ON THE PREDICTION OF GEOMETRIC NONLINEAR EFFECTS ON THE DAMPING OF COMPOSITE STRIPS AND BEAMS: THE CASE OF BUCKLING

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Abstract
A theoretical framework is presented for analyzing the small-amplitude free-vibrational response of composite strips exhibiting significant geometric nonlinearities. Nonlinear Green-Lagrange strains are introduced into the governing equations, assuming a Kelvin viscoelastic solid. A novel beam finite element is developed, which yields new nonlinear damping and stiffness matrices of the structure. The beam element is capable of predicting the damped free-vibration response and the modal characteristics of an in-plane deflected composite strip. Numerical results quantify the geometric nonlinear effect of compressive in-plane loads and the variation of modal damping and natural frequencies of composite strips during buckling and postbuckling response. Experimental measurements of a cross-ply Glass/Epoxy beam subject to buckling were conducted and correlated with the finite element predictions.

1 Introduction
The damping of polymer matrix composite materials and laminates has received substantial attention as it provides an additional functionality to composite structures enabling the passive control of aeroelastic loads. Thus far, the damping behavior of composite laminates and structures loaded within the range of linear response is well-understood and analyzed [1-6]. However, many applications which may benefit from the passive damping of composite materials involve large initial stresses and/or large rotations and displacements, forcing the structural response into the nonlinear regime. Typical examples are pressurized fuselage and wing structures, helicopter blades and large wind-turbine rotor blades exceeding 60m in length. Understanding and predicting the nonlinear damping and stiffness behavior of composite laminates in structures subject to large compressive loads and large displacements are important steps for improving the vibrational and aeroelastic response of many composite structures.

The objectives of the proposed paper include the study and characterization of the nonlinear damping behavior of laminated strips undergoing buckling and post-buckling as well as the formulation of a theoretical framework for modeling the effect of large in-plane compressive loads and large rotations in composite strips. To that direction, a theoretical and computational framework for analyzing the small-amplitude free-vibrational response of composite strips, which exhibit significant geometric nonlinearities, is formulated. A novel beam finite element capable of predicting the damping of composite strips undergoing compressive in-plane stresses is developed. The damping mechanics include multi-scale models for predicting the damping capacity of each composite ply and eventually of the whole beam section. The kinematic equations include nonlinear Green-Lagrange strains in the governing equations and the First order Shear Deformation Theory (FSDT) is implied, assuming a Kelvin viscoelastic solid. The Newton-Raphson incremental-iterative technique is used and the displacement control method is incorporated into the finite element code. The new damped beam finite element provides the effective and tangential (linearized) matrices of the cross-section and predicts the structural stiffness and modal damping of the composite structure subject to in-plane buckling loads. The new contribution of the current work is the prediction of modal damping values of the beam in the pre- and the post-buckling regions.

Numerical results quantify the contribution of first- and second-order nonlinear damping and stiffness laminate terms on the modal characteristics of composite strips subject to in-plane compressive loads. Validations of predicted results with experimentally measured modal damping loss factors of a Glass/Epoxy cross-ply laminated composite strip undergoing buckling are also conducted.

2 Damping mechanics framework

The geometric nonlinear effects are predicted through a multi-scale model [7]. Here we first focus on the beam section level, with proper calculation of the linear and nonlinear stiffness and damping terms, and then on the structural stiffness and damping matrices of the beam.

