FATIGUE SIMULATIONS OF MULTIPLE-FIBRE UNIT CELLS AND MESO-STRUCTURE MODELS OF UNIDIRECTIONAL GFRP COMPOSITES

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Abstract

This paper aims to provide the building blocks for fatigue simulations on glass-fibre reinforced polymer (GRFP) coupons. To this end, 3D simulations of Multiple-Fibre (MF) unit cells consisting of 7 or 45 fibres are used, including the prediction of fibre failure. These simulations are part of a multi-scale simulation approach. The MF unit cells are used to represent the elements of a geometrically larger finite element model (so called Meso-Structure model or MS model). It is shown that the fatigue lives of the MS models with similar geometrical size but different meshing size are comparable, while the normalized axial Young's modulus degradation curves are different.

1 Introduction

Micro-mechanical models are tools in the fatigue behaviour characterization of glass fibre reinforced polymer (GFRP) composites. These materials are frequently used in modern wind turbine rotor blades, which are subjected to severe fatigue loading. Various approaches to the micro-mechanical modelling of tension fatigue behaviour of GFRP composites are possible, depending on the assumptions used to reduce the composite structural complexity and to describe the material characteristics. In this study, a multi-scale modelling methodology is developed for the numerical fatigue characterization of unidirectional GFRP dog-bone specimens. The hierarchical structure of the approach consists of 3 stages, as shown in Figure 1. Firstly it is assumed that, in the first stage of fatigue life, fibre breakages originate independently in random locations [1]. The initiation of these initial microscopic failures is modelled in Multiple-Fibre (MF) unit cells using the Monte-Carlo method. In the next stage, with these initial failures, FE simulations are used to characterize the stress distribution of neighbouring fibres. The fatigue simulations of the MF unit cells are used to predict unit cell fatigue lives and the stiffness degradation. In the final stage, fatigue behaviour of the composite coupons are predicted based on the MF unit cells simulation results by geometrically up-scaling in another two steps, that are Meso-Structure (MS) model fatigue simulations and Coupon Size (CS) fatigue simulations.

In this paper, fatigue simulations on the 7-fibre and 45-fibre unit cells are performed first. The simulation results are used as input parameters for fatigue simulations of two sets of MS models with similar size. Then a comparison between the fatigue simulation results of these two sets of MS models are done. The purpose is to look for the unit cells with a suitable size that can be used in the MS models.

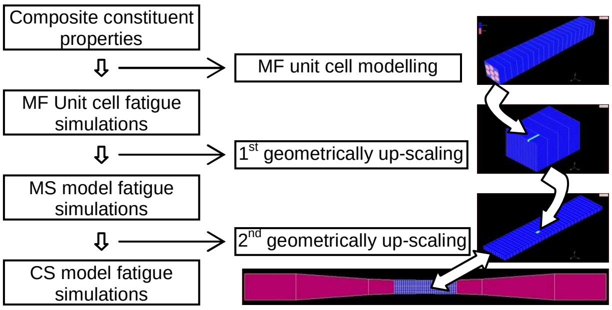


Figure 1. Flow chart of the micro-mechanical fatigue modelling of a UD composite coupon

2 Multiple-Fibre unit cell models

The MF unit cell model is created using a FORTRAN programme "Meso3DFibrer" developed by Mishnaevsky Jr and Brøndsted [1]. A fibre volume fraction of 50% was used. Two types of MF unit cells consisting of 7 and 45 fibres are generated by using the finite element software MSC.MARC, and called 7-fibre unit cell and 45-fibre unit cell respectively. The dimensions of 7-fibre and 45-fibre unit cells are shown in Table 1, in which the length, width and thickness are measured along the x axis (fibre orientation), y axis, and z axis.

	7-fibre unit cell	**45-fibre unit cell**
Length [μm]	400	400
Width [μm]	57	145
Thickness [μm]	57	145

Table 1. Dimensions of MF unit cells

Fibres in a unit cell have a constant diameter of 17.2 μm. Both fibres and matrix are meshed by three-dimensional 20-node brick elements (element type 21 in MSC.MARC), which provide a good trade-off between computational efficiency and accuracy of the strain field calculations [3]. In order to allow distributed fracture locations along fibre axial length, each fibre is evenly divided into 20 segments with the same Young's modulus but Weibull distributed tensile failure strains. The overall view of both unit cell models is illustrated in Figure 2. In both unit cell models, fibres are positioned symmetrically in a square matrix box, in a near hexagonal array with a constant distance from the outmost boundary of surrounding fibres to the corresponding matrix border.

