APPLYING THE SOLID-SHELL CONCEPT TO SANDWICH STRUCTURES

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Abstract

Finite element (FE) simulation of thin sandwich structures with intricate, three-dimensional material behaviour puts a high demand on the underlying element technology. Common solid elements require for very fine discretisations, rendering them numerically inefficient. A recently published novel solid-shell element is applied to an example of sandwich core compression in order to demonstrate the applicability of the formulation to adequately represent the bending dominated deformation mode of the core member struts. The element allows for the use of a fully 3D constitutive law and treats undesirable locking effects, thus permitting coarse discretisations. A study of convergence compares the performance of the proposed formulation to several commercially available element formulations.

1 Introduction

The development of lighter, cheaper and mechanically beneficial structures is constantly pushed forward, particularly by the transportation industry. Sandwich elements match these criteria due to their superior weight-specific stiffness and strength, compared to monolithic structures. Furthermore, their functionality can be extended by including appliances for temperature and moisture regulation or exploiting their outstanding capability of dissipating impact energy.

Structural sandwich elements typically consist of thin top and bottom sheets with a thick and lightweight cellular core placed in between. In order to increase the second moment of area, it is desirable to design the core as thick as possible. However, as the core thickness increases, so does the slenderness of the core members, which increases their susceptibility to stability problems. As can be seen in Fig. 1, the large design space allows for many different shapes, which are typically tailored to a specific application. Hence, research in sandwich core construction involves a large amount of specimen manufacturing and testing, since there is a strong coupling between core cell geometry parameters and bulk mechanical properties.

Numerical simulation of sandwich cores can alleviate this time- and cost-intensive process, given that structure and material are adequately represented. Recently, fibre reinforced composite materials have been introduced in the design of sandwich structures. These materials require for a through the thickness representation of the stress and strain field. Thus, the required constitutive law has to be formulated in 3D.

Classical shell finite elements are advantageous given the thin core and face members, but do not allow for the use of three-dimensional material formulations. Brick elements, however, require very fine discretisations in order to maintain a reasonable aspect ratio.

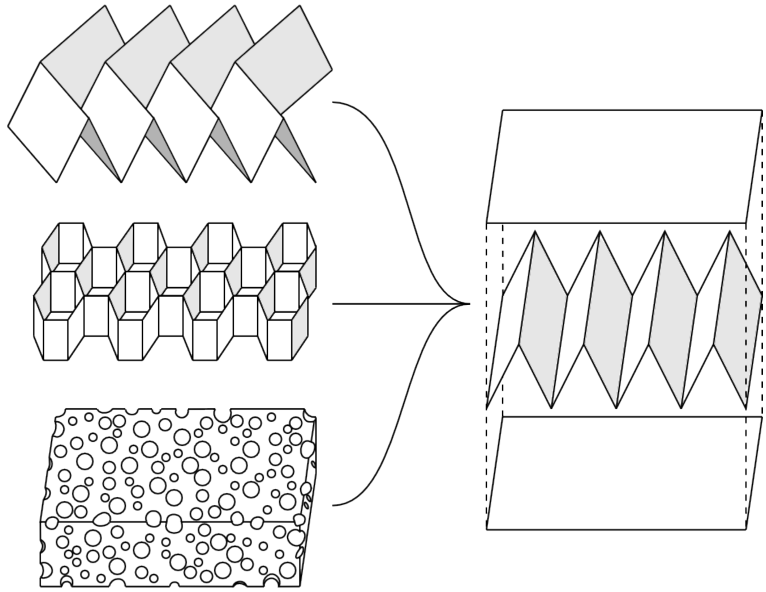

Figure 1: Sandwich panel with different core topologies. Chevron, Honeycomb and Foam cores on the left, top to bottom. The right side depicts the corrugated core used in the present investigation with top and bottom faces.

