HIGH ORDER ANALYTICAL AND 3D FINITE ELEMENT SOLUTION FOR BIAXIAL WRINKLING ANALYSIS OF COMPOSITE-FACED SANDWICH PLATES WITH SOFT CORE

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Abstract

In this paper, a known improved high-order theory and an accurate 3D finite element model are used for biaxial wrinkling analysis of composite sandwich plates with soft orthotropic core. The equations of motion and boundary conditions are derived by principle of minimum potential energy. Analytical solution for static analysis of simply supported sandwich plates under biaxial in-plane compressive loads is presented using Navier's solution. Also, for either geometrical parameter, 3D finite element model of the problem has been constructed in the ANSYS 11.0 standard code. Finally, effect of geometrical parameters and biaxial loading ratio are studied on the wrinkling loads using both models. Comparison of the present finite element results with analytical solutions in special case confirms the accuracy of the present finite element model.

1 Introduction

Sandwich plates are widely used in many engineering applications such as aerospace, because of their high strength and stiffness, low weight and durability. Sandwich plates experience some failure modes not occurred in metallic sheets or laminated plates. Face wrinkling is one of the important behaviors of these plates subjected to in-plane compressive loads. In this phenomenon, the faces buckle in shorter wavelength than those associated with overall buckling of the plate [1].

The wrinkling for sandwich struts with isotropic facings and solid cores were investigated by Hoff and Mautner [2], using a new model. In this model, the through thickness deformation decays linearly from the face sheet into the core. Plantema [3] proposed the exponential decay for the through thickness deformation in his book. Also, Vinson [4] summarized sandwich wrinkling statements in their textbooks. An extended unified theory for overall buckling and face wrinkling of sandwich panels with anisotropic facings was used by Hadi and Matthews [5]. Dawe and Yuan [6] provided a model which uses a quadratic and linear expansion of the in-plane and transverse displacements of the core and represented the face sheets as either FDST or CLPT. A B-spline finite stripe method (FSM) was used for buckling and wrinkling of rectangular sandwich plates subjected to in-plane compressive and shears loads applied to the face sheets. A high-order layer-wise model was proposed by Dafedar et al. [7] for

buckling analysis of multi-core sandwich plates. They assumed the cubic polynomial functions for all displacement components in any layer. As a large number of unknowns is involves, they proposed a simplified model and calculated critical loads based on the geometric stiffness matrix concept.
Biaxial wrinkling of sandwich panels with composite face sheets was investigated by Birman and Bert [1] using three different models. Fagerberg and Zenkert [8] studied the imperfection-induced wrinkling material failure in sandwich panels. Kardomateas [9] presented a 2D elasticity solution for the wrinkling analysis of sandwich beams or wide sandwich panels subjected to axially compressive loading.
Shariyat [10] studied the nonlinear dynamic thermo-mechanical buckling and wrinkling of the imperfect sandwich plates using finite element method. He introduced a generalized global-local plate theory (GLPT) that guarantees the continuity conditions of all displacements and transverse stress components and considered the transverse flexibility of sandwich plates. More recently, Kheirikhah et al. [11, 12] introduced a new high-order theory for buckling analysis of soft-core sandwich plates. They presented analytical solution for uniaxial wrinkling and biaxial overall buckling analysis of composite-faced sandwich plates using energy method.
In the present paper, a new improved high-order theory is presented for biaxial wrinkling analysis of sandwich plates with orthotropic soft core. Third-order plate theory is used for the face sheets and quadratic and cubic functions are assumed for the transverse and in-plane displacements of the core. The nonlinear Von-Karman type relations are used to obtain strains. Continuity conditions of transverse shear stresses at the interfaces as well as the conditions of zero transverse shear stresses on the upper and lower surfaces of the plate are satisfied. Also, transverse flexibility and transverse normal strain and stress of the core are considered. The equations of motion and boundary conditions are derived via principle of minimum potential energy. Analytical solution for static analysis of simply supported sandwich plates under biaxial in-plane compressive loads is presented using Navier's solution. Biaxial wrinkling loads are obtained for various sandwich plates. Effect of geometrical parameters of face sheets and core and biaxial loads ratio are studied on wrinkling behavior of sandwich plates.

