DEBONDING IN REINFORCED COMPOSITE BY AN XFEM COHESIVE MODEL

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Keywords: debonding, XFEM, cohesive model, composite

Abstract

Debonding in long fiber reinforced composites (FRC) is studied by the eXtended Finite Element Method (XFEM) and the cohesive zone model. The Level-set function is used to localize the fiber/matrix interfaces and to enrich the discontinuities. For instance, the jump in deformation at the interface is enriched by the absolute function while the jump in displacement is enriched by the Sign function. Hence, in presence of debonding the weak enrichment is automatically replaced by the strong enrichment. The cohesive model ensures the transition between the perfectly bonded interfaces and the debonded ones. This approach provides high performances to simulate the interfacial debonding crack. In addition, it is sensitive to the interaction between the fibers. The obtained results are compared with the existing analytical and others numerical methods.

1. Introduction

In this study we focus on fiber/matrix debonding which is a complex micro-structural cracking process and affects locally the composite mechanical behavior. Regarding the existing numerical studies of fiber reinforced composites (FRC), the conventional finite elements methods was widely used. Numerical methods are of large interest in dealing with complex geometries and/or inhomogeneous media, mainly in presence of evolving geometries or growing cracks [1]. In these extreme cases, even algorithms using remeshing suffer from difficulties e.g. elements inadmissibly distorted, high computational time, and requirement of the mesh to conform to cracks. To overcome the burdensome task of remeshing, projection of variables between the different meshes and difficulties in post-process, the extended finite element method (XFEM) is successfully introduced; see [2-5]. It alleviates the FEM drawbacks, enhances accuracy and reduces computational time. The present study focuses on fibers/matrix debonding using the XEFM and the cohesive model. The Level-set function is used to localize the position of the fibers within the matrix, to enrich the weak discontinuity along the interfaces and to enrich the strong discontinuity where the cracks occur. To highlight the effect of fibers interaction on the fiber/matrix debonding, the simulation considers nucleation of crack and its growth along the fibers/matrix interfaces for several example tests.

2. XFEM approximation

The displacement approximation using XFEM consists of three terms; the first term represents a continuous contribution, the second one represents the displacement jump through cracks, and the third one is used to describe the jump in deformation (displacement gradient) at the interface.

$$\mathbf{u}^{\mathbf{h}}(\mathbf{x}) = \sum_{\mathbf{i}\in \mathbf{N_u}} \mathbf{N_i}(\mathbf{x})\mathbf{u_i} + \sum_{\mathbf{i}\in \mathbf{N}_a} \mathbf{N_i}(\mathbf{x})\mathrm{H}_\mathbf{i}(\mathbf{x})\mathbf{a_i} + \sum_{\mathbf{i}\in \mathbf{N}_b} \mathbf{N_i}(\mathbf{x})\ \mathbf{G_i}(\mathbf{x})\mathbf{b_i} \tag{1}$$

$N_i(\mathbf{x})$ are the standard shape functions, N_u is the total number of nodes within the domain and $\mathbf{u}_i$ are nodal displacement, H_i is the displacement enrichment function of nodes in the vicinity of cracks, and G_i is the displacement enrichment function of nodes in the vicinity of interfaces, $\mathbf{a}_i$, $\mathbf{b}_i$ are the added degrees of freedom, H_i is represented by the sign of the level-set function. While the absolute function of the level set function [6] is used as deformation enrichment function G_i at the interface. The sign function, the level set function, the strong enrichment function and the weak enrichment (Absolute function) are given respectively by:

$$\mathrm{sign}(\mathbf{x}) = \begin{cases} 1 & \text{if} \quad \mathbf{x}>0 \\ -1 & \text{if} \quad \mathbf{x}<0 \end{cases} \tag{2}$$

$$\phi(\mathbf{x}) = \mathrm{sign}\left[n\cdot\ \mathbf{x}-\mathbf{x}_I \ \right] \tag{3}$$

$$H_i(\mathbf{x}) = \mathrm{sign}(\phi(\mathbf{x})) - \mathrm{sign}(\phi(\mathbf{x}_i)) \tag{4}$$

$$G(\mathbf{x}) = \sum_i N_i \left|\phi_i(\mathbf{x})\right| - \left|\sum_i N_i \phi_i(\mathbf{x})\right| \tag{5}$$