Hence, the use of solid-shell finite elements is proposed for reasons of computational efficiency and investigated using an example of out-of-plane compression of a prismatic sandwich core in this work. It is based on an 8-node hexahedral element and exploits a two field variational formulation curing volumetric and Poisson thickness locking directly. The performance of the current formulation is assessed by conducting a study of convergence, comparing the solid-shell element to several element formulations available in Abaqus® [10]. The geometry for the numerical example is taken from Coté et al. [5], who performed numerical and analytical investigations on steel lattice sandwich structures. Valdevit et al. [6] investigated the optimal design of sandwich panels. Both studies focus on metallic materials. Fischer et al. [7] and Heimbs [8] perform simulations of fibre reinforced materials for chevron and honeycomb cores. All of the above studies use shell elements to perform FE calculations.

2 Finite Element Technology

Ease of meshing and the use of contact algorithms favour linear ansatz functions, hence they are used to discretise both the displacement and the position field. However, linear ansatz functions are known to exhibit several unphysical stiffening phenomena, particularly shear locking when used with bending deformation of thin structures. To overcome this deficiency, two methods are applied.

Firstly, additional internal variables are introduced by enriching the standard displacement-based low order formulation, thus alleviating constraints. The formulation of the solid-shell finite element Q1SPs used in the subsequent investigation is that of Reese [1]. The enhanced assumed strain (EAS) method based on a two field variational formulation applied here has been proposed by Simo and coworkers [3,4]. It is used in the following formulation:

$$g_1(\boldsymbol{u},\boldsymbol{H_e})=\int_{B_0}\boldsymbol{P}(\boldsymbol{H}):\mathrm{Grad}\,\delta\boldsymbol{u}\,dV+g_{ext}=0$$
$$g_2(\boldsymbol{u},\boldsymbol{H_e})=\int_{B_0}\boldsymbol{P}(\boldsymbol{H}):\delta\boldsymbol{H_e}\,dV \tag{1}$$

where the first functional represents the balance of linear momentum defined with respect to the first Piola-Kirchhoff stress tensor $\boldsymbol{P}$ – dependent on the total strain $\boldsymbol{H}$ – and the compatible strain $\mathrm{Grad}\,\boldsymbol{u}$. The total strain $\boldsymbol{H}=\underbrace{\mathrm{Grad}\,\boldsymbol{u}}_{compatible}+\underbrace{\boldsymbol{H_e}}_{enhanced}$ is additively decomposed into a compatible part, representing the standard definition in terms of the displacement degrees of freedom $\boldsymbol{u}$, and an incompatible, or enhanced, strain $\boldsymbol{H_e}$, defined with respect to additional degrees of freedom. The term g_{ext} represents the virtual work of the external forces. The second functional requires the enhanced strain $\boldsymbol{H_e}$ field to be L_2 orthogonal to the stress field.

As a second means to treat locking phenomena in thin structures under bending, it is necessary to capture nonlinearities in the stress-strain behaviour over the thickness. The present formulation exploits a Taylor expansion of $\boldsymbol{P}$ with respect to a straight line in thickness direction through the element centre. The expansion around $\xi_0=(0,0,\zeta)^T$, as depicted in Fig. 2, retains a non-linear dependence of the stress measure on the thickness coordinate ζ. Thus the first Piola-Kirchhoff stress tensor takes on the form:

$$\boldsymbol{P}\approx\boldsymbol{P}|_{\xi=\xi_0}+\frac{\partial\boldsymbol{P}}{\partial\xi}|_{\xi=\xi_0}(\xi-0)+\frac{\partial\boldsymbol{P}}{\partial\eta}|_{\xi=\xi_0}(\eta-0) \tag{2}$$

