SOLID-SHELL CONCEPT APPLIED TO THIN FIBRE COMPOSITE STRUCTURES

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Abstract

In this paper we present a micromechanically motivated material law that is implemented in a solid-shell element to model the mechanical behavior of thin, multilayered composite structures until the onset of delamination. An accurate prediction of the stresses that lead to delamination needs a fully three dimensional constitutive law. Determining the correct position of the crack initiation needs a high resolution of the stress distribution throughout the thickness direction. Here we employ a solid shell formulation that is capable of dealing with both mentioned requirements and still is very efficient from the numerical point of view. The anisotropy and heterogeneity of the composite material is taken into account using the concept of structural tensors and the split of the Helmholtz free energy into fiber and matrix parts.

1 Introduction

One of the most significant problems of fiber reinforced composites is their complex crack behavior. This is mainly caused by their heterogeneity – due to fiber and matrix parts – as well as the interaction of these at the interface. Each of the materials has its own stress-strain relation and different failure behavior. In order to capture this complexity one could use a micromechanical approach. Tackling the problem numerically, e.g. using the finite element method, would lead to a very fine mesh. Considering the complex behavior which requires to use sophisticated constitutive models one arrives at a rather time consuming computation. To reduce the computational effort it is common to work with phenomenological models These micromechanically motivated constitutive laws allow for the use of a coarse mesh but do not have a direct link to the microstructure. Many of these laws based on the concept of structural tensors where developed in the field of biomechanics e.g. to compute the axisymmetric orthotropic behavior of blood vessels [1] or the different orthotropic layers of the aorta [2]. [3] used a similar approach to describe inflatable membranes, which are made of textile reinforced rubber. In this model the strain energy is described as a sum of the strain energies of the different materials. By taking the volume fractions of the considered materials into account, the (homogenized) stress state of every material can be used to evaluate suitable crack criteria. This enables us to differentiate between materials and their crack behavior.

A big disadvantage regarding homogenized materials in combination with standard elements is the low resolution of the position of crack initiation especially in thickness direction.

Increasing the number of elements in this direction would overcome this problem but would also lead to a rapid increase of the total number of elements. Another point is that locking effects occur, when using solid elements with bad aspect ratio [4]. Classical shell element formulations do not show this locking effect but cannot deal with a fully three dimensional material law. One way to overcome this problem is the use of so called “solid-shell” elements. In particular the here used reduced integration solid-shell element formulation of [5] enables an arbitrary number of Gauss points in thickness direction which ensures a high resolution of the stress state in this direction. In addition, many locking effects like volumetric locking, transverse shear locking, curvature locking as well as Poisson thickness locking are cured by using both EAS and ANS concepts.

The combination of both the micromechanically motivated constitutive law and the reduced integration solid-shell formulation is a good set up for the accurate and time efficient investigation of multilayered composite parts for the onset of the in-plane matrix crack – delamination.

2 Solid-shell

As already mentioned, standard solid elements in thin structures would require a very fine mesh and thus a dramatically high number of degrees of freedom. To perform a numerically efficient calculation, we use the reduced integrated solid shell element based on the work of Schwarze and Reese [5]. The formulation is based on the two field variational functional introduced by Simo [6]. Following the enhanced assumed strain (EAS) concept, an additive split of the Green-Lagrange strain into a compatible and an incompatible part includes one internal degree of freedom. The isoparametric eight node element uses the well-known tri-linear ansatz functions. In bending dominated problems, ansatz functions of low order cause parasitic shear terms. The so called transverse shear locking phenomenon occurs. This problem is cured by a bi-linear reinterpolation of the transverse shear strains. Therefore, two assumed natural strain (ANS) degrees of freedom are introduced. Another problem of linear ansatz functions is that the true thickness of curved structures is only represented correctly at the transversal edges of the element. When the strain in thickness direction is evaluated at any other position, the element reacts too stiffly – and causes curvature thickness locking. To avoid this phenomenon, another bi-linear ANS reinterpolation at the shell midplane is performed to replace the strain in thickness direction. The replaced thickness strain is independent of the thickness coordinate. Due to the Poisson effect, the thickness strain must be at least linear. The EAS degree of freedom mentioned above remedies this Poisson thickness locking effect by adding a linear term to the thickness strain.

