SELECTION OF WOVEN FABRIC COMPOSITES USING A MULTI-CRITERIA DECISION MAKING TECHNIQUE AND MESO-LEVEL FINITE ELEMENT SIMULATION

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Abstract

Various types of woven fabrics have found considerable attention in modern industries. Although they have brought design flexibilities for composite manufacturers, often the selection of the most suitable fabric that can meet all design requirements for a given product may be challenging, especially due to the potential presence of conflicting criteria. As a result, the application of Multi-Criteria Decision Making (MCDM) techniques is deemed necessary. In this article, a well-established MCDM technique is incorporated into a woven composite material selection problem, along with the capabilities of meso-scale finite element modeling which eliminates the need for time-consuming or expensive experiments. In implementing the MCDM, a new combination of the modified digital logic weighting factors (defined by the designer) with entropy weights (defined based on material data) is introduced.

1 Introduction

The application of woven fabric composites in high tech industries is rapidly increasing as a result of their superior mechanical properties compared to convectional engineering materials and even unidirectional composites. There exists a multitude of composite fabrics with different constituent (fiber and matrix) material options, geometrical parameters, and weave patterns. This has brought the advantage of providing high level of flexibility in design of composite structures across engineering applications. Essentially, structures made of woven composites are produced from dry fabrics that that are formed into desired 3D shapes and impregnated with a resin. After curing/consolidation, the entire structure finds a solid and stable shape, which can then tolerate external loading. Dry fabrics themselves are made of interlacing sets of yarns known as warps and wefts. Depending on the interlacing pattern of warps and wefts, different types of fabrics and eventually different types of composites can be produced [1]. The most common patterns of woven fabrics include: Plain weave, Twill weave and Stain weave. In each case, depending on the material type and dimensions of the warp and weft yarns, a balanced or unbalanced fabric may be fabricated. Nonetheless, every woven fabric composite can be argued to be made from a geometrical repetition of specific architectural pattern (also known as unit cell) in two planar directions. Compared to dimensions of an actual structural component that is often at a macro scale, the unit cell is considered to be a meso-level material system, which is assumed to have the same properties of the fabric at macro-level.

1.1 Meso-level modeling of fabrics

Prediction of mechanical properties of a woven fabric can be done by studying one unit cell of the material under specific boundary conditions, known as periodic boundary conditions [2]. The numerical modeling of woven fabric composites began with the so called 'mosaic models' [1], [3], [4]. In a mosaic model, the unit cell of the impregnated woven fabric is idealized as an assemblage of asymmetrical cross-ply laminate and the geometry is simplified by neglecting yarns' waviness. Later on, a fabric undulation model and the bridge model were developed as extensions to the mosaic modeling approach to improve the resulting predictions; however, some drawbacks were still found mostly due to the simplification of yarn geometries and the assumptions made for extracting the total stiffness matrix of the unit cell [5]. Later on, advancements in computational capabilities of digital computers made it easier to use more complex finite element codes. Accordingly, numerous finite element models were reported in the literature on the prediction of mechanical response of woven fabrics using standard unit cells [6–8], as well as imperfect (non-standard) unit cells with microstructural defects [9], [10]. Moreover, a variety of software packages (e.g., TexGen [11]) were developed for easy generation and meshing of various fabric patterns, and for assigning accurate material orientations in the yarns. Subsequently, today the above techniques and tools have made it possible to estimate mechanical properties of different fabrics with a reasonably low computational cost, without actually producing and testing the fabrics.

1.2 Decision making and fabric material selection

As mentioned above, there exists a broad variety of fabric types in the composite market. On the other hand, quite often the optimum selection of a suitable fabric that can meet all the design requirements of a given product may be challenging, especially due to the presence of conflicting criteria in the material response under different deformation modes, ease of formability, low material cost, etc. Therefore, the application of Multi-Criteria Decision Making (MCDM) techniques is deemed necessary in such selection processes [12]. In general, weighting the decision criteria in an MCDM method can be tackled either via purely statistical techniques or by trusting designer's intuition over criteria priorities. A combination of these two methods may also be employed for more reliable decision making processes. The following sections of this article will discuss the application of two weighting methods known as "Entropy method" and the "modified digital logic" method. The former is more based on the statistical properties of the decision matrix (i.e., material data), whereas the latter is more based on preferences of the designer from one criterion to another. Subsequently, a combination of the two methods is presented as a third method that can rely on both objective (Entropy) and subjective (designer's) weights. In conjunction with the MCDM approach, unit cell finite element modeling is employed to make a sample library of mechanical properties of six different woven fabrics with different material and weave patterns. For demonstration purposes, the case study considers the selection of the best fabric in the library for a final product that will primarily undergo uniaxial tension (e.g., a composite bar).