In the present study, an exponential cohesive traction-separation law is used. The damage starts when the maximum interfacial strength σ_c is reached which corresponds to the opening δ_0 and ends when the opening reaches the maximum value of δ_c which corresponds to zero traction.

3. Governing and discretized equations

Using the constitutive law, the energy balance and the XFEM approximation, the weak form of the equilibrium state can be simplified as:

$$\int_{\Omega} \mathbf{B}^T C\, \mathbf{B}\, d\Omega.\mathbf{v} = \lambda \int_{\Gamma_t} \mathbf{N}^T \cdot \bar{\boldsymbol{t}}\; d\Gamma - 2\int_{\Gamma_c} \tau(\delta)\, \mathbf{N}\, \mathbf{n}\; d\Gamma = 0 \qquad (6)$$

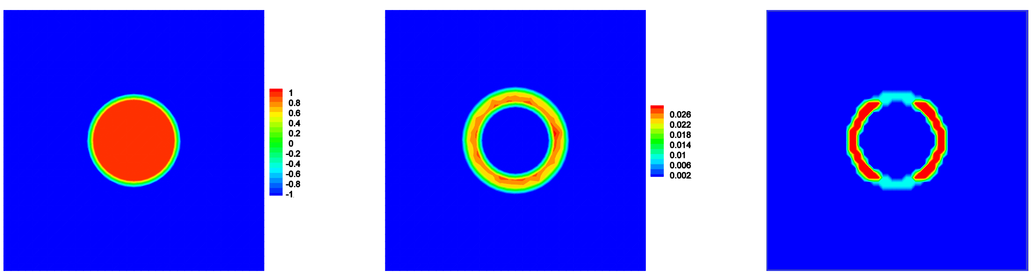
Where: C is the material stiffness tensors Γ_t, Γ_c are boundaries of the applied traction forces and crack surface respectively, **v** is the vector of the generalized displacements, **B** is the strain matrix, **N** is the shape function, λ is a parameter to control the loading, **n** is the normal vector.
The crack opening in the local coordinates δ is calculated from the jump in displacement as follow:

$$\delta = \mathbf{n}.(\mathbf{u}^+ - \mathbf{u}^-) = 2\mathbf{n} \sum_{i \in N_a} N_i(\mathbf{x})\mathbf{a}_i \qquad (7)$$

The integration is performed using Gauss quadratures and element subdivided technique where the intersection of the zero-level set function with the element edges is find using the following equation:

$$\sum_i N_i(\mathbf{x}_{\text{int}})\phi_i = 0 \qquad (8)$$

The obtained non-linear problem is solved by iterative schemes.



(a) Sign function (b) Absolute function (c) Debonding evolution

Figure 1: One singular fiber model

3.1. Crack growth criterion:

For perfectly bonded interfaces, we use the normal and the shear (σ_n, σ_t) tractions resulting at the interface. Then, the failure initiation is given by:

$$\sqrt{\left(\frac{\sigma_n}{\sigma_{nc}}\right)^2 + \left(\frac{\sigma_t}{\sigma_{tc}}\right)^2} \geq 1 \qquad (9)$$