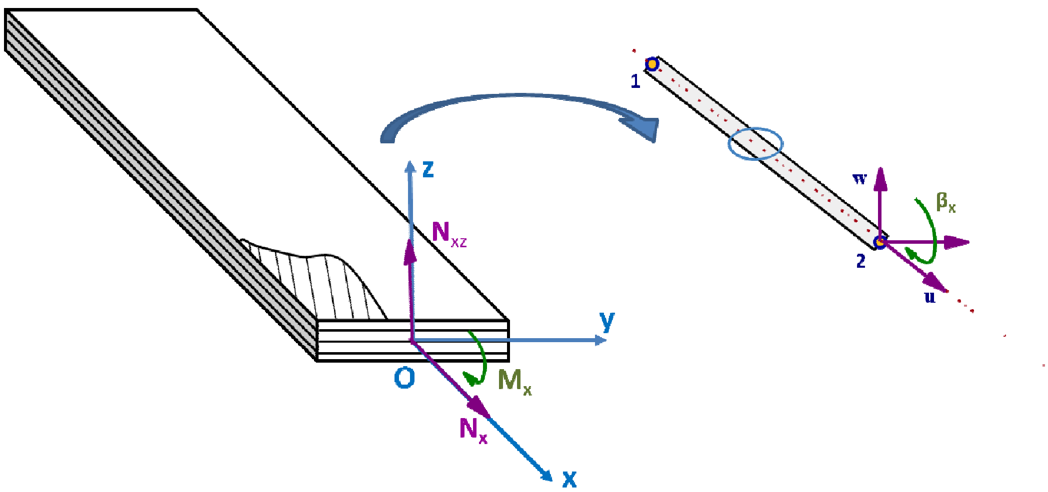

Figure 1. Laminated composite beam-strip element: (a) Cross-section; (b) Finite element and nodal degrees of freedom.

To that direction, a plate-beam or strip with arbitrary lamination is considered (Fig. 1a). The beam is assumed to be neither curved nor pre-twisted.

2.1 Section kinematics

The first-order shear section deformation theory (FSDT) was considered, which admits extension and bending along the x-axis and out of plane shear on x, z directions (Fig 1a). The kinematic assumptions are the first step in order to build the nonlinear beam finite element formulation and have the following form,

$$u(x,z,t) = u^0(x,t) + z\beta_x(x,t)$$
$$w(x,z,t) = w^0(x,t) \tag{1}$$

where: u, w are the displacement components of the section and β_x is the bending rotation angle around y-axis; superscript 0 indicates mid-section and the comma in the subscripts indicates differentiation.

In order to capture the effect of initial in-plane loads, a nonlinear Green-Lagrange normal strain component was considered. The shear strain acting on the cross-section is assumed to remain linear. Thus, the engineering strains acting on the section have the following form:

$$\varepsilon_x = u_{,x} + \frac{1}{2}w_{,x}^2$$
$$\varepsilon_{xz} = u_{,z} + w_{,x} \tag{2}$$

Combining Eqs. (1) and (2), the detailed normal and shear strains of the section are expressed as follows:

$$\varepsilon_x(x,z) = \varepsilon_x^0(x) + \frac{1}{2}w_{,x}^{0^2}(x) + zk_x(x)$$
$$\varepsilon_{xz}(x,z) = w_{,x}^0 + \beta_x \tag{3}$$

The previous generalized strains, which equivalently describe the deformation of the section, include the linear axial strain ε_x^0, the transverse shear strain, ε_{xz}^0 the bending curvature k_x and the nonlinear axial strain due to large deformations $\varepsilon_x^L(x)$.

2.2 Equation of motion

The equations of motion of the beam could be described by the variational form:

$$\int_0^L \mathrm{d}x \int_A \delta H \mathrm{d}A + \int_0^L \mathrm{d}x \int_A \delta \mathrm{T} \mathrm{d}A + \oint_\Gamma \delta \bar{\mathbf{u}}^{\mathbf{T}} \bar{\boldsymbol{\tau}} \mathrm{d}\Gamma = 0 \tag{4}$$

where H and T are the strain and kinetic energy; $\bar{\tau}$ are surface tractions on the free surface Γ; A is the cross-sectional area covered by material and L is the length of the beam.

The strain energy variation of the section $\delta H^{\sec}$ is represented by the integral over the cross-sectional area as follows:

$$\delta H^{\sec} = b\int_h \delta \boldsymbol{\varepsilon}_{\mathbf{c}}^{\mathbf{T}} \boldsymbol{\sigma}_{\mathbf{c}} \mathrm{d}z \tag{5}$$

where $\boldsymbol{\varepsilon}_\mathbf{c}$ and $\boldsymbol{\sigma}_\mathbf{c}$ are the off-axis strains and stresses of a rotated composite ply, respectively; c indicates off-axis ply and b is the width of the composite beam section.