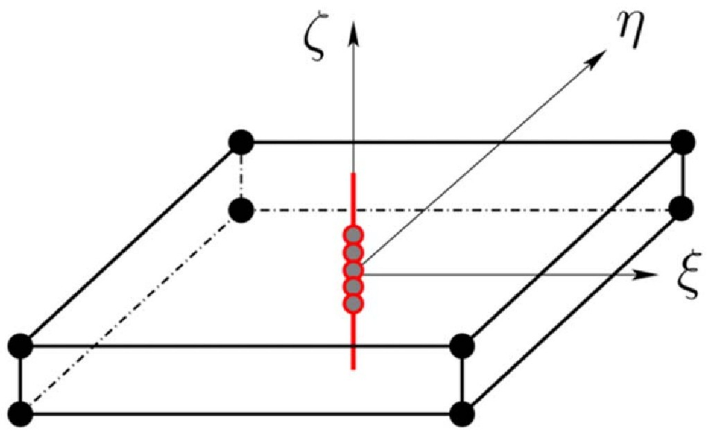

Figure 2: The line $\xi_0=(0,0,\zeta)^T$ indicating the location of Taylor series expansion of the stress measure.

In order to increase the numerical efficiency, a reduced integration scheme of the element stiffness matrices is employed. This well-known technique is accompanied by zero energy deformation modes, for which reason a stabilisation procedure is established.

The structure is discretised using the presented solid-shell element, implemented in the finite element programme FEAP [9] and three different elements in Abaqus®. These are the reduced integration S4R shell, SC8R continuum shell elements and the EAS C3D8I solid element. An overview of the elements, their applicability and some of the underlying technology is given in Table 1.

Element	S4R	SC8R	C3D8I	Q1SPs
Solid geometry	-	√	√	√
3D material	-	-	√	√
EAS	-	-	√	√
Red. Int.	√	√	-	√

Table 1: The elements used in this investigation and their dominant features.

3 NUMERICAL INVESTIGATIONS

As mentioned before, the current investigation focuses on a corrugated sandwich core. A single core member strut – depicted in Fig. 3 – is modelled and loaded as if the whole sandwich would undergo out-of-plane compression, yielding a global strain in y-direction of 0.3. The sum of the reaction forces of the nodes at the top end are compared between the various element formulations.

3.1 Structure and boundary conditions

The length of the single strut is l = 12 mm, it has a width of w = 6 mm and t = 0.3 mm thickness, see Fig. 4. In order to take account for manufacturing imperfections, an initial transversal deflection Δ of the core member strut is assumed. Note, that this also enforces well defined buckling.

All displacement degrees of freedom of the bottom end of the strut are fixed. The top end is fixed in x- and z-direction but allowed to move vertically. For the S4R shell elements, also the rotational degrees of freedom on both sides have to be restrained.

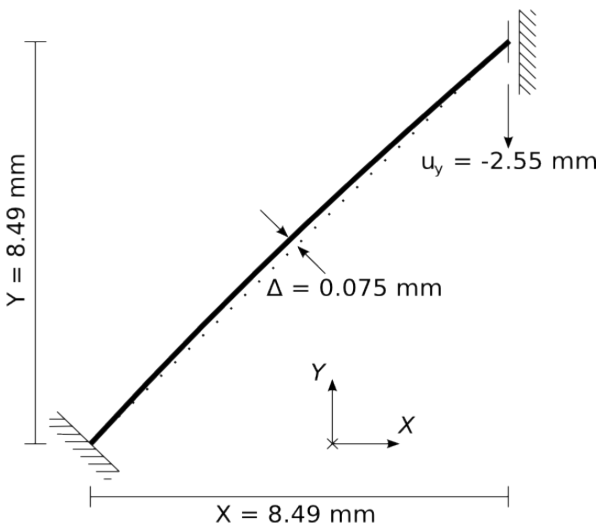

Figure 3: Free-body-diagram of the investigated structure. The top end of the core strut is moved downwards.

