Weibull statistics of bamboo fibre bundles: methodology for tensile testing of natural fibres

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Abstract

In order to characterize bamboo (Guadua angustifolia) fibres mechanically, single fibre (SFT), dry fibre bundle (DFB) and impregnated bundle (IFB) tests were performed. It was found that the strength of the fibres follows a Weibull distribution, furthermore showing that the variability of the material (obtained from the shape parameter m=8.3) is relatively low. The tensile strength of the fibres was around 800 MPa (SFT). In order to determine experimentally the two Weibull parameters from load-strain (P-ε) curves in the DFB, it was found that the alignment of the fibres and therefore the uniform tensile loading of the fibres is a critical aspect in order to obtain consistent results. For this, a methodology including techniques to evenly spread the fibres in a UD configuration and strain measurements with an optical extensometer, were developed. For the DFB configuration the tensile strength of the bamboo fibres dropped 20% in comparison with the SFT and the Weibull shape parameter reduced to 5.3, indicating higher variability. In the case of the IFB, the characteristic tensile strength (σ_o=254 MPa) was found 14% higher than the results from standard UD bamboo fibre/epoxy composites previously reported. The results allow formulating guidelines regarding methodology and practical feasibility of the different tensile tests (i.e. maximizing reliability with minimal preparation time and amount of material) in order to characterize not only bamboo fibres but also, other natural fibres.

1 Introduction

Natural fibres are in comparison with synthetic fibres emerging as good alternative reinforcing materials because of their bio-based character, low density and in some cases better specific mechanical properties. Nevertheless, one of the main concerns for massive industrial application of natural fibres is the variability in their mechanical properties from batch to batch depending on the time and place of harvest [1, 2]. An inverse fibre strength-diameter correlation [3] has been found and a certain stress/span length dependency of the fibres under tensile loading [4] that can be used as a measurement of the defect density in the fibres and can be calculated as the slope of the plot of the tensile strength vs. test length [5].

Apart of this, in contrast to synthetic fibres such as carbon and glass, natural fibres exhibit much larger variation not only in terms of morphology, surface characteristics, mechanical, physical and chemical properties but also, they present a geometry variation between fibres and within a certain fibre diameter [3-7], which also can affect their properties.

To fully explore the potential of using bamboo fibres for composite materials, a novel process for extracting bamboo fibres has been developed [8] in order to obtain high quality long fibres. However, these new fibres require a complete strength statistical mechanical characterization for better understanding of the fibre properties and good estimation of the composite behaviour.

As for other natural fibres, a single (technical) bamboo fibre can be considered as a concatenation of small pieces (elementary fibres). A very common way to process and describe the "weakest link character" of natural fibres in single fibre tensile testing, is the use of the Weibull distribution [9]. The application of Weibull statistics can be made only if certain requirements are fulfilled. One of the most important is the brittleness of the material, meaning that the material does not show plastic deformation before it breaks and the assumption that the strength is governed by the presence of the most serious flaw; like a chain breaking if the weakest link fails [10,11].

Weibull distribution is widely used to describe tensile strength of brittle materials such as glass fibres [18]; recently this theory has been used to analyze the properties of a wide range of natural fibres such as jute [12,13], hemp [14], flax [11] and coconut [15] and bamboo [17] fibres. Strength distribution σ for natural fibres is usually described by means of a two parameter Weibull distribution [9]. Its general form:

$$P(\sigma, L) = 1 - exp\left[-\frac{L}{L_o}\left(\frac{\sigma}{\sigma_o}\right)^m\right] \quad (1)$$

Where P is the probability of failure of a fibre of length L at a stress less than or equal to σ. The constants σ_0 and m are the scale parameter or characteristic strength of life and the shape parameter, respectively. The scale parameter is called as such because it scales sigma (σ) corresponding to the fracture stress of the fibre. m is the shape parameter, also called the Weibull modulus, which defines how wide (shape) is the function. L_0 is the reference length (the minimum length to find a flaw); usually it is considered as unity or as the same length as the scale parameter, in order to simplify the calculations; in this last case the fraction $L/L_0 = L$ [11,12,17].