A strain based Kelvin viscoelastic constitutive model was considered, next. Thus, the ply stresses are related to the strain in the form:

$$\boldsymbol{\sigma}_\mathbf{c} = \left[\mathbf{Q}_\mathbf{cs}\right]\boldsymbol{\varepsilon}_\mathbf{c} + \left[\mathbf{Q}_\mathbf{cd}\right]\dot{\boldsymbol{\varepsilon}}_\mathbf{c} \tag{6}$$

where $\mathbf{Q}_\mathbf{cs}$ and $\mathbf{Q}_\mathbf{cd}$ are the reduced off-axis stiffness and damping matrices of the composite ply, indicated by the subscripts s and d, respectively. Substituting Eq. (6) into Eq. (5), the final expression for the strain energy variation over the cross-sectional area takes the form:

$$\delta H^{\mathrm{sec}} = b\int_h \left(\delta\boldsymbol{\varepsilon}_\mathbf{c}^\mathbf{T}\left(\left[\mathbf{Q}_\mathbf{cs}\right]\boldsymbol{\varepsilon}_\mathbf{c} + \left[\mathbf{Q}_\mathbf{cd}\right]\dot{\boldsymbol{\varepsilon}}_\mathbf{c}\right)\right)\mathrm{d}z = \delta H_s + \delta H_{ds} \tag{7}$$

where δH_s and δH_{ds} are the expressions for the strain and dissipated energy variation of the cross-section, respectively.

2.3 Section stiffness and damping terms

Replacing the normal and shear strain expressions provided by Eq.(3), into Eq. (7), integrating firstly over the laminate thickness and assuming negligible transverse normal and shear laminate stresses N_y, N_{xy}, N_{yz} and transverse and shear moments M_y, M_{xy} along the coordinate axes, O_{xyz}, the stored and the dissipated strain energy in the section takes the form:

$$\begin{aligned} \delta H_s &= \delta H_{s_0} + \delta H_{s_1} + \delta H_{s_2} \\ \delta H_{ds} &= \delta H_{ds_0} + \delta H_{ds_1} + \delta H_{ds_2} \end{aligned} \tag{8}$$

where the subscripts *s*, *ds* indicate the cross-section strain and dissipated energy terms, whereas the subscripts *0, 1, 2* represent the terms containing linear, nonlinear first- and second-order components.

3 Damped beam finite element

A three-dimensional shear beam finite element was developed for the nonlinear quasi-static damped dynamic analysis of composite beams encompassing the aforementioned nonlinear mechanics (Fig. 1b). The element has 3 DOFs at each node (indicated with superscript i), and approximates the generalized displacements by c^0 continuous shape functions $N^i(x)$,

$$\left\langle u^o(x),\quad w^o(x),\quad \beta_x(x)\right\rangle \cong \sum_{i=1}^{n} N^i(x)\left\langle u^{oi},\quad w^{oi},\quad \beta_x^i\right\rangle \tag{9}$$

where, n is the number of element nodes.

Combining the previous kinematic assumptions and collecting the common coefficients of the total stiffness $[\mathbf{K}]$, damping $[\mathbf{C}]$ and mass $[\mathbf{M}]$ matrices respectively of the beam, the equilibrium $\mathbf{u(t)}$ is provided by the following equation:

$$\boldsymbol{\Psi}(\mathbf{u,t})=[\mathbf{M}]\ddot{\mathbf{u}}(\mathbf{t})+[\mathbf{C}]\dot{\mathbf{u}}(\mathbf{t})+[\mathbf{K}]\mathbf{u}(\mathbf{t})-\mathbf{F}(\mathbf{t}) \tag{10}$$

3.1 Small-amplitude free-vibration of composite strip

For vibrating beams subject to large deformations, we specialize their motion to the case of a perturbation vibration around a nonlinear static equilibrium point $\mathbf{u_s}$, such that:

$$\mathbf{u}(\mathbf{t})=\mathbf{u_s}+\bar{\mathbf{u}}(\mathbf{t}) \tag{11}$$

where overbar indicates perturbation quantities and $\mathbf{u_s} \gg \bar{\mathbf{u}}(\mathbf{t})$. In this case the equilibrium takes the following form:

$$\boldsymbol{\Psi}(\mathbf{u,t})=[\mathbf{M}]\ddot{\bar{\mathbf{u}}}+\frac{\partial([\mathbf{C}]\dot{\mathbf{u}})}{\partial\dot{\mathbf{u}}}\dot{\bar{\mathbf{u}}}+([\mathbf{K}]\mathbf{u_s}-\mathbf{F_s})+\frac{\partial([\mathbf{K}]\mathbf{u})}{\partial\mathbf{u}}\bar{\mathbf{u}}-\bar{\mathbf{F}}(\mathbf{t})=\mathbf{0} \tag{12}$$

Since $\mathbf{u_s}$ is the point of static equilibrium, the imbalance force vector between the internal forces and externally applied mechanical loads, vanishes,

$$\boldsymbol{\Psi}_\mathbf{s}=([\mathbf{K}]\mathbf{u_s}-\mathbf{F_s})=\mathbf{0} \tag{13}$$