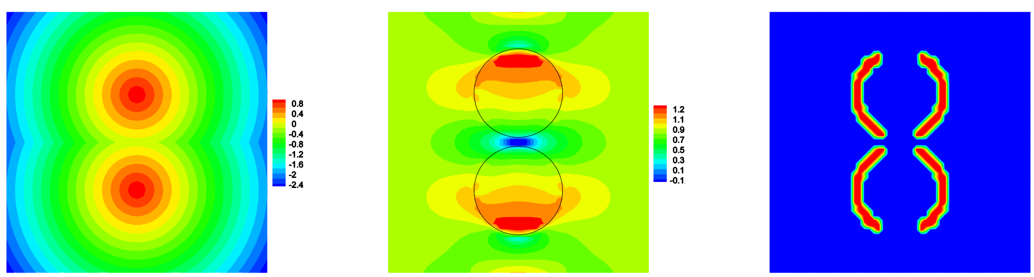
Where σ_{nc} and σ_{nt} are strengths of the interface in the normal and the tangential directions. The commonly way to involve both the toughness and the strength is the use of the cohesive model.

4. Numerical results

4.1. Single fiber model:

The problem studied in [7] is reinvestigated in this test where the composite is subject to a remote force along the horizontal direction. The absolute function is active from the beginning of the simulation since the fiber/matrix interface is perfectly bounded. Once the crack occurs in the fiber/matrix interface, the Heaviside function becomes active and the absolute function deactivate along the cracked part. The permutation between the absolute function and Heaviside function is ensured by the cohesive law.

Fig. 1-c shows the crack growth at the fiber/matrix interface. The crack onset takes place where the concentration of stresses is maximal. Due to uniform distribution of the calculated stresses on both sides of the fiber symmetry is found. The maximum fiber/matrix debonding semi-angle obtained is about $\alpha = \pi / 3$.

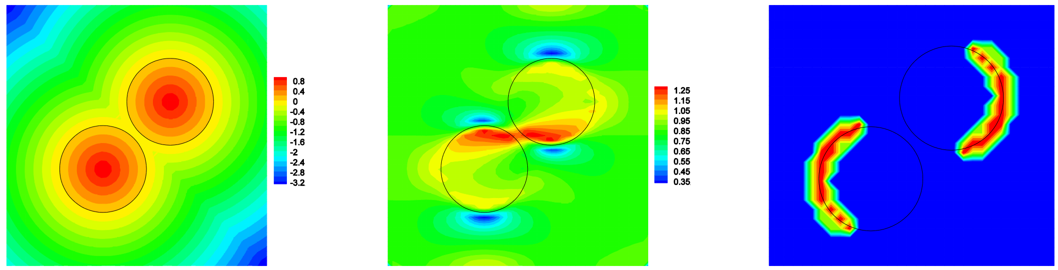


(a) Level set function (b) Stress contour in load direction (c) Debonding evolution

Figure 2: Vertical double fibers model $\alpha = \pi / 2$

4.2. Double inclusions model:

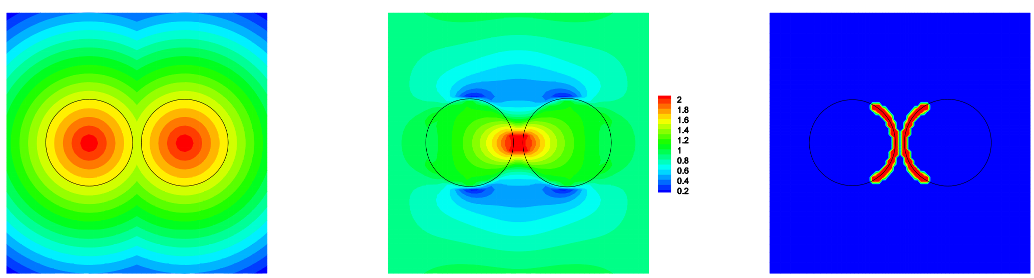
To take into account the interactions between fibers, a 2D model consists of two fibers reinforced matrix is considered. The goal is to show the influence of the fibers configuration within the matrix on the debonding evolution. Different scenarios are supposed and handled by the parameter α which is the angle between the line connecting the two fiber centers and the direction of loading.



