A MULTIFIBRE MULTILAYER REPRESENTATIVE VOLUME ELEMENT (M^{2}RVE) FOR PREDICTION OF MATRIX AND INTERFACIAL DAMAGE IN COMPOSITE LAMINATES

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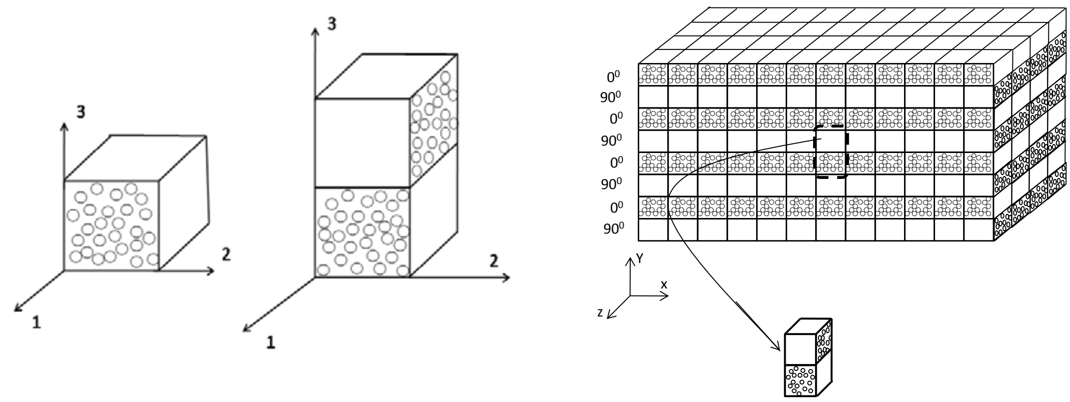
Abstract
Multiscale micro-mechanics theory is extensively used for the prediction of the material response and damage analysis of unidirectional lamina using a representative volume element (RVE). This paper presents a RVE-based approach to characterize the material response of a multi-fibre cross-ply laminate considering the effect of matrix damage and fibre-matrix interfacial strength. The framework of the homogenization theory for periodic media has been used for the analysis of a 'multi-fibre multi-layer representative volume element' (M^{2}RVE) representing cross-ply laminate. The non-homogeneous stress-strain fields within the M^{2}RVE are related to the average stresses and strains by using Gauss theorem and the Hill-Mandal strain energy equivalence principle. The interfacial bonding strength affects the in-plane shear stress-strain response significantly. The material response predicted by M^{2}RVE is in good agreement with the experimental results available in the literature. The maximum difference between the shear stress predicted using M^{2}RVE and the experimental results is ~15% for the bonding strength of 30MPa at the strain value of 1.1%.

1 Introduction
It is important to accurately predict the mechanical behavior of fibre reinforced plastic (FRP) composite laminates for their efficient and reliable use in structural applications. Different failure mechanisms like matrix failure, fibre fracture, and fibre matrix debonding takes place at different scales during the damage process of composites, this leads to complex fracture patterns even for simple loading conditions. Therefore, it is important to carefully study this complex local damage initiation and evolution at the fibre and matrix level. Use of finite element analysis (FEA) based on micro-mechanics theory is a powerful tool, which utilizes a 'unit cell' or 'representative volume element'(RVE)'model for predicting local as well as global behavior of the composite[1].It is possible to simulate failure of the fibre, matrix and interface at the same time using RVE. It is also convenient to study the effect of various parameters on damage response of the composite.
At present, the application of micro-mechanics via a RVE-based model is limited to unidirectional lamina [1].The results obtained for unidirectional lamina are used to predict the