By definition the terms $[\bar{\mathbf{K}}]=(\partial([\mathbf{K}]\mathbf{u})/\partial\mathbf{u})$, $[\bar{\mathbf{C}}]=(\partial([\mathbf{C}]\dot{\mathbf{u}})/\partial\dot{\mathbf{u}})$ are the tangential or linearized stiffness and damping of the structure at the point of static equilibrium. Hence, Eq. (12) takes its final form which describes the small vibration of the beam:

$$\boldsymbol{\Psi}(\bar{\mathbf{u}},\mathbf{u_s},\mathbf{t})=[\mathbf{M}]\ddot{\bar{\mathbf{u}}}(\mathbf{t})+[\bar{\mathbf{C}}(\mathbf{u_s})]\dot{\bar{\mathbf{u}}}(\mathbf{t})+[\bar{\mathbf{K}}(\mathbf{u_s})]\bar{\mathbf{u}}(\mathbf{t})-\bar{\mathbf{F}}(\mathbf{t})=\mathbf{0} \tag{14}$$

3.2 Modal damping calculation

Assuming harmonic motion Eq. (14) may be solved either directly to yield the complex eigenvalues of the system or by using an energy approach for the calculation of structural damping. In the present paper the second method is used, where the numerical solution of the undamped system provides the undamped modal frequencies and the relative mode shapes of the beam structure. The modal loss factor for the assumed Kelvin damping is calculated as the following ratio of the dissipated to the maximum stored modal energy in the structure:

$$\eta_m=\frac{\omega_m}{2\pi}\frac{\bar{\mathbf{U}}_\mathbf{m}^\mathbf{T}[\mathbf{C}(\mathbf{u})]\bar{\mathbf{U}}_\mathbf{m}}{\bar{\mathbf{U}}_\mathbf{m}^\mathbf{T}[\bar{\mathbf{K}}(\mathbf{u})]\bar{\mathbf{U}}_\mathbf{m}} \tag{15}$$

where ω_m and $\bar{\mathbf{U}}_\mathbf{m}$ are the undamped modal frequency and modal displacement vector, respectively.

4 Numerical results

The developed beam finite element was evaluated through a series of experimental cases on a composite $[0_2/90_2]_s$ Glass/Epoxy cross-ply specimen [7]. The finite element code was formulated using the displacement control method and the Newton-Raphson iterative technique. Regarding the experimental procedure (Fig. 2), the beam was attached on a hydraulic uniaxial testing machine MEYES 100KN with both ends being clamped by hydraulic wedge grips; one remaining immovable while an in-plane displacement was applied to the other end at a rate of 0.01mm/min and during the load application vibration analysis tests were performed.

Figure 2. Testing apparatus for the buckling experiments of Glass/Epoxy beam specimen.

Fig.3 shows the transverse deflection versus the applied compressive displacement for two sets of measured data (I and II) and finite element predictions. The displacement was calculated as the reaction force at the node where the imposed compressive displacement was applied. An initial w_0 range of 0.1-0.3mm was observed in the tested beam. In order to identify the sensitivity of the beam response to the initial imperfection, predicted results for w_0=0.14mm, w_0=0.28mm and w_0=0.55mm are presented.

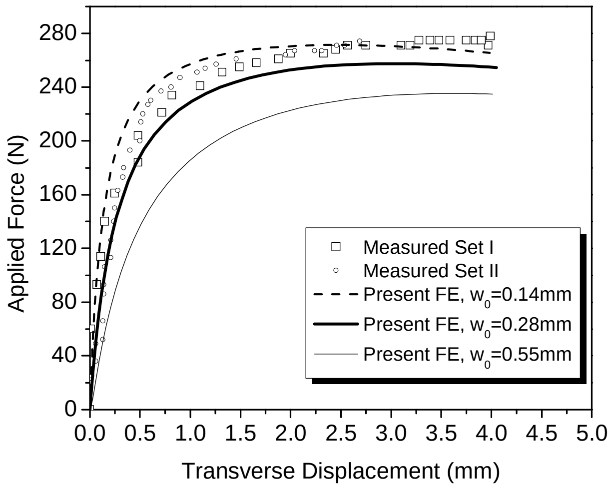

Figure 3. Predicted and measured transverse displacement at the midspan of the $[0_2/90_2]_s$ clamped-free Glass/Epoxy plate-strip subject to in-plane compressive displacement along its axis.