For reasons of numerical efficiency, the reduced integration scheme is implemented. To treat zero energy modes, the element is suitably stabilized. Simple procedures require an analytical integration of the hourglass stabilization matrix. Though the element has an arbitrary shape, the Jacobian matrix as well as the compatible strain is a rational tensor function, which makes the integration of the hourglass stabilization matrix very complex. Many authors like Reese [7] and Zhang [8] approximate the Jacobian matrix by its value in the element center. To improve geometric accuracy, Schwarze carried out a Taylor series expansion of the Jacobian matrix with respect to the element center so that the coefficients are polynomials. In this way, an analytical integration of the hourglass matrix is possible. To complete the reduced integration procedure, a Taylor series of the first Piola-Kirchhoff stress tensor with respect to the shell director (normal through the element center) is carried out. These steps are explained in more detail in [5].

3 Material model

The fiber composites examined in this paper are composed by various layers which consist of a matrix phase and an arbitrary number of fiber phases. The basis of the following continuum model is the anisotropic material law proposed in [3].

Starting with the deformation gradient **F**, one can determine the right Cauchy-Green tensor **C**, which represents the deformation of a continuous body, using

$$\mathbf{C} = \mathbf{F}^T\mathbf{F} \tag{1}$$

For every hyperelastic material, there exists a scalar strain energy density function *W*. Then the second Piola-Kirchhoff stress tensor reads

$$\mathbf{S} = 2\frac{\partial W(\mathbf{C})}{\partial \mathbf{C}} \tag{2}$$

Introducing the structural vectors $\boldsymbol{n}_{\mathrm{i}}$, which are associated with the main fiber direction at a certain position, the structural tensors are defined by

$$\mathbf{M}_{\mathrm{i}} = \mathbf{n}_i \otimes \mathbf{n}_i \tag{3}$$

where *i* is the number of main fiber directions.

Besides the first three well known isotropic invariants, one can define a number of pseudo invariants.

$$I_k = tr(\mathbf{C}\mathbf{M}_{\mathrm{i}}) \tag{4}$$

$$I_l = tr(\mathbf{C}^2\mathbf{M}_{\mathrm{i}}) \tag{5}$$

with k = 2 [(2i+1)/2]+1 and l = k+1

Picking up Reese´s idea for orthotropic materials, we can define the strain energy density function in terms of the invariants. Together with the fiber densities φ_{i} of each fiber family with the restrictions: $0 \leq \varphi_{\mathrm{i}}\,;\ \sum \varphi_{\mathrm{i}} \leq 1$, we slightly modify the ansatz of Reese to:

$$W = (1 - \sum \varphi_i) W_{NH}(I_1, I_3) + W_{ani}(\varphi_i, I_k, I_l) \tag{6}$$

W_{NH} represents the matrix part, modeled as an isotropic Neo-Hookean material

$$W_{NH}(I_1, I_3) = \frac{\mu}{2}(I_1 - 3) - \mu \ln\sqrt{I_3} + \frac{\lambda}{4}(I_3 - 1 - 2\ln\sqrt{I_3}) \tag{7}$$

where λ and μ are the Lamè parameters. The anisotropic part corresponding to the fibers is defined by

$$W_{ani}(\varphi_i, I_k, I_l) = \sum_i \varphi_i [\frac{1}{\alpha_i} K_i^{ani1} (I_k - 3)^{\alpha_i} + \frac{1}{\beta_i} K_i^{ani2} (I_l - 3)^{\beta_i}] \tag{8}$$

In this equation α_i, β_i , K_i^{ani1} *and* K_i^{ani2} are material parameters. In the special case of a linear behavior of the fibrous part, K_i^{ani1} can be associated with the Young´s modulus of the fiber, then $\alpha_i = 2$ and $K_i^{ani2} = 0$ must be chosen.