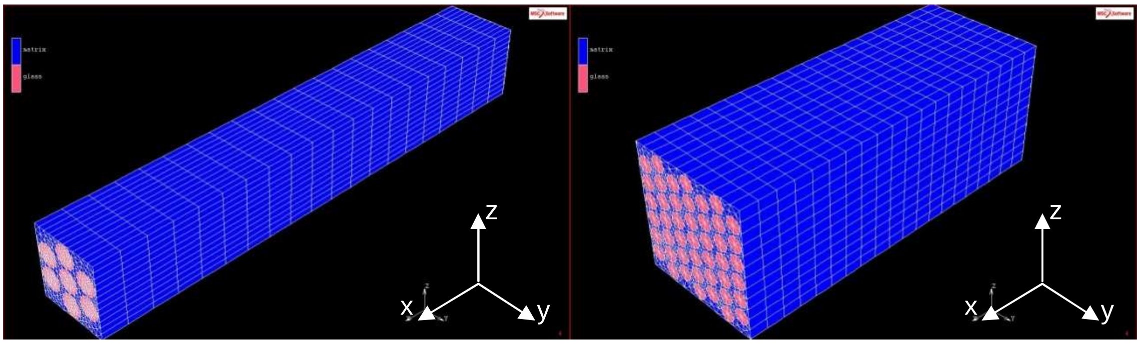


Figure 2. The overall view of 7-fibre unit cell (left) and 45-fibre unit cell (right)

For simplicity's sake, the failure evolution in MF unit cell fatigue simulations is assumed only to occur by the accumulation of fibre breakages and resulting matrix failure. Fatigue propagation of matrix cracks or interface failure is not taken into account. Furthermore, fibres are assumed to fail by either tension or fatigue, while matrix is assumed to fail only due to the breakage of epoxy chemical bonds, rather than fatigue damage. The maximum strain criterion is used to determine the failures of fibre segments and matrix elements. The matrix failure strain is a theoretical maximum failure strain 0.093, which is much higher than the macroscopic failure strain measured experimentally [4].

	Fibre	**Matrix**
Young's modulus [MPa]	73000	3640
Poisson's ratio	0.22	0.34

Table 2. Static mechanical properties of fibres and matrix

The input material properties for fatigue simulations of MF unit cells include the static properties of fibre segments and matrix shown in Table 2, and fibre tensile properties calculated by Eq. (1) and fatigue properties calculated by Eq. (3).

$$P(\varepsilon_t) = 1 - \exp\left\{-\left(\frac{\varepsilon_t}{\varepsilon_0}\right)^{\beta}\right\} \quad (1)$$

where P is the fibre failure probability at tensile strain ε_t, β and ε_0 are Weibull parameters. It is assumed that fibre segment fatigue lives are characterized by a power law S-N model Eq. (2).

$$\log(N) = C_1 \times \log(\varepsilon_{\max}) + C_2 \text{ where } C_2 = -C_1 \log(\varepsilon_t) \quad (2)$$

where ε_t is the fibre tensile failure strain, N is the fatigue life of the fibre segment subjected to the maximum fatigue loads ε_{max}, C_1 and C_2 are S-N slope and intercept. In other words, fibre fatigue is regarded as a deterministic process and the fatigue life of a fibre segment is only determined by its initial static failure strain and the slope of the S-N line. By substituting Eq. (1) into a S-N form model Eq. (2), Eq. (3) is obtained for random fatigue lives calculation of fibre segments.

$$P(N) = 1 - \exp\left\{-\left(\frac{N}{N_0}\right)^{\beta_N}\right\} \text{ where } \beta_N = \frac{\beta}{-C_1} \text{ and } N_0 = \left(\frac{\varepsilon_0}{\varepsilon_{\max}}\right)^{-C_1} \quad (3)$$

where β_N and N_0 are Weibull parameters of fibre segment fatigue lives subjected to the maximum fatigue loads ε_{max}.

The parameters values for Eq. (1) to Eq. (3) are shown in Table 3. The high ε_0 is used because the length of fibre segments is only 20 μm.

Tensile strain parameters	
β	4.9
ε_0 [%]	11.5
Fatigue life parameters	
C_1	-30.0

C_2	28.2

Table 3. Parameters of fibre properties characterization

From the MF unit cell fatigue simulation, the fatigue lifetime and stiffness degradation curves for the unit cell are obtained. Repeated simulations are used in order to characterize the fatigue life distribution of the unit cell, and generate a data pool of stiffness degradation curves. These data are used as input of the Meso-Structure models.