2 Selected MCDM techniques

In any MCDM method, the goal is to chose the top alternative, or ranks the alternatives (here different fabrics) against a set of design criteria. In doing this, the first step would be to collect information regarding the performance values of each alternative (here fabric properties). The nature/type of criteria can be "the higher the better" (like stiffness, strength,…), or "the lower the better" (like the material density, cost,...), or they can be qualitative in which case their judgmental/linguistic assessments can be converted into numerical numbers (e.g., 0 for a poor performance, 1 for neutral, 2 for an acceptable performance, etc). Eventually, after all numerical data are collected, a decision table (matrix) is formed where the rows represent the selection options (materials) and the columns contain the criteria values (Table 4 will show an example of such matrix). Keeping in mind that in general the magnitude or units of the criteria are different, a normalized form of the decision matrix is required for subsequent calculations. The normalization can be done using [13]:

$$p_{ij} = \frac{y_{ij}}{\sum_{i=1}^{n} y_{ij}} \text{ for all } i, j \tag{1}$$

where y_{ij} are the actual (non-normalized) values of the decision matrix, p_{ij} are the normalized values and n is the number of materials in the matrix. Materials that are in the selection pool can be ranked based on a score that they receive from performance values under different criteria as defined by the designer. The simplest method for defining this score for each materiel is summing the corresponding normalized properties as follows.

$$\gamma_i = \sum_{j=1}^{k} p_{ij} \tag{2}$$

where γ_i is the score of the i-th material and k is the number of criteria. However, the calculated scores via Eq. (2) neglect the relative importance of criteria that may be of interest to the designer for a given application; for example more emphasize on the density of a material may be given in an application compared to its shear stiffness. Therefore, the weighted properties is suggested as a better option as follows [13]:

$$\gamma_i = \sum_{j=1}^{k} (-1)^{m j}\, w_j\, p_{ij} \tag{3}$$

w_j are the weighting factors, and m_j is included to distinguish the desirable (i.e., the higher the better) and undesirable (the lower the better) criteria (also called attributes in MCDM). If an attribute is desirable, m_j =0 and if it is undesirable, m_j =1. The latter method in MCDM is referred to as the "weighed sum method" or WSM and has been most widely used in different disciplines (see, e.g., [12] for an application in composite gear selection).

In practice, there are numerous methods for finding the weighting factors. They can be based on pure statistical processes (e.g., the entropy method [14]) or based on designer's input (e.g., the digital logic method [15]). In addition, one may combine these two categories of weights to arrive at a more comprehensive set of weights as will be shown in Sections 2.3 and 3.2.

2.1 Entropy method

The entropy method is particularly useful for investigating contrasts in discriminations between sets of data [14]. In this method, the entropy of each column (criteria) is calculated via:

$$E_j = -\alpha \sum_{i=1}^{n} p_{ij} \ln p_{ij} \text{ for all } j \tag{4.a}$$

$$\alpha = 1/\ln(n) \tag{4.b}$$

If no priori from designer's decisions is included, the following formula can be used for finding the final weights via the entropy method:

$$w_j = \frac{1 - E_j}{\sum_{i=1}^{k}(1 - E_i)} \tag{5}$$

2.2 Modified digital logic method

In the (original) digital logic method, only two attributes are compared at a time by the designer and a value of 1 or 0 is assigned to each, based on their relative importance (e.g., the designer may prefer the high shear stiffness over the low density for a given application, thus he/she assigns 1 and 0 to these two criteria respectively). After all pair-wise comparisons are made, the assigned 0/1values are summed and normalized for each criterion to find its relative weight. As one can argue, this method is completely based on the intuition and experience of the decision maker (designer). Dehghan-Manshadi et al. [15] suggested a modified version of the method (called the modified digital logic/MDL method) to assign a value of 3 for strict preference, 2 for an equal preference, and 1 for loose preference. In other words, they added the 'equally important' option so that when the designer is not sure to judge between two criteria, he/she can assign a value of 2 to both of them. This method has proven to be more flexible and accurate in design case studies [15].