To check whether delamination occurs, only the stress caused by the matrix part is taken into account. Our investigation is based on the onset criterion of Ye [9], in which the quadratic interaction of the stresses leading to delamination is formulated as

$$\left(\frac{\langle \sigma_{33} \rangle}{Z_{33}}\right)^2 + \left(\frac{\sigma_{31}}{Z_{31}}\right)^2 + \left(\frac{\sigma_{32}}{Z_{32}}\right)^2 \geq 1 \tag{9}$$

when the 3-direction is the thickness direction. This is a reliable criterion for formulations at small strains. Here the Z_{3i} are the crack resistances of the matrix. In the case of large deformation, notably large rotation, the onset criterion should be formulated in terms of the Cauchy stresses. To calculate the Cauchy stress $\boldsymbol{\sigma}$ the second Piola-Kirchhoff stress tensor must be pushed forward to the current configuration:

$$\boldsymbol{\sigma} = \frac{1}{\det \mathbf{F}} \mathbf{F}\, \mathbf{S}\, \mathbf{F}^T \tag{10}$$

Using Cauchy´s theorem $\mathbf{t} = \mathbf{n}\,\boldsymbol{\sigma}$ and the normal vector $\mathbf{n}$ of the shell in current configuration, the traction

$$\sigma_n = \mathbf{n} \cdot \boldsymbol{\sigma}\, \mathbf{n} \tag{11}$$

and resultant shear

$$\tau_n = \sqrt{\|\mathbf{n}\,\boldsymbol{\sigma}\|^2 - \sigma_n^{\ 2}} \tag{12}$$

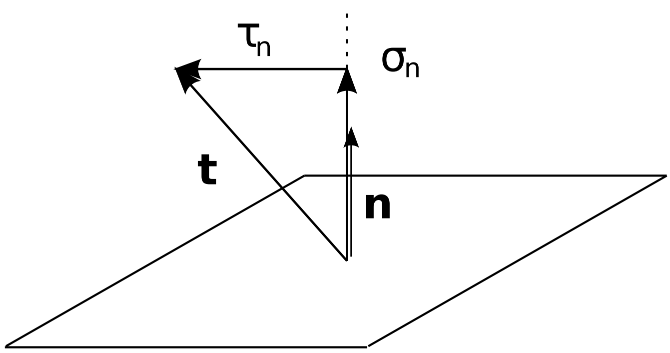

Figure 1. Traction vectors acting on infinitesimal surface element

can be achieved. Since τ_n is the resultant shear stress, a resultant shear resistance $Z_\tau = \sqrt{{Z_{31}}^2 + {Z_{32}}^2}$ is determined as the geometric average of the shear resistances above. Consequently we call the resistance for traction $Z_\sigma = Z_{33}$. Hence we can define an onset criterion as follows:

for $\sigma_n > 0$: $$\left(\frac{\sigma_n}{Z_\sigma}\right)^2 + \left(\frac{\tau_n}{Z_\tau}\right)^2 \geq 1 \qquad (13a)$$

for $\sigma_n \leq 0$: $$\left(\frac{\tau_n}{Z_\tau}\right)^2 \geq 1 \qquad (13b)$$

This condition must be checked at every load increment in a numerical calculation to predict the load at which the crack criterion is fulfilled.

4 Numerical example

We performed a simulation according to the investigations by A. Turon et al. in [10] using the methods proposed in section 2 and 3. At the middle of the considered composite plate a beveled composite flange is applied.