2 Mathematical formulations

A rectangular sandwich plate with the plane dimensions of $a \times b$ and the total thickness of h is considered as shown in Figure 1. The sandwich is composed of three layers: the top and the bottom face sheets and the core layer. All layers are assumed with uniform thickness and the z coordinate of each layer is measured downward from its mid-plane.

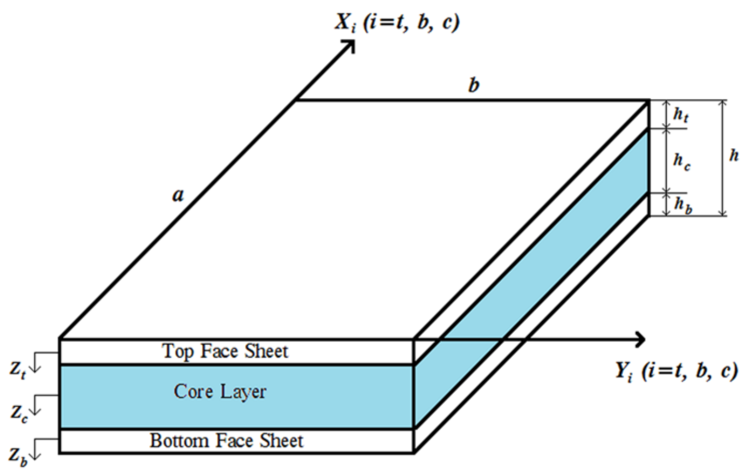

Figure 1. A typical sandwich plate and its dimensions

The face sheets are generally unequal in thickness, i.e., h_t and h_b are the thicknesses of the top and the bottom face sheets, respectively. The face sheets are assumed to be laminated composites. The core is also assumed as soft orthotropic material with thickness h_c.
In the present structural model for sandwich plates, the third-order shear deformable theory is adopted for the face sheets. Hence, the displacement components of the top and bottom face sheets (j = t, b) are represented as [11]:

$$\begin{aligned} u_j(x,y,z_t) &= u_{0j}(x,y) + z_j u_{1j}(x,y) + z_j^2 u_{2j}(x,y) + z_j^3 u_{3j}(x,y) \\ v_j(x,y.z_t) &= v_{0j}(x,y) + z_j v_{1j}(x,y) + z_j^2 v_{2j}(x,y) + z_j^3 v3j(x,y) \\ w_j(x,y,z_t) &= w_{0j}(x,y) \end{aligned} \tag{1}$$

where u_{Kj} and v_{Kj} (k = 0, 1, 2, 3) are the unknowns of the in-plane displacements of each face sheet and w_{0j} are the unknowns of its vertical displacements, respectively.
The core layer is much thicker and softer than the face sheets. Thus, the displacements fields for the core are assumed as a cubic pattern for the in-plane displacement components and as a quadratic one for the vertical component:

$$\begin{aligned} u_c(x,y,z_c) &= u_{0c}(x,y) + z_c u_{1c}(x.y) + z_c^2 u_{2c}(x,y) + z_c^3 u_{3c}(x,y) \\ v_c(x,y,z_c) &= v_{0c}(x,y) + z_c v_{1c}(x,y) + z_c^2 v_{2c}(x,y) + z_c^3 v_{3c}(x,y) \\ w_c(x,y,z_c) &= w_{0c}(x,y) + z_c w_{1c}(x,y) + z_c^2 w_{2c}(x,y) \end{aligned} \tag{2}$$