properties of the laminate at different orientations using various laminate theories [2]. Use of such approximate theories restricts the possibility of accurate local and global damage prediction. Most of the RVE-based models typically use a single fibre representing the volume fraction of the fibre in the composite which is not a very accurate representation [1]. However, a few studies have been reported on the prediction of micro-damage via a RVE with multiple fibres [3-4]. The multifibre RVE approach has been used for the prediction of in-plane shear strain response of a cross-ply laminate [3]. This can be achieved by applying a shear loading parallel to the fibre direction in the RVE and subsequently applying a shear loading perpendicular to the fibre direction in the RVE using two different simulations. The shear loading parallel to the fibre is expected to offer less resistance to the deformation as compared with the shear loading perpendicular to the fibres, where rotation of the fibres takes place under this loading. Both the responses are then averaged out to predict the in-plane shear stress-stain response of a cross-ply laminate [3]. Another approach to predict the material response of a laminate is to use a multilayer RVE. Studies on multilayer RVEs have been reported in the literature with an equivalent single fibre representing the entire volume fraction of fibres in the lamina [5-6]. Periodic boundary conditions have been used to obtain the global material response.



(a) **(b)**

Figure 1.(a) Typical RVE and M^{2}RVE (b) Micromechanical representation of an interior element of $[0//90]_{ns}$ cross-ply element

Based on a literature review, it can be inferred that a detailed micro-mechanical model consisting of multiple fibres and multiple layers representing a composite laminate has not yet been implemented. A multilayer multifibre representative volume element (M^{2}RVE) could potentially capture all the damage mechanisms, viz., fibre breakage, fibre-matrix debonding, matrix cracking and delamination for any symmetric laminate. Additionally, it is a better geometrical representation of the lamina as compared to an equivalent single fibre multilayer RVE. The effect of one fibre on the response of an adjutant fibre is considered in a multifibre representation, which is closer to reality. This paper explores the possibility of predicting the mechanical properties and the damage response at laminate level using the M^{2}RVE approach. A 3D two-cell, lay-up structure of a cross-ply laminate is simulated via a multilayer multifibre representative volume element (M^{2}RVE) with periodic boundary conditions as shown in Figure 1(a) was created in DIGIMAT®. The thickness of each cube was kept as the thickness of the lamina to ensure full periodicity in all three directions. An elastic-plastic material model for the epoxy matrix was implemented in ABAQUS Standard®. An M^{2}RVE was subjected to an in-plane shear loading in the present paper. The predictions from the M^{2}RVE model was validated with experimental results reported in the literature [7].

The proposed model was used to predict the material response of a cross-ply laminate incorporating the effect of fibre-matrix bonding strength and matrix failure.