Rearranging the two parameter Weibull distribution by taking twice the natural logarithm, Equation 1 can be rewritten in a linear form $y=mx + c$:

$$\ln(\ln(1/1 - P)) = m\ln(\sigma) - m\ln(\sigma) + \ln\left(\frac{L}{L_0}\right) \quad (2)$$

A straight line should be found when ln(σ) is plotted against ln(ln(1-P)). The Weibull modulus (m) corresponds with the slope of the plot and the characteristic strength (σ_0) can be calculated based on the intersection with the y-axis via linear regression. The P-value is estimated by a function known as the probability index (Equation 3).

$$P = \frac{i - (3/8)}{n + 0.25} \quad (3)$$

The expected average strength <σ> is calculated following Equation 4, were Γ is the gamma function:

$$\langle\sigma\rangle = \sigma_0\left(\frac{L}{L_0}\right)^{\frac{-1}{m}} \Gamma\left(1 + \frac{1}{m}\right) \quad (4)$$

According to Equation 1, the flaw distribution per unit length is independent of specimen length. Thus, the length dependence of the average strength σ (Equation 4), may be characterized only by the shape parameter, *m*. This parameter is an indicator of the variation in the data: the bigger the value for the shape factor m, the smaller the variation in the data. Synthetic fibres usually have shape factors between 5 and 15, while for natural fibres which intrinsically have more variation in properties, this value ranges between 1 and 6 [16,17].

Apart from the SFT, testing complete fibre bundles has gained increased support in recent years, because the statistical data concerning fibre strength are much more conveniently obtained. This technique has been applied mainly for synthetic fibres such as carbon and glass fibres [18-20]. For natural fibres, Weibull parameters under DBT have not been widely reported in literature, probably because of the difficulty to perform the experiment (e.g. adequate alignment of the fibres) and the processing of the data, taking into account for example the cross sectional variability in natural fibres. The aim of this study is to compare data obtained from different tensile test methods and to establish fibre properties more conveniently and more representative of the situation that prevails in a finished fibre-reinforced composite material.

2 Materials and methods

2.1 Single fibre test:

Bamboo fibres of the species *Guadua angustifolia* (Figure 1a) were obtained from well defined locations in Colombia. Technical fibres (which will be referred to as fibres in this paper), were extracted from the bamboo culms using a newly developed and proprietary extraction process, giving a maximum fibre length between 20 and 35 cm. Figure 1b shows a group of mechanically extracted fibres whose diameter has a range between 90 and 250 µm; the main diameter concentration is around 150 µm. Before tensile testing (SFT), the fibres were visually selected in order to verify the absence of defects along the length of the fibres (Figure 1c).

Figure 1. a) Bamboo *Guadua angustifolia* culm, b) group of technical bamboo fibres after extraction with an average diameter around 150µm and a length of 35 cm and c) single fibres.

The cross sectional area of each individual fibre was determined using both the apparent density (1.4 g/cm^3) and the weight as well as the length of the fibre. The single fibre tensile test was carried out under standard environmental conditions (21 ± 2 °C and 50 ±2 %RH), in a mini-Instron 5943 universal tensile testing machine using rubber-faced clamp grips with 10x30 mm grip faces and a 1kN load cell. Single fibres at different span lengths (5, 10, 30 and 40 mm) were tested in order to evaluate the influence of the gauge length. The crosshead speed was set at 0.85 mm/min. The fibres were pneumatically clamped using a pressure of 5 bar, using rubber clamps specially designed for single fibres, avoiding the use of a paper frame and reducing manipulation during the test. The load was registered during the complete test. At least 35 successful tests were performed for each span length, excluding fibres that broke near the edge of the clamps.

2.2 Dry fibre bundle test:

For the dry fibre bundle test (DFB) (see Figure 2), bamboo fibres where carefully aligned and evenly spread by hand reaching an average areal density of 0.020 ± 0.002 g/cm^2 and thickness of around 0.35 mm, in order to obtain samples of 10 mm width. The bundles were glued at the ends between aluminium foils (griping zone) in order to load as uniformly as possible all the fibres during the tensile test. The speed of the test was set at 0.3 mm/sec with a load cell of 30 kN with 100 mm gauge length. The strain measurement for the bundles was registered with an optical extensometer (LIMESS).

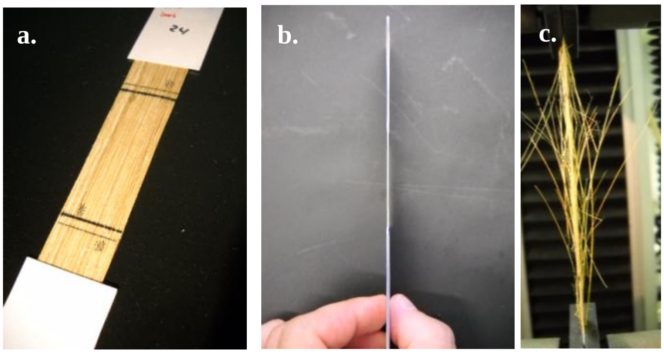

Figure 2. a) Front and b) side view of a dry fibre bundle (DFB). c) Dry fibre bundle after tensile failure.

For the determination of the fibre strength of the fibres derived from the bundle behaviour using Weibull statistics, the methodology described by [20,21] was followed.