where u_{KC} and v_{KC} (*k = 0, 1, 2, 3*) are the unknowns of the in-plane displacement components of the core and w_{lC} (*l = 0, 1, 2*) are the unknowns of its vertical displacements, respectively. Therefore, the face sheets are assumed as in-plane flexible and transversely rigid plates. Also, the core is assumed as in-plane and transversely flexible layer. Finally, in this model there are twenty nine displacement unknowns: nine unknowns for each face sheet and eleven unknowns for the core. The nonlinear Von-Karman strain–displacement relations are used to obtain strains of the face sheets and core.
In the present sandwich plate theory, the core is perfectly bonded to the face sheets. Hence, there are three interface displacement continuity requirements in each face sheet-core interface which were presented in Ref. [11]. Also, eight equations were obtained for satisfying the continuity conditions of transverse shear stresses at the interfaces as well as the conditions of zero transverse shear stresses on the upper and the lower surfaces of the sandwich plate.
The governing equations of motion for the face sheets and the core are derived through the principle of minimum potential energy:

$$\delta \Pi = \delta U + \delta V = 0 \tag{3}$$

where U is the total strain energy, V is the potential of the external loads and δ denotes the variation operator. The first variation of the total strain energy can be expressed in terms of all stresses and strains of the face sheets and the core. In addition, six compatibility conditions at the interfaces, four conditions of zero transverse shear stresses on the upper and the lower surfaces of the plate and four continuity conditions of transverse shear stresses at the interfaces are fulfilled by using fourteen Lagrange multipliers. Thus, integrating by part and doing some mathematical operations, the equations of motion for the top and bottom face sheets (j = t, b) can be calculated as [11]:

$$
\begin{aligned}
&-N^{j}_{xx,x}-N^{j}_{xy,y}+\lambda^{j}_{x}-\bar{n}_{xj}=0\\
&-M^{j}_{xx,x}-M^{j}_{xy,y}+Q^{j}_{xz}+\frac{h_t}{2}\lambda^{j}_{x}+\lambda^{j}_{xz}+\frac{h_t}{2}\bar{n}_{xj}=0\\
&-P^{j}_{xx,x}-P^{j}_{xy,y}+2S^{j}_{xz}+\frac{h_t^2}{4}\lambda^{j}_{x}-h_t\lambda^{j}_{xz}+\lambda^{jc}_{xz}-\frac{h_t^2}{4}\bar{n}_{xj}=0\\
&-R^{j}_{xx,x}-R^{j}_{xy,y}+3T^{j}_{xz}+\frac{h_t^3}{8}\lambda^{j}_{x}+\frac{3h_t^2}{4}\lambda^{j}_{xz}+\frac{h_t^3}{8}\bar{n}_{xj}=0\\
&-N^{j}_{yy,y}-N^{j}_{xy,x}+\lambda^{j}_{y}-\bar{n}_{yj}=0\\
&-M^{j}_{yy,y}-M^{j}_{xy,y}+Q^{j}_{yz}+\frac{h_t}{2}\lambda^{j}_{y}+\lambda^{j}_{yz}+\frac{h_t}{2}\bar{n}_{yj}=0\\
&-P^{j}_{yy,y}-P^{j}_{xy,x}+2S^{j}_{yz}+\frac{h_t^2}{4}\lambda^{j}_{y}-h_t\lambda^{j}_{yz}+\lambda^{jc}_{yz}-\frac{h_t^2}{4}\bar{n}_{yj}=0\\
&-R^{j}_{yy,y}-R^{j}_{xy,x}+3T^{j}_{yz}+\frac{h_t^3}{8}\lambda^{j}_{y}+\frac{3h_t^2}{4}\lambda^{j}_{yz}+\frac{h_t^3}{8}\bar{n}_{yj}=0\\
&-Q^{j}_{xz,x}-Q^{j}_{yz,y}-\mathrm{N}(w^{j}_{0})+\lambda^{j}_{x}-\lambda^{j}_{xz,x}-\lambda^{j}_{yz,y}-q_j=0
\end{aligned}
\tag{4}
$$

and also for the core as [11]:

$$
\begin{aligned}
&-N^{c}_{xx,x}-N^{c}_{xy,y}-\lambda^{t}_{x}-\lambda^{b}_{x}=0\\
&-M^{c}_{xx,x}-M^{c}_{xy,y}+Q^{c}_{xz}+\frac{h_c}{2}\lambda^{t}_{x}-\frac{h_c}{2}\lambda^{b}_{x}-\frac{G^{c}_{xz}}{2h_tQ^{t}_{55}}\lambda^{tc}_{xz}+\frac{G^{c}_{xz}}{2h_bQ^{b}_{55}}\lambda^{bc}_{xz}=0\\
&-P^{c}_{xx,x}-P^{c}_{xy,y}+2S^{c}_{xz}-\frac{h_c^2}{4}\lambda^{t}_{x}-\frac{h_c^2}{4}\lambda^{b}_{x}+\frac{G^{c}_{xz}h_c}{2h_tQ^{t}_{55}}\lambda^{tc}_{xz}+\frac{G^{c}_{xz}h_c}{2h_bQ^{b}_{55}}\lambda^{bc}_{xz}=0\\
&-R^{c}_{xx,x}-R^{c}_{xy,y}+3T^{c}_{xz}+\frac{h_c^3}{8}\lambda^{t}_{x}-\frac{h_c^3}{8}\lambda^{b}_{x}-\frac{3G^{c}_{xz}h_c^2}{8h_tQ^{t}_{55}}\lambda^{tc}_{xz}+\frac{3G^{c}_{xz}h_c^2}{8h_bQ^{b}_{55}}\lambda^{bc}_{xz}=0\\
&-N^{c}_{yy,y}-N^{c}_{xy,x}-\lambda^{t}_{y}-\lambda^{b}_{y}=0\\
&-M^{c}_{yy,y}-M^{c}_{xy,x}+Q^{c}_{yz}+\frac{h_c}{2}\lambda^{t}_{y}-\frac{h_c}{2}\lambda^{b}_{y}-\frac{G^{c}_{xz}}{2h_tQ^{t}_{44}}\lambda^{tc}_{yz}+\frac{G^{c}_{xz}}{2h_bQ^{b}_{44}}\lambda^{bc}_{yz}=0\\
&-P^{c}_{yy,y}-P^{c}_{xy,x}+2S^{c}_{yz}-\frac{h_c^2}{4}\lambda^{t}_{y}-\frac{h_c^2}{4}\lambda^{b}_{y}+\frac{G^{c}_{xz}h_c}{2h_tQ^{t}_{44}}\lambda^{tc}_{yz}+\frac{G^{c}_{xz}h_c}{2h_bQ^{b}_{44}}\lambda^{bc}_{yz}=0\\
&-R^{c}_{yy,y}-R^{c}_{xy,x}+3T^{c}_{yz}+\frac{h_c^3}{8}\lambda^{t}_{y}-\frac{h_c^3}{8}\lambda^{b}_{y}-\frac{3G^{c}_{yz}h_c^2}{8h_tQ^{t}_{44}}\lambda^{tc}_{yz}+\frac{3G^{c}_{yz}h_c^2}{8h_bQ^{b}_{44}}\lambda^{bc}_{yz}=0\\
&-Q^{c}_{xz,x}-Q^{c}_{yz,y}-\mathrm{N}(w^{c}_{0})-\lambda^{t}_{z}-\lambda^{b}_{z}+\frac{G^{c}_{xz}}{2h_tQ^{t}_{55}}\lambda^{tc}_{xz}-\frac{G^{c}_{xz}}{2h_bQ^{b}_{55}}\lambda^{bc}_{xz}+\frac{G^{c}_{xz}}{2h_tQ^{t}_{44}}\lambda^{tc}_{yz}-\frac{G^{c}_{xz}}{2h_bQ^{b}_{44}}\lambda^{bc}_{yz}=0\\
&-S^{c}_{xz,x}-S^{c}_{yz,y}+N^{c}_{zz}+\frac{h_c}{2}\lambda^{t}_{z}-\frac{h_c}{2}\lambda^{b}_{z}-\frac{G^{c}_{xz}h_c}{4h_tQ^{t}_{55}}\lambda^{tc}_{xz,x}-\frac{G^{c}_{xz}h_c}{4h_bQ^{b}_{55}}\lambda^{bc}_{xz,x}-\frac{G^{c}_{xz}h_c}{4h_tQ^{t}_{44}}\lambda^{tc}_{yz,y}+\frac{G^{c}_{xz}h_c}{2h_bQ^{b}_{44}}\lambda^{bc}_{yz,y}=0\\
&-T^{c}_{xz,x}-T^{c}_{yz,y}+2M^{c}_{zz}-\frac{h_c^2}{4}\lambda^{t}_{z}-\frac{h_c^2}{4}\lambda^{b}_{z}+\frac{G^{c}_{xz}h_c^2}{8h_tQ^{t}_{55}}\lambda^{tc}_{xz,x}-\frac{G^{c}_{xz}h_c^2}{8h_bQ^{b}_{55}}\lambda^{bc}_{xz,x}+\frac{G^{c}_{yz}h_c^2}{8h_tQ^{t}_{44}}\lambda^{tc}_{yz,y}-\frac{G^{c}_{yz}h_c^2}{8h_bQ^{b}_{44}}\lambda^{bc}_{yz,y}=0
\end{aligned}
\tag{5}
$$