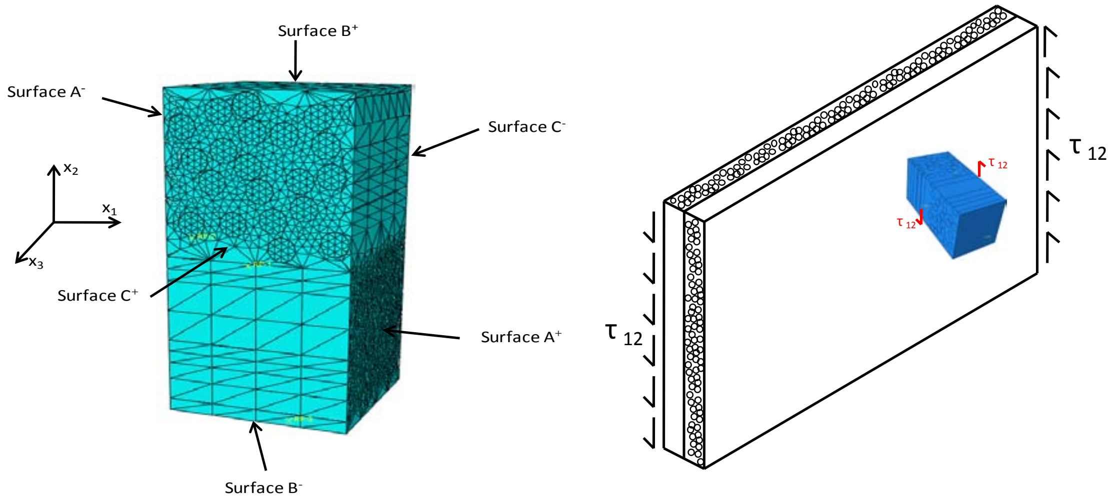
2 Finite Element Modeling of M^2RVE

2.1 Numerical formulation of M^2RVE

A random distribution of random fibres, 17 μm in diameter were generated using a fibre randomization algorithm in DIGIMAT® [12] to obtain a nominal fibre volume fraction of 42.67% [7]. The M^2RVE (matrix and fibres) was meshed using standard tetrahedral C3D4 elements in ABAQUS Standard® [11]. The FE mesh contains 15491 nodes and 54122 elements as shown in Figure 2 (a). Refined elements were used near the fibre/matrix interface to capture the stress gradients. Cohesive surfaces were used at the fibre/matrix interface to simulate the effect of fibre-matrix debonding.

2.2 Boundary and loading conditions

In a composite material, non-uniform stress and strain states will exist even under uniform loading as it is composed of fibres and matrix with vastly different mechanical properties [1-4]. As mentioned previously, the M^2RVE is a representative unit for the cross-ply laminate as shown in Figure 1(b) and therefore, can be treated as a periodic array implemented using periodic boundary conditions. Periodicity implies that each M^2RVE in the composite has the same deformation mode and there is no separation or overlap between the neighboring M^2RVEs. The positions of split fibre sections have been copied on the opposite face of the M^2RVE to ensure the periodicity condition as shown in Figure 2 (a). Perfect bonding has been assumed between the plies.



(a) **(b)**

Figure 2. (a) Schematic of the meshed M^2RVE used for implementation of periodic boundary conditions (b) In-plane shear loading using M^2RVE

The periodic boundary condition applied on the proposed M^2RVE is shown in Figure 2(a). Eq. (1) shows the displacement 'u_i' as a function of applied global loads

$$u_i = S_{ij} x_j + v_i \qquad (1)$$

where S_{ij} is the average strain and v_i is the periodic part of the displacement components u_i on the boundary surfaces (local fluctuation). The indices i and j denote the global three-dimensional coordinate directions 1,2,3. An explicit form of periodic boundary conditions suitable for the proposed M^{2}RVE model has been derived from the above general expression. For the M^{2}RVE as shown in Figure 2(a), the displacements u_i on a pair of opposite boundary surfaces are

$$u_i^{K+} = S_{ij}x_j^{K+} + v_i^{K+} \quad (2)$$
$$u_i^{K-} = S_{ij}x_j^{K-} + v_i^{K-} \quad (3)$$

where 'k$^+$' means displacement along the positive x_j direction and 'k$^-$' means displacement along negative x_j direction on the corresponding surfaces A$^-$/A$^+$, B$^-$/B$^+$, and C$^-$/C$^+$ (see Figure 2(a)). The local fluctuations v_i^{K+} and v_i^{K-} around the average macroscopic value are identical on two opposing faces due to the periodic condition. Hence, the difference between the above two equations is the applied macroscopic strain condition, given as

$$u_i^{K+} - u_i^{K+} = S_{ij}(x_j^{K+} - x_j^{K+}) \quad (4)$$

The non-homogeneous stress and strain fields obtained are reduced to a volume-averaged stress and strain by using Gauss theorem in conjunction with the Hill-Mandal strain energy equivalence principle. Finally, the elastic modulus is obtained as the ratio of the average stress to the average strain. The average stresses and strains in the M^{2}RVE are defined by [8-10]

$$S_{ij} = \frac{1}{2}\int_v s_{ij}\, dV \quad (5)$$
$$E_{ij} = \frac{1}{2}\int_v e_{ij}\, dV \quad (6)$$

where V is the volume of the periodic representative volume element, S_{ij} and E_{ij} are average strains and average stresses in the M^{2}RVE, respectively. Here, s_{ij} and e_{ij} represents local strains and stresses.
Figure 2(b) shows the in-plane shear loading using the M^{2}RVE. The left face of the M^{2}RVE is subjected to a fixed boundary condition. A displacement of 1 mm is applied to all the nodes on the right face of the M^{2}RVE. The material response of the M^{2}RVE was used with periodic homogenization to predict the global response of the structure.