2.3 Impregnated bundle test:

The Light RTM technique was used to impregnate UD bamboo fibre bundles with epoxy resin (Araldite LY564) for the IFB test with good surface quality, dimensional accuracy and good degree of alignment (see Figure 3). A fibre volume fraction of 40% was reached, as confirmed by weight measurements.

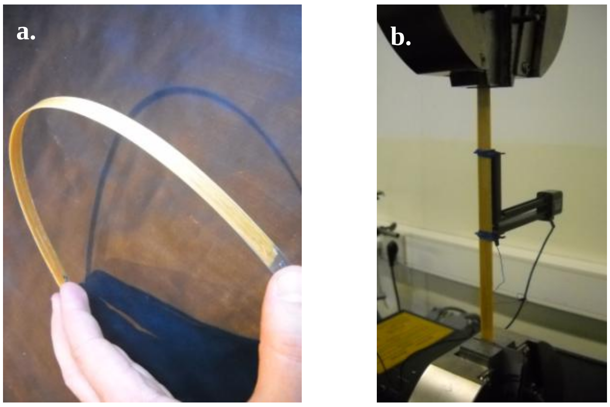

Figure 3. a) Impregnated fibre bundle (IFB) and b) bundle during tensile test

The multicavity mould used had the final sample dimensions (10 mm width and 250 mm length), in order to avoid flaws (cracks) that can act as stress concentrations along the edges after cutting. In total 16 samples with an average thickness of 0.76 mm were tested. Tensile tests were performed on an Instron 4467 with a 30kN loadcell with a gage length of 150 mm. Cross speed was set at 2 mm/min.

3 Results and discussion

A summary of Weibull modulus, m, and the characteristic strength (σ_o) for single fibre, dry fibre bundle and impregnated fibre bundle tests are shown in Table 1.

Sample	Gauge length (mm)	No. of observations	Characteristic strength σ_o (MPa)	Weibull shape parameter (m)
Single fibre	5	40	833±101	9.9
	10	40	836±119	8.3
	35	40	778±122	7.5
	40	35	772±113	8.1
	All fibres	155	804±115	8.2
Dry bundle test	40	4	640±54	5.7
Impregnated bundle	150	16	254±18	15

Table 1. A summary of the Weibull modulus (m) and the characteristic strength (σ_o), for failure stress of bamboo (*G. angustifolia*) materials, obtained from single fibre, dry bundle and impregnated bundle tests.

For single fibres, the characteristic strength as observed at different span lengths remains rather constant; nevertheless a slight decrease is observed when the gauge length increases. This can be explained by the fact that the average flaw size is independent of the gauge length, but the number of defects increases with the length. An Analysis of Variance (ANOVA) indicates that there is not a significantly strength difference between these four single fibre groups (four different span lengths); indicating not only that this property, but also the shape parameter does not appear to be a function of the gauge length.

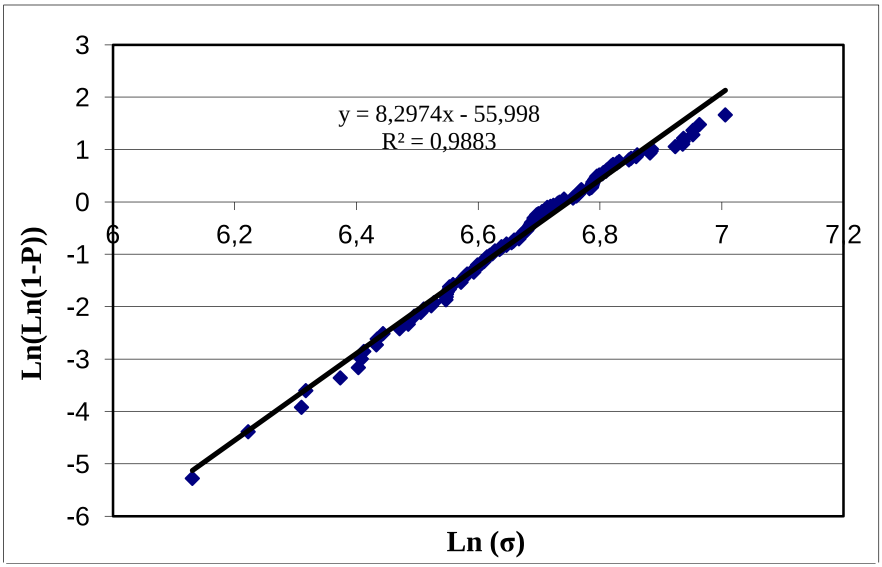

Figure 4. Plot Ln(Ln(1-P)) vs Ln (σ) for determination of Weibull parameters for all single bamboo fibres

