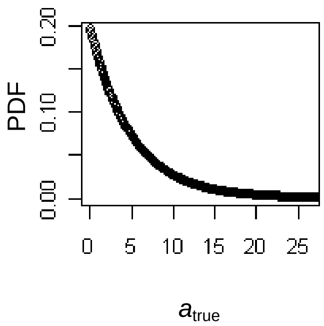
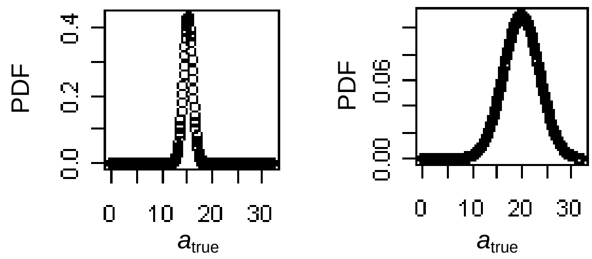


Figure 4. Prior distribution



(a) a_{est}= 15 (b) a_{est}= 20

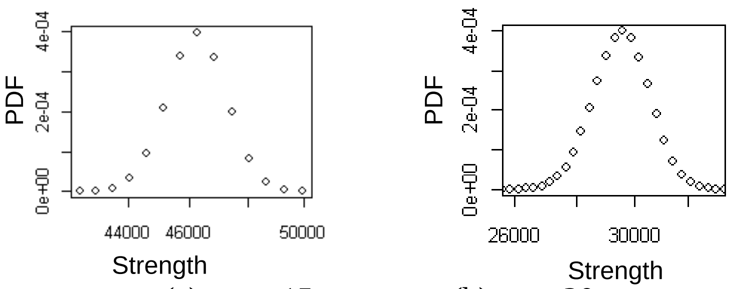
Figure 5. Posterior distribution

3.4 Estimation of distribution of the buckling strength

Delamination crack causes the deterioration of compressive strength. In this study, the buckling failure of the surface layer by the delamination was considered. Buckling strength serves as the following formula.

$$p_w = \frac{4\pi^2 EI}{a^2} \tag{6}$$

where, p_w is the buckling strength, E is the stiffness and I is the moment of inertia of areas. The estimated result of the distribution of strength when delamination sizes are estimated to 15mm and 20mm is shown in Figure 6(a) and 6(b). Abscissa of the figure shows the buckling strength. As shown in the figure, the strength decreases with the increase of the delamination size.



(a) a_{est}= 15 (b) a_{est}= 20

Figure 6. Estimated distribution of the residual strength

3.5 Estimation of the probability of failure

The external force is assumed in order that buckling failure might arise in an average of 20mm delamination in this case. The result is shown in Fig. 7. The abscissa is the estimated size, and the vertical axis is the probability of failure. Because external force is not constant, it is thought that the failure is caused at least 20mm or less. As shown in the figure, the probability of failure started the lifting by the damage smaller than 20mm, and it is saturated with about 23mm. It can be said that it is possible to evaluate the probability of failure over the estimated result by the proposed method.

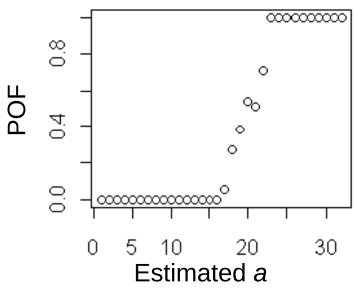


Figure 7. Estimated probability of failure over estimated size

4 Conclusion

For the evaluation of the reliability of the structure from the diagnostic results, evaluation of the probability of underestimation of the severe damage, which cause the failure of the structure, is important. In this study, the method of estimating the probability distribution of occurrence of true damage properties over the arbitrary estimated results is proposed, using occurrence probability estimation of the cause event via the Bayesian theorem. From the distribution, residual strength of the damaged structure is estimated and estimation of the probability of failure is performed using the limit state function method. As the result, the probability of failure started the lifting by the damage smaller than critical level, and it is saturated with over than critical level. It can be said that it is possible to evaluate the probability of failure over the estimated result by the proposed method.

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