2.3 Material model and failure criteria

All the simulations were carried out in ABAQUS Standard®[11], within the framework of finite deformations and rotation theory with an initial unstressed state as the reference. In the FE analysis, E-glass fibres were modeled as linear elastic, and isotropic solids and the epoxy matrix was assumed to behave as an isotropic, elasto-plastic solid. Elastic constants of fibre and matrix are provided in Table 1 [7].

	E (GPa)	v
E-glass fibres	**72.4**	**0.22**
Epoxy matrix	**3.2**	**0.36**

Table 1. Elastic properties of matrix and fibres [7]

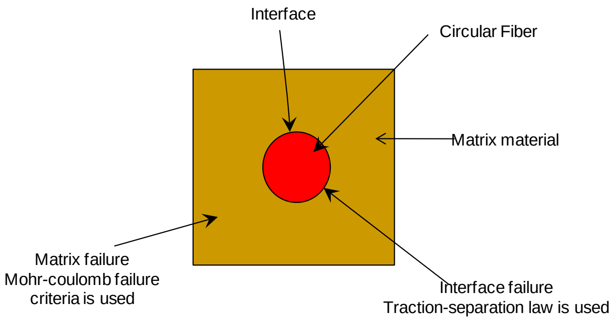


Figure 3. Schematic representation of the failure criterion used for matrix and fibre matrix debonding

During the damage process of the laminates in shear, matrix cracking (transverse cracking) is the first damage phenomenon to take place since the matrix has the lowest stress to failure of all the composite constituents [1-4]. Therefore, for the cross-ply laminate structure, the dominant damage mode is the matrix transverse cracking followed by fibre matrix debonding. Although the M^{2}RVE model discussed here is subjected to uniform in-plane shear loading, a tri-axial stress state exists in the individual elements of the model. Consequently, the Mohr–Coulomb multi-axial damage criterion is used to model the matrix damage as shown in Figure 3. The Mohr-Coulomb criterion assumes that yielding takes place when the shear stress, τ, acting on a specific plane reaches a critical value, which is a function of the normal stress, σ_n ,acting on that plane [3-4].The Mohr-Coulomb criterion can be expressed as,

$$\tau = c - \sigma \tan \varphi \tag{6}$$

where c and φ stand for the cohesion and the friction angle, respectively. These two material parameters control the plastic behavior of the matrix. Physically, the cohesion ,c, represents the yield stress of the matrix under pure shear while the friction angle takes into account the effect of the hydrostatic stresses. In the present work, $\varphi = 15^0$ is used to represent the matrix behavior which is within the range determined by Puck and Schürmann [13] and González and Llorca [3].The value of cohesion , c, is taken as 34.5MPa which is the yield strength of the epoxy [7]. It is assumed that c and φ are constant and independent of the accumulated plastic strain.

The fibre-matrix debonding were simulated using standard traction-separation law using cohesive surface elements with standard traction-separation law . In the absence of damage, the interface behavior is linear with very high initial stiffness to ensure the displacement continuity at the interface. It also avoids any modification of the stress fields around the fibres in the absence of damage.

3 Model Validations

At the end of each load step in non-linear analysis, volume average stresses and strains obtained by using equation (5) and (6) were plotted. The in-plane shear stress–strain curves for the perfect bonding case, obtained from the numerical simulations for the composite, are plotted for two different cases (Figure 4) along with the experimental data for the cross-ply laminates. Due to perfect bonding, the stresses developed in the matrix material gets easily transferred to the fibre material. Consequently, the fibres take more load as compared to a bonding having a finite interfacial strength. The differences between simulations and experiments could be attributed to the assumption of perfect bonding, underestimation of the matrix yield stress and the assumption of no inter-ply delamination.