3 Meso-Structure models

The MS models are created by stacking elements of the size of the MF unit cell models. Two types of MS models built up of 7-fibre unit cells and 45-fibre unit cells are generated by using the finite element software MSC.MARC, and called MS7 model and MS45 model respectively. The dimensions of MS7 and MS45 models are shown in Table 4, in which the length, width and thickness are measured along the x axis (fibre orientation), y axis and z axis.

	MS7 model	**MS45 model**
Length [μm]	1600	1600
Width [μm]	1141	1157
Thickness [μm]	971	1012
Number of elements	1360	224

Table 4. Dimensions of MS models

The overall view of both unit cell models is illustrated in Figure 3. Identical to the elements in MF unit cells, the MS model is meshed by three-dimensional 20-node brick elements. Each element represents a typical 7-fiber or 45-fibre unit cell.

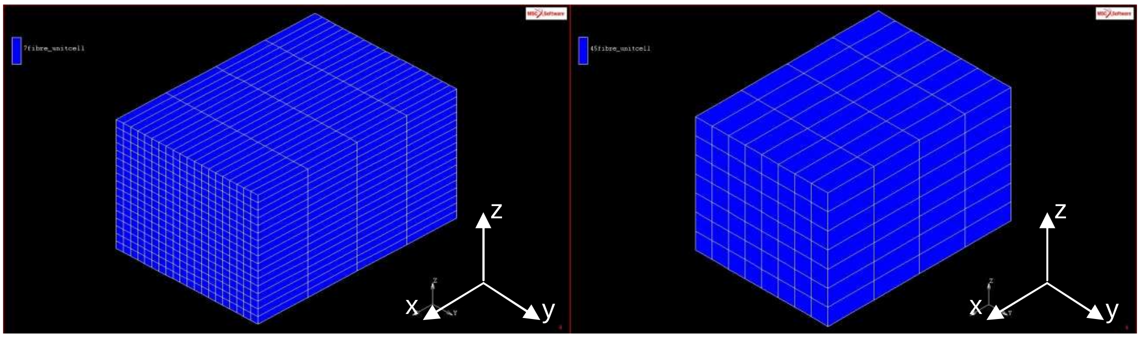


Figure 3. The overall view of MS7 models (left) and MS45 models (right)

The element material properties are based on the simulation results of the MF unit cells, and include the initial static properties, fatigue lives, degradation curves of stiffness and Poisson's ratios. The initial static properties are calculated directly from static simulations of MF unit cells. The definition of element properties degradation is done by randomly selecting a set of degradation curves of a MF unit cell from a predefined data pool, which contains the degradation curves of stiffness and Poisson's ratios of all 20 MF unit cells simulation results obtained for the load level ε_{max} of 3%. The fatigue lives are calculated by Eq. (2) together with Eq. (3), of which the fatigue parameters β_N, C_1 and C_2 are extracted from simulated fatigue lives of 20 MF unit cells. Miner's Rule is used to deal with the spectrum ε_{max} loading problem caused by the uneven degradation of mechanical properties of the surviving elements.

Because the properties of the unit cell along the y-axis and the z-axis are slightly different due to the orthogonal asymmetry, an arbitrary number of elements (MF unit cells) are rotated by 90^{o}. For the rotated elements, the mechanical properties along y-axis and z-axis are exchanged. An example of the element rotation is illustrated in Figure 4, in which a random 7-fibre unit cell is rotated in the MS7 model.

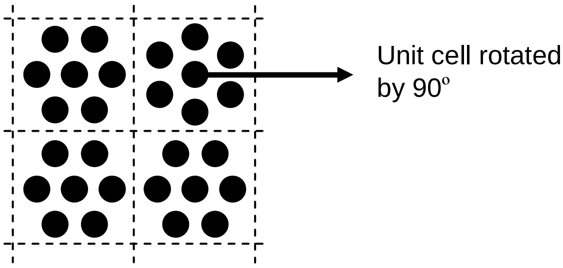


Figure 4. Illustration of a random 7-fibre unit cell rotated by 90°

4 Modelling procedure

The fatigue simulation procedure on MF unit cells and MS models are comparable. Figure 5 shows the general flow chart for both modelling procedures.