3 Analytical solution

The exact analytical solutions of equations (4) and (5) exist for the simply supported rectangular sandwich plate with cross-ply face sheets. Both face sheets are considered as a cross-ply laminated composite. For simply supported plates, the tangential displacements on the boundary are admissible, but the transverse displacements are not as such [11]. The applied boundary conditions of simply supported plates are presented in Table 1.
For sandwich plate which is subjected to biaxial compressive loading:

$$
N_{xx}=-N_0\ ,\ N_{yy}=-KN_0\ ,\ N_{xy}=\bar{n}_{xt}=\bar{n}_{yt}=\bar{n}_{xb}=\bar{n}_{yb}=q_t=q_b=0 \tag{6}
$$

In the above equation, K is compressive loads ratio. If $K = 0$, the uniaxial buckling occurred. For $K > 1$ or $K < 1$, the compressive load along y direction is greater or smaller than that along x direction, respectively.

At edges $x = 0$ and $x = a$:	**At edges $y = 0$ and $y = b$:**
$v_{Kj} = 0, \quad k = 0,1,2,3 \;, \quad j = t,b,c$	$u_{Kj} = 0, \quad k = 0,1,2,3 \;, \quad j = t,b,c$
$w_{0t} = 0, \quad w_{0b} = 0, \quad w_{0c} = 0, \quad w_{1c} = 0, \quad w_{2c} = 0$	$w_{0t} = 0, \quad w_{0b} = 0, \quad w_{0c} = 0, \quad w_{1c} = 0, \quad w_{2c} = 0$

Table 1. Applied boundary conditions of simply supported plates

In the buckling analysis, if a uniform state of strain is assumed, the relative edge stresses in the individual layers are proportional to the respective elastic modulus. The in-plane flexural rigidity of the soft cores is comparatively very small and hence the condition of uniform strain state is more realistic for sandwich plates [7]. Therefore, in this analysis the uniform strain state is assumed. Hence, the external in-plane loads exerted to the top and the bottom face sheets and the core along x direction can be defined as [11]:

$$\begin{aligned}
\hat{N}^{t}_{xx} &= \frac{N_0\left(h_b + h_c\right)\left(K^0_{t,12}K^0_{b,11} - K^0_{t,11}K^0_{b,12}\right)}{\left(h_t + h_b + 2h_c\right)\left(K^0_{t,12}K^0_{b,11} - K^0_{t,11}K^0_{b,12}\right) + K^0_c\left(h_b + h_c\right)\left(K^0_{b,11}C^c_{12} - K^0_{b,12}C^c_{11}\right) + K^0_c\left(h_t + h_c\right)\left(K^0_{t,12}C^c_{11} - K^0_{t,11}C^c_{12}\right)} \\
\hat{N}^{b}_{xx} &= \frac{N_0\left(h_t + h_c\right)\left(K^0_{t,12}K^0_{b,11} - K^0_{t,11}K^0_{b,12}\right)}{\left(h_t + h_b + 2h_c\right)\left(K^0_{t,12}K^0_{b,11} - K^0_{t,11}K^0_{b,12}\right) + K^0_c\left(h_b + h_c\right)\left(K^0_{b,11}C^c_{12} - K^0_{b,12}C^c_{11}\right) + K^0_c\left(h_t + h_c\right)\left(K^0_{t,12}C^c_{11} - K^0_{t,11}C^c_{12}\right)} \\
\hat{N}^{c}_{xx} &= \frac{K^0_c N_0\left[\left(h_b + h_c\right)\left(K^0_{b,11}C^c_{12} - K^0_{b,12}C^c_{11}\right) + \left(h_t + h_c\right)\left(K^0_{t,12}C^c_{11} - K^0_{t,11}C^c_{12}\right)\right]}{\left(h_t + h_b + 2h_c\right)\left(K^0_{t,12}K^0_{b,11} - K^0_{t,11}K^0_{b,12}\right) + K^0_c\left(h_b + h_c\right)\left(K^0_{b,11}C^c_{12} - K^0_{b,12}C^c_{11}\right) + K^0_c\left(h_t + h_c\right)\left(K^0_{t,12}C^c_{11} - K^0_{t,11}C^c_{12}\right)}
\end{aligned} \tag{7}$$

where $\hat{N}^{t}_{xx}$, $\hat{N}^{b}_{xx}$ and $\hat{N}^{c}_{xx}$ are the parts of the total load which are exerted to the top face sheet, the bottom face sheet and the core along x direction, respectively. The partial loads along y direction can be calculated using above procedure. Using Navier's procedure, the solution of the displacement variables satisfying the above boundary conditions can be obtained [11]. By substituting the Navier's solution into equations (4) and (5) and collecting the coefficients, the final equations of motion in matrix form can be determined as:

$$[A]_{43\times 43}\{X\}_{43\times 1} = \{0\}_{43\times 1} \tag{8}$$

where [A] is coefficients matrix and {X} is Unknown Vector [11]. The nonzero result and buckling load is obtained when the determinant of [A] is set to be zero.

4 Finite element modeling

For either selected values of *a/h* and *a/b*, finite element model of the problem has been constructed by employing 20-nodes isoparametric hexahedral elements (solid 95) for discretizing the orthotropic core and 20-nodes layer elements (solid 191) for the laminated face sheets in the ANSYS 11.0 standard code. The constructed finite element model of a sandwich plate is shown in Fig. 2.

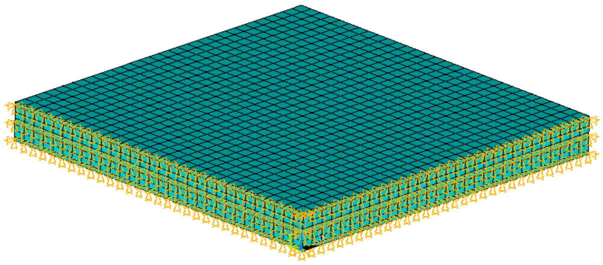

Figure 2. The finite element model of sandwich plate

5 Numerical results and discussion

In this section, several examples of the overall buckling and face wrinkling problems of the sandwich plates are studied to verify the accuracy and applicability of the present higher order theory. The results obtained by the present theory are compared with the results in the literature. The following dimensionless buckling load used in the present analysis is defined as [7]:

$$\overline{N} = \frac{a^2\, N_0}{E_2\, h^3} \tag{9}$$

where E_2 is the transverse elastic modulus of the face sheets. Two types of buckling modes are studied for sandwich plates: overall buckling and wrinkling modes. Generally, the overall buckling load corresponds to both wave numbers equal to unity ($m = n = 1$). If the buckling load of higher wave number is less than the overall buckling load, the sandwich plate would fail in the wrinkling mode, although, it is not a general case. For assessing of uniaxial wrinkling possibility, the wave number m should be increased in steps of one, when the wave number n is considered to be unity. But, biaxial wrinkling loads can be obtained by increasing both wave numbers in steps of one.