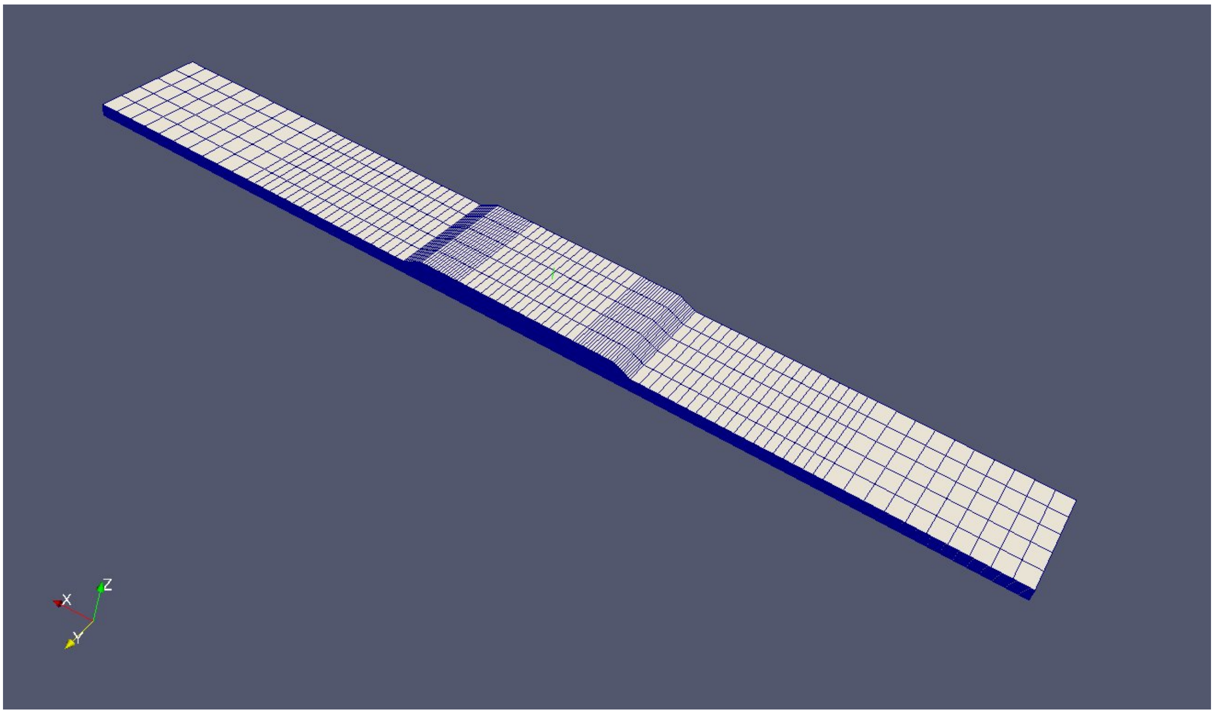

Figure 2. Meshed carbon fiber plate with attached flange

In Figure 2., the discretized composite layup is shown. For every layer, one element in thickness direction is used with five Gauss points over the thickness. The mesh is refined at the zone, where onset of delamination is expected. The investigated system has 9600 elements and 24576 nodes with three DOF each.

The composite plate (203 mm x 25.4 mm) has 14 unidirectional carbon fiber reinforced epoxy resin layers $(0°/45°/90°/-45°/45°/-45°/0°)_S$ whereas the flange (50 mm x 25.4 mm) has the layup $(45°/90°/-45°/0°/90°)_S$ out of the same material with a fiber volume fraction of 60%. Every layer has a thickness of 0.188 mm. Based on the parameters

Tensile modulus fibre	[GPa]	230
Tensile modulus matrix	[GPa]	3.9
Shear modulus matrix	[GPa]	3.4
Poisson´s ratio matrix		0.45
Z_{33}	[MPa]	76
$Z_{13} = Z_{23}$	[MPa]	79

Table 1. Engineering material parameters

of the materials of the composite we determine the parameters for our model in the case of linear fiber behavior and one family of fibers as

λ	[MPa]	6300
μ	[MPa]	1383
K_1^{ani1}	[GPa]	230
α_1		2
Z_σ	[MPa]	76
Z_τ	[MPa]	$79\sqrt{2}$

Table 2. Material parameters

In [10], the experimental and numerical investigation started with a thermal loading of the specimen. This caused a precondition of the shape and internal stresses. Afterwards a tensile force is applied to the specimen.