(a) Level set function (b) Stress contour in load direction (c) Debonding evolution

Figure 3: Inclined double fibers model $\alpha = \pi / 4$

Three cases are considered: $\alpha = \pi / 2$, $\alpha = \pi / 4$ and $\alpha = 0$, but the distance between the center of the two fibers is kept the same. The results are compared with those in [8]. In case of $\alpha = \pi / 2$, the progressive debonding is shown in Fig. 2-c, where the debonding behavior and the maximum length of the debonded arcs are not sensitive to the interactions between the two fibers. In case of $\alpha = \pi / 4$, Fig. 3 shows the results relative to this current configuration. Interface debonding occurs only in the two external arcs. In case $\alpha = 0$, (Fig. 4), interface debonding occurs also in two arcs but the internal ones.



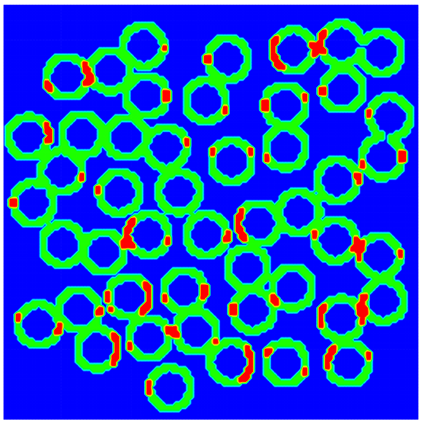
(a) Level set function (b) Stress contour in load direction (c) Debonding evolution

Figure 4: Horizontal double fibers model $\alpha = 0$

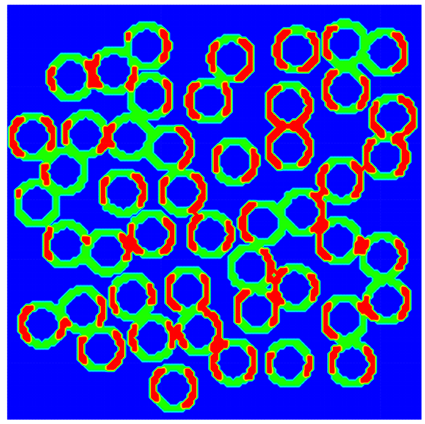
4.3. Multi-inclusions model:

This section shows the robustness of the XFEM method in modeling multi-fibrous composites. We consider a finite matrix containing a large number of fibers randomly distributed. The objective is to visualize the fibers distribution in the matrix using the preceding functions i.e. the sign function and the Level-set function. To analyze the debonding crack, we consider two plates containing 48 randomly distributed fibers having the same dimensions but different fibers diameters. Furthermore, the properties of all the samples are kept the same as in the single model. The samples are subjected to two different loading tensile tests where ($\sigma^{\infty} = 0.15$) and

($\sigma^{\infty} = 0.25$). This parametric study shows the effect of the fibers size and distribution on the growth of debonding, see Fig. 5. Thus, for the same loading the response is different from a configuration to another, this is due to the change in interaction between the fibers. In Fig. 5 the debonding is random and some fibers are stressed more than others. Under low loading some of them remain perfectly bonded to the matrix. These last agree well with experimental observations.



(a) $\sigma^{\infty} = 0.15$



(b) $\sigma^{\infty} = 0.25$

Figure 5: Multi fibers randomly distributed model

5. Conclusion

The eXtended Finite Element Method is implemented to study the debonding growth in Fiber Reinforced Composites. The cohesive model was used to growth progressively the debonding. The fiber/matrix interface is defined implicitly by the Level-set and the bonding is enforced by the absolute function. However, the Heaviside function substitutes the Absolute function where the crack occurs to enforce the jump in displacement. This technique is efficient to track the debonding without any external intervention. It is found that the fibers configurations influences the stress distribution in the vicinity of the interfaces and changes the debonding growth process. Moreover, the inter-fiber distance plays a decisive role for the debonding nucleation and onset. The obtained results are in close agreement with those that use other classical techniques.

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