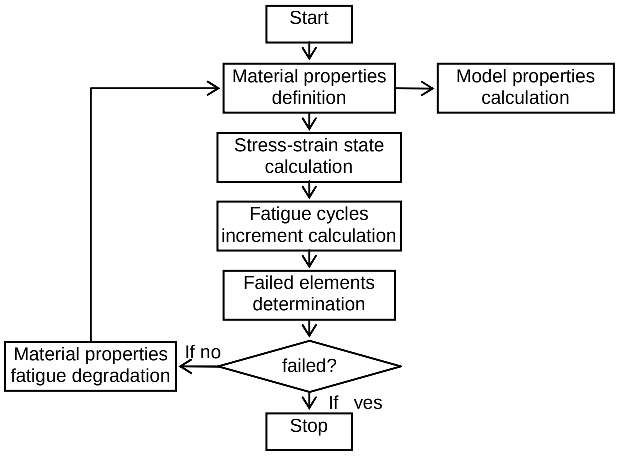
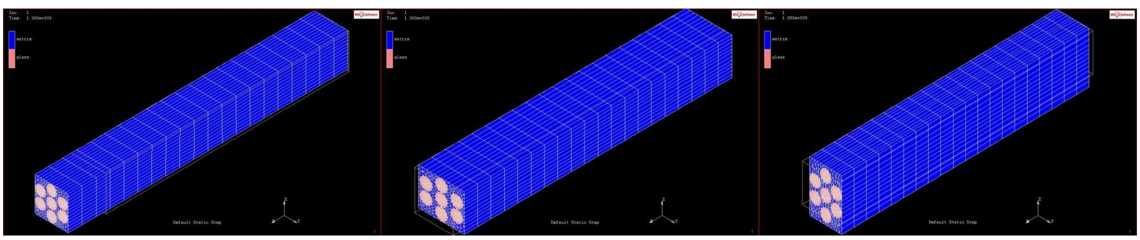


Figure 5. Flow chart of MF unit cell and MS model fatigue simulation

At each iteration, the output material properties of MF unit cells or MS models involve longitudinal Young's modulus E_{11}, transverse Young's moduli E_{22}, E_{33}, Poisson's ratios ν_{13}, ν_{31}, ν_{23}, ν_{32}, ν_{12}, ν_{21}, longitudinal shear moduli G_{23}, G_{32} and transverse shear moduli G_{13}, G_{31}, G_{12}, G_{21} which are calculated from 9 independent finite element simulations subjected to periodical boundary conditions. Tensile and shear deformations of the MF unit cells or MS models are obtained under these boundary conditions, as shown in Figure 6.