Fig. 4 shows the variation of the first bending modal frequency for increasing compressive load. It is obvious that as the buckling path transitions from the pre- to post-buckling region, the natural frequency decreases and then increases, respectively. The higher the initial imperfection at the midspan the less severe is the aforementioned transition in modal frequency, a conclusion reported also by Kosmatka [8]. The credibility of the developed beam finite element is validated by the excellent correlation of the predicted results with the experimental measurements, for the case of initial w_0=0.28mm.

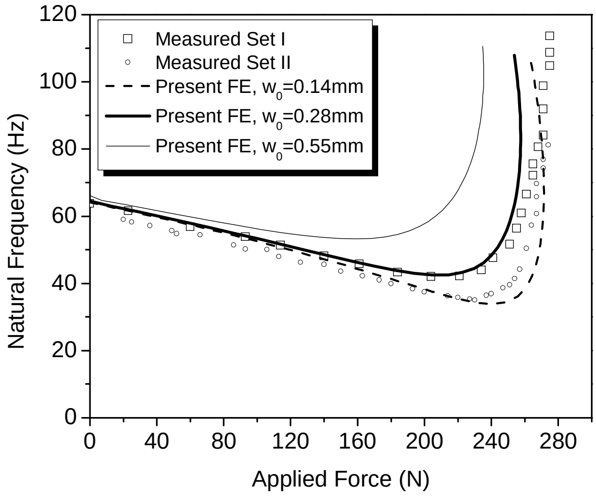

Figure 4. Predicted and measured first bending modal natural frequency of the $[0_2/90_2]_s$ clamped-free Glass/Epoxy plate-strip subject to in-plane compressive displacement along its axis.

The new capabilities of the developed beam element are clearly illustrated in Fig. 5, where the first modal loss factor of the composite beam is shown.

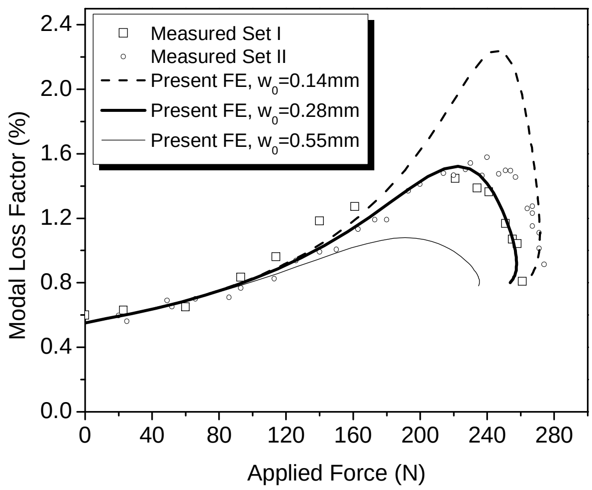

Figure 5. Predicted and measured first bending modal loss factor of the $[0_2/90_2]_s$ clamped-free Glass/Epoxy plate-strip subject to in-plane compressive displacement along its axis.

The variation of the modal damping is not monotonic. Within the pre-buckling region the modal damping gradually increases, reaches its maximum value near the critical load and thereafter it follows a decreasing path as the beam regains stiffness in the post-buckling regime. The predicted results are in excellent agreement with the experimental measurements for the case of initial w_0=0.28mm.

5 Concluding remarks

The theoretical and computational framework of a damped nonlinear beam finite element was presented to predict the dynamic response of composite beams under large in-plane buckling loads. The aforementioned nonlinear damping mechanics were incorporated into an updated research finite element code enabling computational prediction of the nonlinear damping and stiffness of composite laminated strips. New first- and second-order nonlinear damping and stiffness terms were formulated to predict the small-amplitude free-vibration response of composite strips in the pre- and post-buckling region. The new beam finite element captures the effect of stress-stiffening and large rotations on the natural frequencies and especially on modal loss factor values of composite strips subject to in-plane buckling loading.

Both analytical and experimental results show that compressive loads may drastically change the damping of composite structures. The modal damping increases monotonically in the pre-buckling range, reaches a maximum at the critical load and then decreases in the post-buckling region. The credibility of the new finite element is further highlighted by the good correlations between predicted results and experimental measurements which also give credence to the Kelvin viscoelastic strain model, to provide good modal damping predictions in the buckled strip.

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