Weibull modulus values between 10 and 8 and characteristic strength values of approximately 800 MPa were obtained by SFT. When the four groups of fibres are re-grouped by fibre diameter (4 groups of defined diameter range) and not by gauge length as done before, again the ANOVA analysis shows no statistical difference for the strength values. As stated by [11], the calculated "m" and σ_0 should be independent of the tested fibre length. A variation of the Weibull parameters with length or diameter of the fibres is one of the main concerns when this statistical distribution is applied to natural fibres. The closeness of the data to a straight line (see Figure 4) indicates the applicability of Weibull statistics to the current bamboo fibres. Significant variations in properties are often related to the harshness of the extraction process and the amount of defects that are introduced when the fibres are damaged, lowering their mechanical properties. In this case the newly developed extraction process [8] shows to be gentle with the fibres.

In the case of the DFB it was found that the alignment of the fibres and consequently the uniform tensile loading of the fibres are critical aspects in order to obtain consistent results.

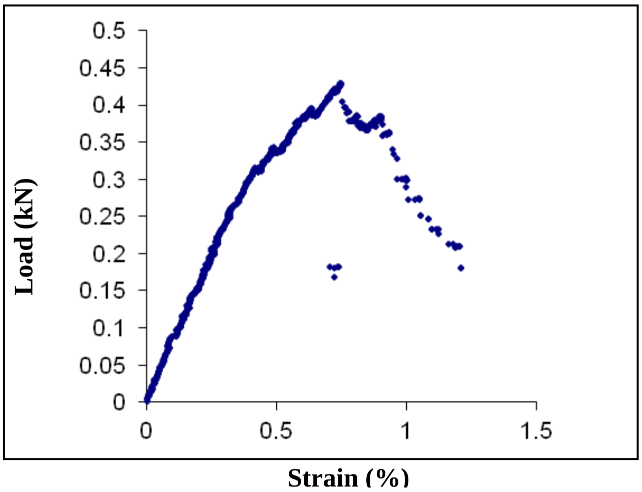

Figure 5. Typical load-strain curve for dry fibre bundle test

The obtained characteristic fibre strength of 640 MPa means a drop of around 20% in tensile strength in comparison with SFT. This result was expected taking into account that there was no pre-selection of the fibres as was done for SFT. It means that all fibres are tested giving a more realistic value or "averaging" of the strength properties because all fibres coming from a specific batch are tested. A typical load-strain curve for DFB is shown in Figure 5.

Figure 6 shows the strength results for the impregnated fibre bundles (IFB) on a Weibull Plot. The bundles showed a linear elastic behaviour until failure. The obtained characteristic tensile strength of 254 MPa is 14% higher in comparison with a typical UD bamboo fire/epoxy composite (also 40% V_f) reported by [21] with the same type of bamboo fibres.

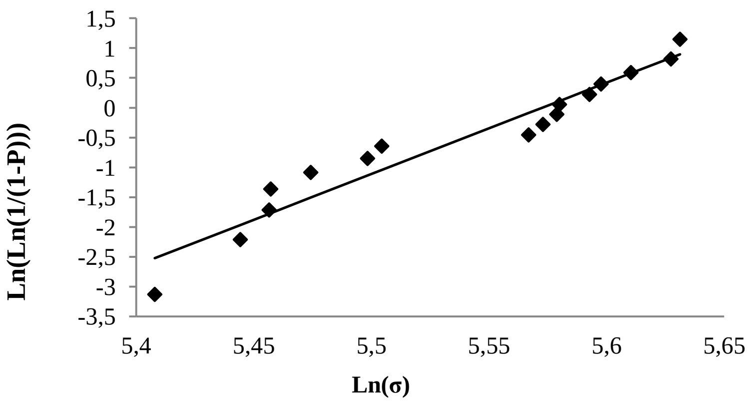

Figure 6. Weibull plot of the strength results for the impregnated bamboo fibre bundle test

This improvement can be attributed to the fact that the volume under tension is much smaller (and therefore the probability of defects in the sample is smaller) and the alignment of the fibres is better controlled.

Finally, the value of the Weibull shape parameter (*m*) of 15, shows that there was relatively little variability in the strength between the different specimens. This may be attributed to an averaging of single fibre properties when a considerable amount of fibres are tested together in a bundle.

4. Conclusions

Methods for determining the Weibull parameters for bamboo fibres were compared. The currently studied bamboo fibres show to follow a Weibull distribution for the strength in single fibre testing (SFT). Also, there was no correlation between the maximum stress and the tested gauge length or fibre diameter. Determination of the Weibull distribution parameters of single fibres from the P-ε curve for a bundle of bamboo fibres (DFB); shows that reliable values of the parameters can be obtained with this method for a bigger population of the fibres with a relatively small amount of experiments. The failure of IFB bundles can also be described using Weibull statistics.

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