In addition to the perfect bonding, a finite cohesive strength (t_c = 40 MPa) condition was also simulated. An interface fracture energy, Γ= 100 J/m^2 was used for all the simulations[3]. As the interface strength is close to the matrix yield strength, the initial region of the stress–strain curves for perfect bonding and finite interfacial strength are similar up to a shear strain of 0.9%. Beyond which, the perfect bonding predicts higher stresses, whereas the response from the finite interfacial bonding strength condition approaches the experimental response after a strain of 2.5%.

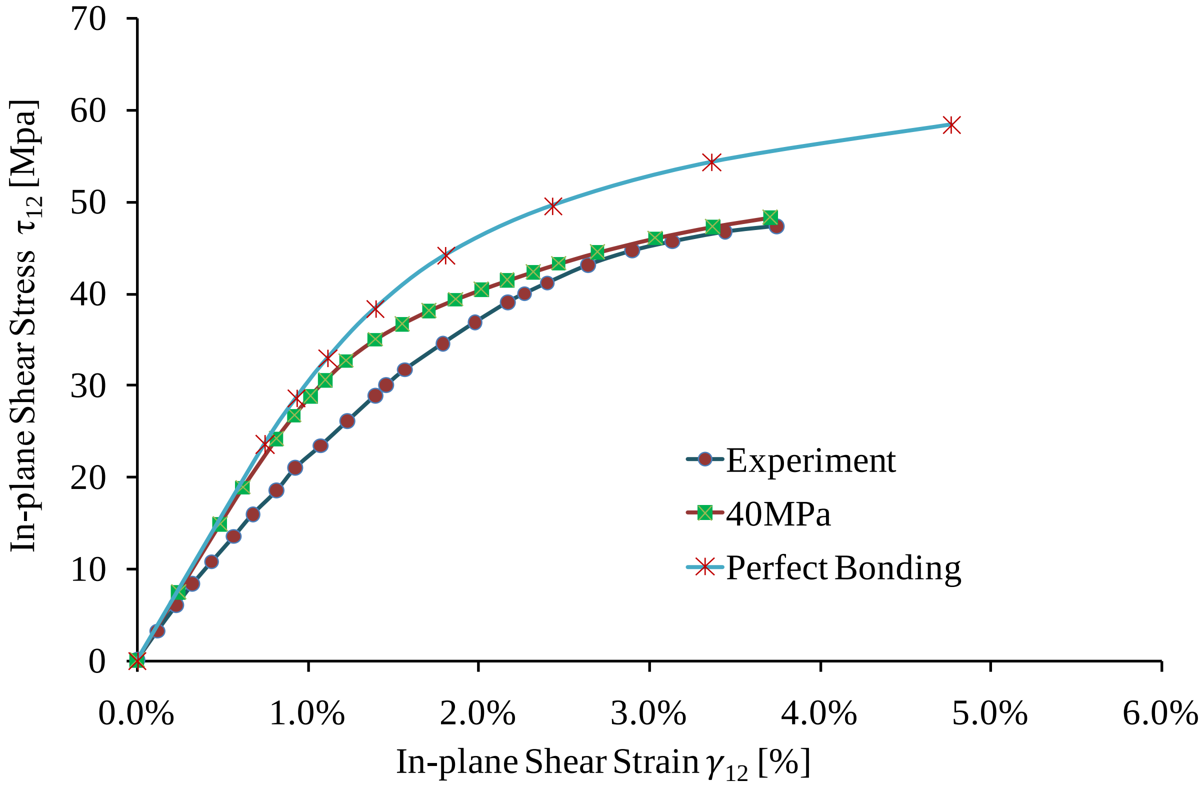
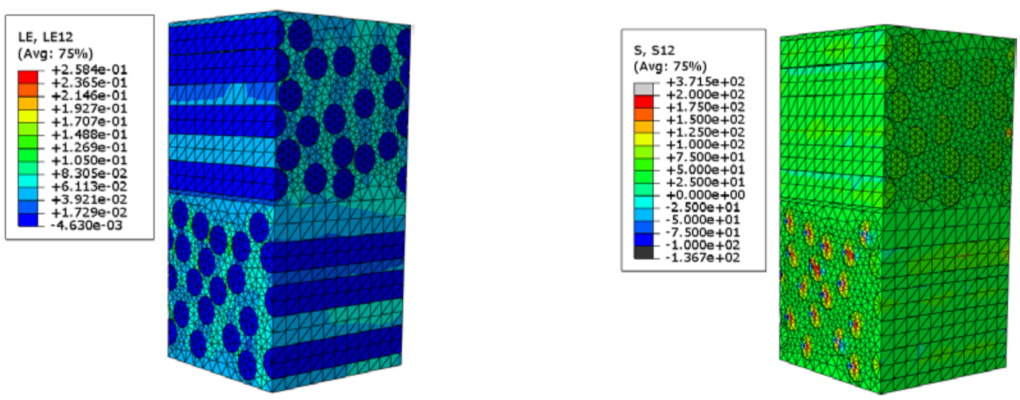