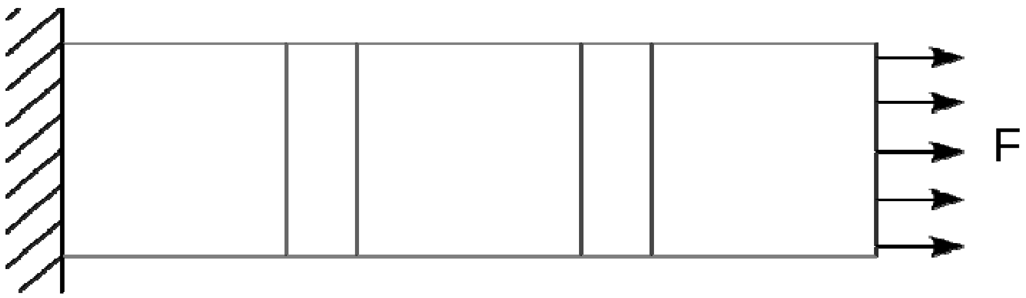

Figure 3. Boundary conditions of present structure

In the experiment, the displacement of two points at the surface of the specimen was measured by an extensometer. The numerical prediction in [10] is consistent with the performed experiments. The delamination showed up within a range of the measured length change of 0.11 mm and 0.15 mm. Unfortunately, the exact position of the observed points is not given.

In our numerical investigation the debonding onset occurred in the region shown in figure 4.

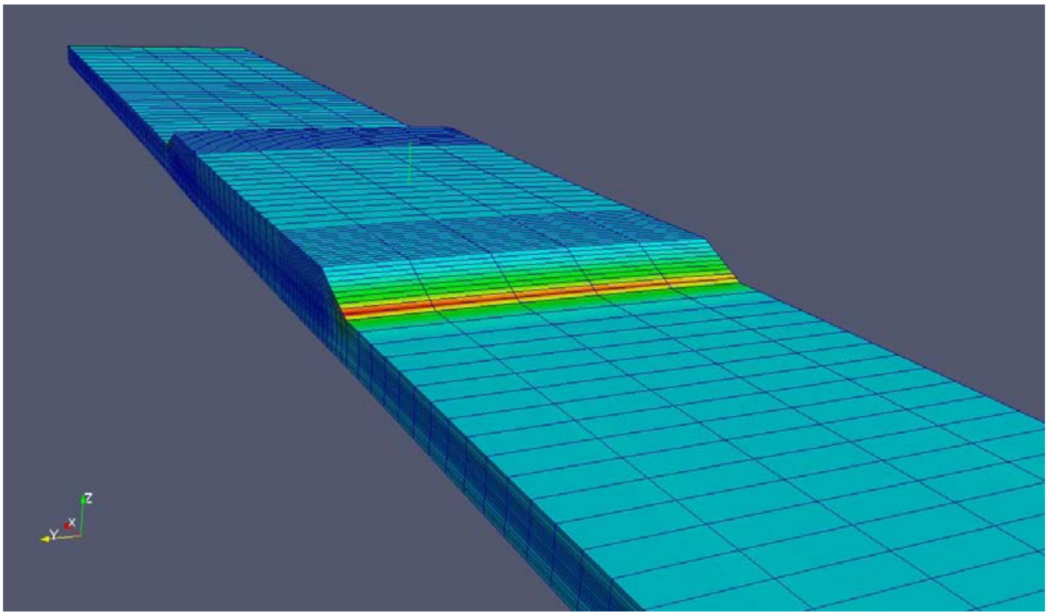

Figure 4. Delamination zone

This is reasonable and in good agreement with the results in Turon´s work. The numerically predicted length change of 0.17 mm is a little higher as the ones measured by the extensometer. This can easily be explained by the fact that in our calculations the thermal precondition is not taken into account.

4 Conclusions

We proposed a meaningful way to apply the solid shell concept to numerical investigations of thin fiber reinforced plastic structures by means of the prediction of delamination. The micromechanically motivated material law implemented in the proposed solid-shell allows for fully three-dimensional investigations. This is needed when crack criteria shall be evaluated. The performed numerical example is in good agreement with reference solutions and experiments found in the literature. Further investigations must include a deeper inspection of the continuity of inter laminar shear stresses, which must be treated carefully to ensure the correct shear traction, in particular for the evaluation of matrix crack initiation criteria.

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