A square symmetric sandwich plate with stack-up sequence of [(0/90)$_5$/Core/(90/0)$_5$] with the total thickness of h is considered. The sandwich plate consists of equal thickness cross-ply laminated face sheets with 10 layers and a soft orthotropic core. The analysis is performed for different thickness ratios (a/h = 20, 10, 20/3 and 5), different face sheet thickness ratios (ht/h = 0.025, 0.05, 0.075 and 0.1) and different biaxial load ratios (K = 0, 0.5, 1). The material constants used in this example are assumed as presented in Table 2.

The composite layer of the face sheets	The orthotropic core
$E_1 = 19E$, $E_2 = E_3 = E$	$E_1 = 3.2\times10^{-5}\,E$, $E_2 = 2.9\times10^{-5}\,E$, $E_3 = 0.4E$
$G_{23} = 0.338E$, $G_{12} = G_{13} = 0.52E$	$G_{12} = 2.4\times10^{-3}\,E$, $G_{23} = 6.6\times10^{-2}\,E$, $G_{13} = 7.9\times10^{-2}\,E$
$\nu_{23} = 0.49$, $\nu_{12} = \nu_{13} = 0.32$	$\nu_{12} = 0.99$, $\nu_{123} = \nu_{13} = 3\times10^{-5}$

Table 2. The material constants [7, 10]

The dimensionless wrinkling loads obtained by the present high-order analytical theory and finite element method are given and compared in Table 3. Also, if the wrinkling mode is possible, the mode numbers of wrinkling loads (m,n) which obtained by analytical solution is presented in parenthesis. The accuracy of present analytical high-order theory is examined by present authors for biaxial overall buckling and uniaxial wrinkling and its precision and efficiency was verified [11, 12]. Therefore, in the case of uniaxial wrinkling, the obtained results by the present finite element method are compared to the published analytical results [12] in the Table 3. This comparison confirms that the present finite element method is accurate. The results show that in constant geometrical parameters, the wrinkling loads decrease with increase in the load ratio (K), but their variations are very small. In constant thickness ratio (a/h), the wrinkling loads increase with increase in the face sheet thickness ratio (ht/h), because the stiffness of the face sheets is much greater than the stiffness of the core. It can be concluded that for thin sandwich plates ($a/h = 20$), the wrinkling behavior is not happened. Also, it can be seen that for all possible cases, the plates wrinkled in the mode number $n = 1$. Therefore it can be concluded that the wrinkling waves are only propagated along x direction. Also, the results indicated that in constant geometrical parameters, the obtained wave numbers are same for all the load ratios. In constant face sheet thickness ratio (ht/h), the wrinkling wave number m increases with increase in the thickness ratio (a/h).

ht / h	*a / h*	Analytical Solution			Finite Element Method		
		K = 0 [12]	*K = 0.5*	*K = 1*	*K = 0*	*K = 0.5*	*K = 1*
0.025	20	-*	-	-	-*	-	-
	10	1.3322 (54,1)	1.3321 (54,1)	1.3320 (54,1)	1.353	1.347	-
	20/3	0.5929 (36,1)	0.5928 (36,1)	0.5926 (36,1)	0.612	0.607	0.601
	5	0.3341 (27,1)	0.3340 (27,1)	0.3339 (27,1)	0.328	0.324	0.313
0.05	20	-	-	-	-	-	-
	10	2.9321 (42,1)	2.9318 (42,1)	-	2.982	2.945	-
	20/3	1.3043 (28,1)	1.3039 (28,1)	1.3035 (28,1)	1.326	1.301	1.277
	5	0.7346 (21,1)	0.7342 (21,1)	0.7337 (21,1)	0.723	0.720	0.711
0.075	20	-	-	-	-	-	-
	10	-	-	-	-	-	-
	20/3	2.1821 (25,1)	2.1812 (25,1)	2.1804 (25,1)	2.240	2.213	-
	5	1.2288 (19,1)	1.2280 (19,1)	1.2271 (19,1)	1.219	1.206	1.198
0.1	20	-	-	-	-	-	-
	10	-	-	-	-	-	-
	20/3	3.1383 (24,1)	-	-	3.129	-	-
	5	1.7667 (18,1)	1.7654 (18,1)	-	1.751	1.747	-