Figure 4. In-plane shear stress-strain response of M^{2}RVE with perfect and imperfect bonding between matrix and fibre

4 Results and Discussions

4.1 Stress and Strain Evolution



(a) **(b)**

Figure 5. (a) Contour plot of the in-plane shear strain in M^{2}RVE with cohesive strength equal to 40 MPa, (b) Contour plot of the in-plane shear stress in M^{2}RVE with cohesive strength equal to 40 MPa.

The top and bottom laminate are referred to as 0^0(parallel to the applied displacement) and , 90^0(perpendicular to the applied displacement) respectively. The strains are shown in Figure 5 (a), it can be seen that similar strain fields are obtained in both the laminae. However, the stress plots shown in Figure 5(b) are very different because the 90^0 lamina is stiffer due to the perpendicular fibre orientation which induces higher stresses as opposed to the 0^0 lamina where the parallel fibres do not provide sufficient stiffness. As expected, the shear stresses developed in the fibres are much higher than those of the matrix in both the cases due to the higher modulus of elasticity (72.40 GPa) for the fibre as compared to matrix (3.2 GPa).

4.2 Interface de-cohesion and the effect of interfacial strength

It can be observed that stresses are transfers to the fibres via the interface and high stresses are developed in the fibres in the case of 90^0 lamina. Interface failure leads to the reduction in the slope of the linear hardening region of the stress–strain curve. The model predictions for the behavior of the cross-ply composite, assuming debonding between matrix and fibre are in very good agreement with the experimental data [7]. In particular, they are able to account for the quantitative effect of damage by interface de-cohesion on the load transfer from the matrix to the fibres.

It is expected that the effect of interfacial bonding strength will affect the in-plane shear response. Hence, different cohesive strength values of 20, 30, 40 and 60 MPa have been used in the simulations. The corresponding response curves are shown in Figure 6. These results shows that the interface de-cohesion limits the load transfer from the matrix to the fibres under in-plane shear loading leading to a reduction in the slope of the linear hardening region after matrix yielding. It is interesting to note that the onset of the softening precedes de-cohesion for all the conditions.