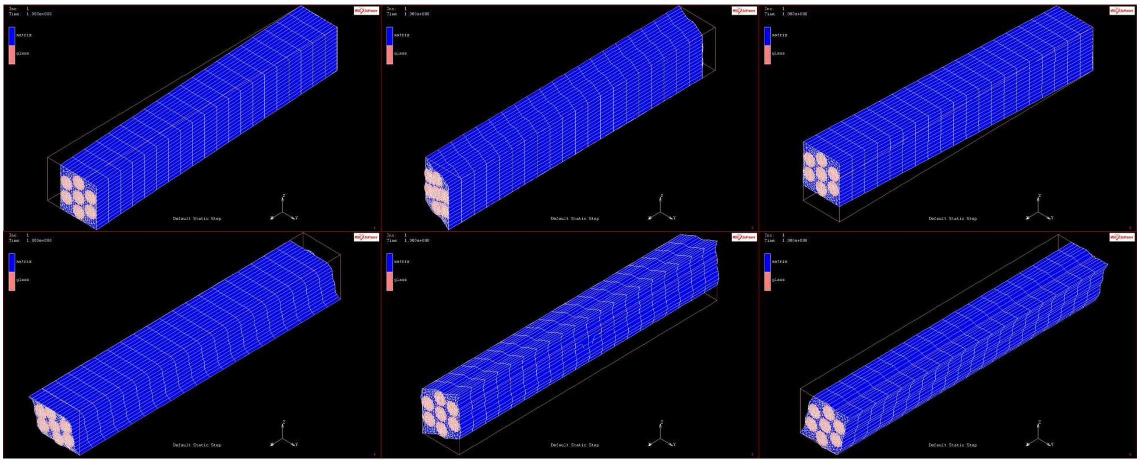


Figure 6. Unit cell deformations under different periodical boundary conditions

The determination of fatigue cycles for each iteration is different between simulations of MF unit cells and MS models. For the MF unit cell simulations, the number of fatigue cycles at each iteration is found by finding the fibre segments with the smallest remaining life. That means, at each iteration, one (or more) fibre segments are indicated broken. For the MS7 model, a similar approach is used by identifying the elements with the smallest remaining life. However, for the MS45 models, the increment of fatigue cycles at an iteration is calculated by 1/20 of the remaining fatigue cycles of the next possible broken element. In the following iterations, if no elements are found broken, the increment of fatigue cycles remains the same. Otherwise, the fatigue cycles increment at that iteration is updated by the remaining fatigue cycles of the new broken elements. Then in the next iteration, the increment of fatigue cycles is re-calculated by 1/20 of the residual fatigue life of the new next possible broken element. This approach is required because otherwise the large element size affects the accuracy of the stress redistribution and stiffness degradation calculations, and consequently of the fatigue life. For the other models this approach is not required because the finer mesh makes them less sensitive to stiffness degradation of individual elements. In all simulations, for practical reasons, the calculated elapsed fatigue life less than 1 cycle is assigned to be 0 cycles.

The simulated model is assumed to fail when all fibres are broken (for MF unit cells) or a throughout elements failure is formed (for MS models). Figure 7 shows iso-surfaces of tensile strain ε_{11} for a 7-fibre unit cell and MS7 model at the final failure. The yellow color indicates the high ε_{11} strains caused by the failure of fibre segments (in the 7-fibre unit cell) or elements (in the MS7 model). Meanwhile, the corresponding normalized E_{11} degradation curves are plotted at the right-down corner, with the black circles showing the stage where the final failure occurs.

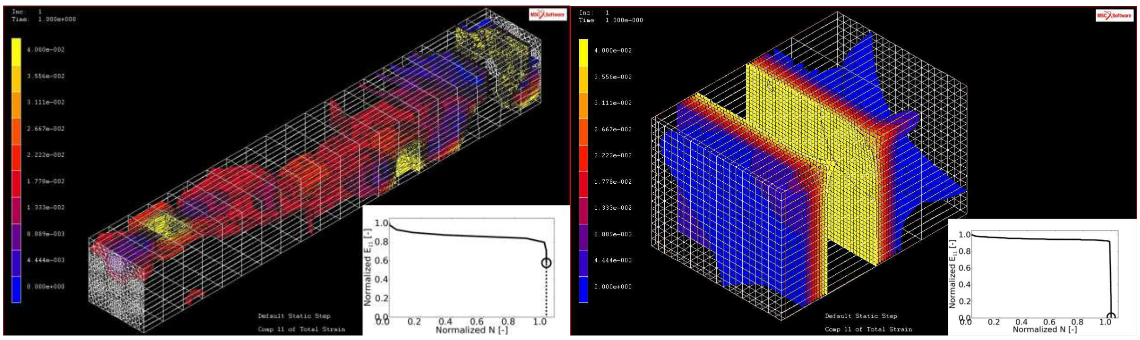


Figure 7. Final failure patterns of a 7-fibre unit cell and MS model

If the final criterion is not reached, the material properties of all broken elements are degraded to a near-zero value (e.g. 0.001). Furthermore, for the MF unit cells, residual tensile strains of surviving fibre elements are calculated by rewriting Eq. (2) to Eq. (4), while the Young's modulus is assumed not to degrade. It is assumed no matrix crack forms and grows in the unit cell. Therefore, all material properties of the surviving matrix elements are not degraded.

$$\log(N_{res}) = -C_1 \times [\log(\varepsilon_{\max}) - \log(\varepsilon_{res})] \tag{4}$$

in which N_{res} and ε_{res} are the residual fatigue life and tensile strain respectively. For MS models, the material properties of the surviving elements are degraded according to the corresponding degradation curves, which are obtained from the MF unit cell fatigue simulations.

5 Results and discussions

The simulated fatigue lives of MF unit cells and MS models are shown in Figure 8 and Figure 9. Extracted fatigue parameters β_N, C_1 and C_2 from these results are shown in Table 5.