* Wrinkling mode is not possible

Table 3. Dimensionless biaxial wrinkling load for symmetric square sandwich plate [$(0/90)_5$/Core/$(90/0)_5$]

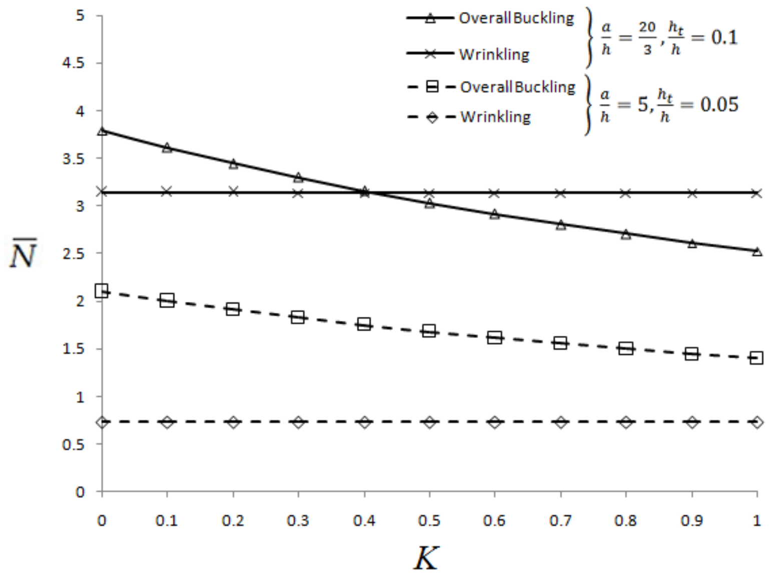

Figure 3. Dimensionless biaxial overall buckling and wrinkling load for symmetric square sandwich plate [$(0/90)_5$/Core/$(90/0)_5$]

Figure 3, shows the variation of dimensionless biaxial overall buckling and wrinkling load versus load ratio (K) for two different symmetric square sandwich plates. In first sandwich plate (*a/h = 20/3, ht/h = 0.1*), for *K < 0.4*, the overall buckling loads are greater than the wrinkling loads and hence, the wrinkling is happened. But for $K > 0.4$, the overall buckling loads are smaller than the wrinkling loads and the wrinkling is not happened. In second one ($a/h = 5$, $ht/h = 0.05$), the overall buckling loads are greater than the wrinkling loads for all the load ratios and therefore, the wrinkling always happened.

6 Conclusion

In this paper, the known accurate high-order theory is used for biaxial faces wrinkling analysis of composite faced sandwich plates with soft core. Analytical solution for face wrinkling analysis of simply supported sandwich plates under various biaxial in-plane compressive loads is presented using Navier's solution.

It can be concluded that the obtained results by present finite element method are in good agreement with the analytical solutions. The results showed that in constant geometrical parameters, the dimensionless wrinkling loads decrease with increase in the load ratio (K). The results indicated that in all sandwich plates which the wrinkling behavior is possible, the wrinkling wave is only propagated along x direction for the load ratios $K \leq 1$. Also, it can be concluded that for thin sandwich plates ($a/h = 20$), the wrinkling behavior is not happened.

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