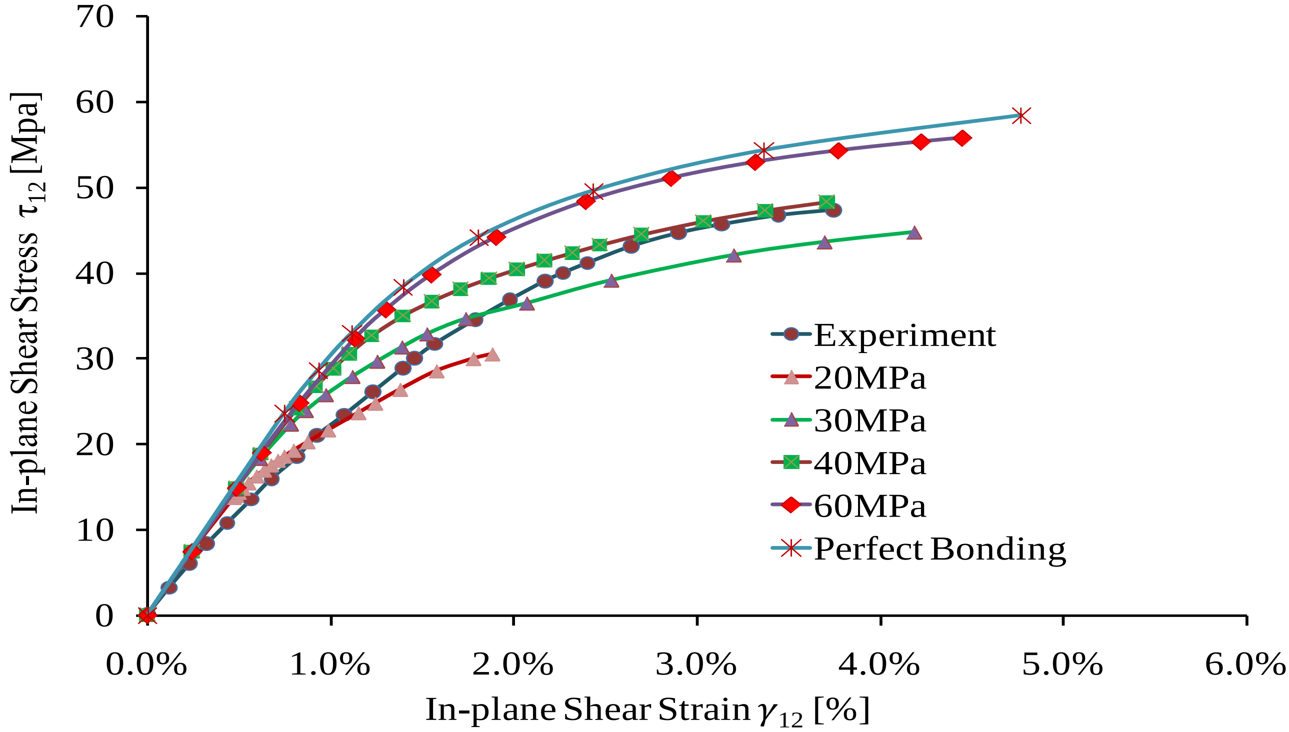


Figure 6. In-plane shear stress-strain response of M^2RVE with different bonding strength between fibres and matrix

5 Conclusions

From the simulations, it can be concluded that the proposed M^2RVE can accurately predict the in-plane stress-strain response of the cross-ply laminates. The in-plane shear stress–strain response of glass-epoxy laminates shows three distinct regimes. The initial, elastic one regime only dependes on the elastic properties and volume fraction of matrix and fibres. It is followed by a non-linear region which begins with the onset of matrix plastic deformation and a plastic regime at shear strains of 3–4%.

Following specific conclusions can be drawn from the current work.

1. In case of perfect bonding between fibres and matrix, the composite shear stresses and stiffness values are fairly high. The use of cohesive surfaces significantly alters the in-plane shear behavior as well as the load transfer between matrix and fibres. Interfacial failure takes place when the cohesive surfaces are used between fibre and matrix.
2. The slope of the linear hardening region and the strain-to-failure decreases rapidly with reduction in interface strength.
3. Laminate failure occurs at very low strain (before matrix yielding) if the fibre–matrix interface shear strength is lower than the matrix yield strength.

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