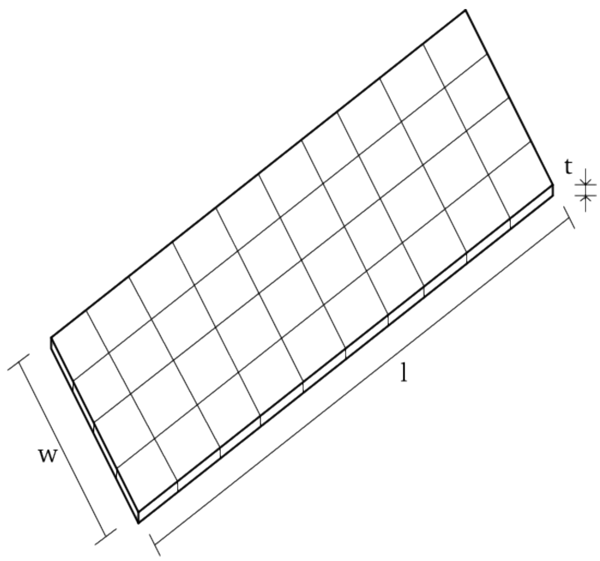

Figure 4: Sketch of the discretisation

3.2 Constitutive Law

For reasons of simplicity, a Neo-Hookean form of the strain energy density function (3) is used, where $\bar{I}_1$ is the first distortional invariant of the deformation gradient $\boldsymbol{F}$, and J is its determinant. The parameters are chosen to represent polypropylene, yielding a shear and bulk modulus of $\mu = 317\,\mathrm{MPa}$ and $\kappa = 1875\,\mathrm{MPa}$, respectively.

$$\Psi_{NH} = \frac{\mu}{2}(\bar{I}_1 - 3) + \Psi_{vol}(\kappa)$$
$$\bar{I}_1 = \mathrm{tr}\,(J^{-\frac{1}{3}}\boldsymbol{F}) \tag{3}$$

4 Results

In order to establish the converged solution, a very fine discretisation of 64 elements in length-direction and 32 elements in width-direction is chosen. A proper aspect ratio is maintained by using 4 elements across the thickness for the C3D8I and Q1SPs elements. All formulations converge to a value of ≈-2.36 N, hence this force is treated as a reference.

The mesh convergence study uses two elements in width-direction and only one element across the thickness. The results of the are depicted in Fig. 5. Both the classical shell S4R and the continuum shell element SC8R exhibit a good convergence behaviour. The solid element C3D8I does not converge using the coarsest discretisation investigated and exhibits errors of ≈35% for less than 12 elements. The presented Q1SPs formulation exhibits good performance except for very coarse discretisations.

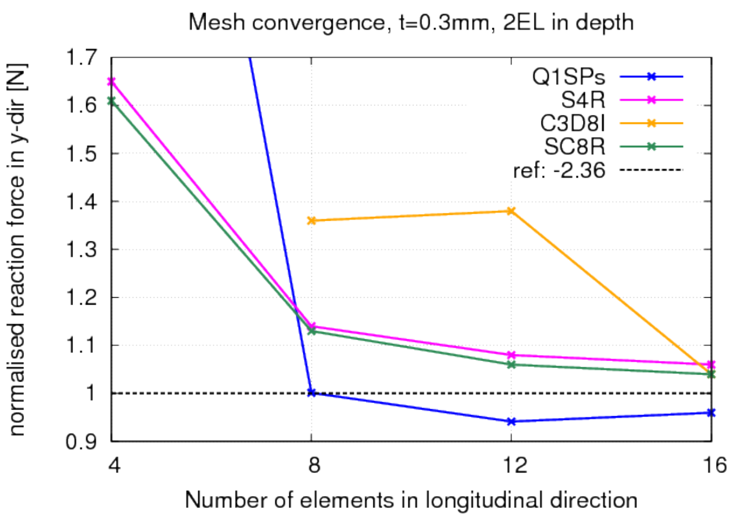

Figure 5: Sum of nodal reactions in y-direction vs. #elements in length-direction of the strut. All values are normalised with respect to the reference solution of -2.36 N. All formulations use 2 elements in width direction and 1 element across the thickness. No solution could be obtained using the C3D8I element with 4 elements along the strut.

5 Conclusions and Outlook

The applied solid-shell formulation shows satisfying results in a mesh convergence study. In direct comparison to the only other element that allows for the usage of a fully 3D material model (C3D8I), its performance is superior. The application of a more recent extension of the current solid-shell formulation by Schwarze and Reese [2], treating additional locking phenomena, is currently in progress. Further investigations will also focus on the influence of varying shell thickness.

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