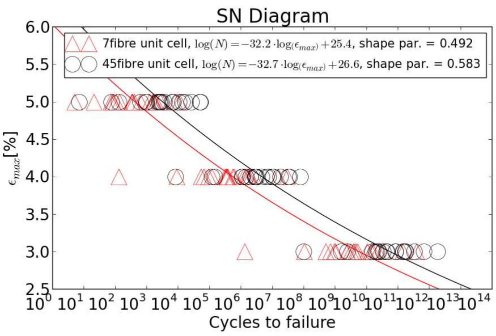


Figure 8. Comparison of fatigue lives of 7-fibre and 45-fibre models

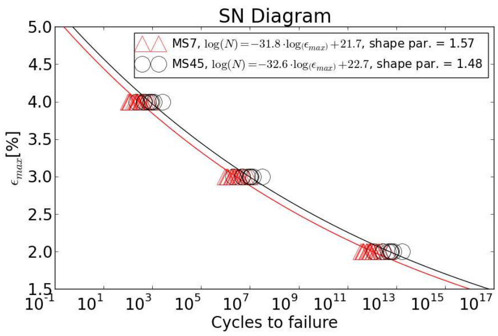


Figure 9. Comparison of fatigue lives of MS7 and MS45 models

For MF unit cells, the 45-fibre unit cell gives higher fatigue life and smaller scatter (higher β_N) than 7-fibre unit cells. The reduced fatigue life scatter can be explained by the large number of fibres, reducing the chance of a unit cell with extreme properties. The longer fatigue lives are caused by the large model size which allows more severe damage progression in the displacement controlled boundary conditions before final failure occurs.

	7-fibre unit cell	45-fibre unit cell	MS7 model	MS45 model
β_N [-]	0.49	0.58	1.83	1.82
C_1 [-]	-32.2	-32.7	-32.2	-32.7
C_2 [-]	25.4	26.6	22.2	22.7

Table 5. Comparison of fatigue parameters between fibres and MF unit cells

For MS models, the fatigue parameters of MS7 and MS45 models are similar. This indicates the modelling approach is consistent within the scale range used. However, difference between the stiffness degradation curves of the MS models can be observed, as in the plots of degradation curves of normalized axial Young's modulus E_{11} shown in Figure 10. The normalized E_{11} are calculated by the ratio between degraded values and initial values, while normalized N are calculated by the ratio between elapsed fatigue lives and final unit cell fatigue lives. It is clearly shown that normalized E_{11} degradation curves of MS7 models are more gradual than those of MS45 models.

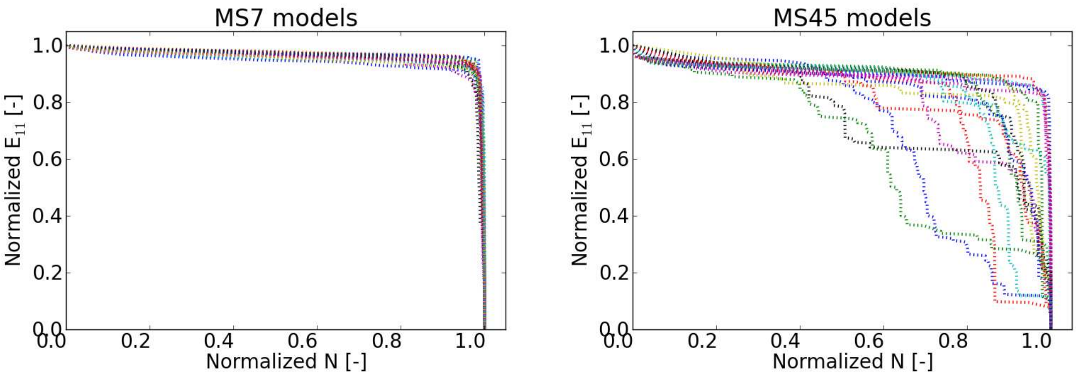


Figure 10. Comparison of normalized E_{11} degradation curves of MS7 and MS45 models

Moreover, it has to be mentioned that the computing time for a 45-fibre unit cell fatigue simulation is much longer than for a 7-fibre unit cell, because the 45-fibre unit cell is larger and more iterations are needed for the fatigue simulations.

6 Concluding remarks

MS7 and MS45 are two types of Meso-Structure numerical models with similar geometrical size but constituted by 7-fibre and 45-fibre unit cells respectively. In a multi-scale micro-mechanical fatigue modelling approach, they are regarded as the basic building block elements of geometrically larger numerical models (i.e. the Coupon Size models).

According to the study in this paper, the fatigue lives of MS7 model and MS45 model are comparable. The use of the MS models based on 7-fibre unit cells is preferred because this size is shown to be sufficient to adequately model the material behaviour and it is computationally more efficient.

Acknowledgement

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