2.3 Combined method

As noted before, it is also possible to combine the effects from weights calculated by the entropy method (or any other objective weighting method in MCDM) with the weights from designer's priori (i.e., subjective weights like those from MDL). To this end, the following formula my be used:

$$w_j = \frac{w_j^{subjective}\ w_j^{objective}}{\sum_{i=1}^{k}\left(w_i^{subjective}\ w_i^{objective}\right)} \tag{6}$$

3 Illustrative example

To demonstrate the application of the MCDM along with the introduced weighting techniques, a basic case study is considered in this section. The goal is to find a suitable fabric composite among a pool of six different candidates, for a structure that presumably will undergo uniaxial tension during service (e.g., laminated elements of a truss). Low weight is also highly desired. The pool of fabrics include three different weave architectures (1*1Plain, 2*1 Twill and 2*2 Twill) along with the option of E-glass yarns and AS4 carbon yarns, both with 70% volume fraction and consolidated with Epoxy resin over the entire unit cell volume.

3.1 Characterization of woven fabric composite candidates

As addressed in Section 1, meso-level finite element models can be effectively used to evaluate the mechanical response of woven fabric composites. For the composites in this case study, three different unit cell architectures were constructed using TexGen and imported into the Abaqus finite element software. Identical yarns size and distance between the cross over points are used in all the composite unit cells as shown in Table 1. A depiction of modeled yarns and resin is also shown in Figure 1. Moreover, Table 2 shows the mechanical constants used for the constituent materials of composites (E-glass, AS4 carbon and epoxy resin) where L and T subscripts refer to the longitudinal and transverse properties along yarn direction. Table 3 summarizes the material properties of the composite yarns, where the effective mechanical properties are taken from other references (as cited in the table). The standard rule of mixture is used for estimating the ultimate strength of composite yarns composed of 70%

fiber and 30% resin in volume. The rule of mixture has proven to be a fair approximation for calculating the composite strength as long as defects in the yarns are negligible [16].

Yarn spacing	**Yarn width**	**Yarns height**
5.14 (mm)	3.72 (mm)	0.39 (mm)

Table 1. The unit cell dimensions for the woven fabric composites

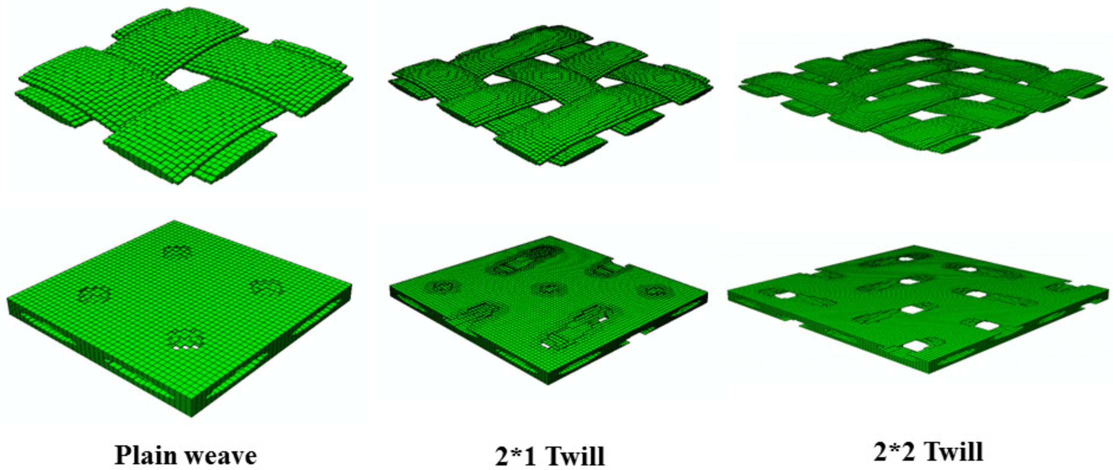


Figure 1. Three yarns and resin elements used in the 3D unit cells of the selected fabric architectures

	E_L (GPa)	E_T (GPa)	G_{LT} (GPa)	G_{TT} (GPa)	ν_{LT}	ν_{TT}	$\sigma_{L,ult}$ (MPa)
E-glass [17], [18]	73.1	73.10	30.19	30.19	0.22	0.22	2413
AS4 carbon [18]	221	13.80	13.8	5.50	0.20	0.25	3585
Epoxy [17], [18]	3.45	3.45	1.83	1.83	0.35	0.35	103

Table 2. Material constants of the individual constituents in the selected composites

	E_L (GPa)	E_T (GPa)	G_{LT} (GPa)	G_{TT} (GPa)	ν_{LT}	ν_{TT}
E-glass/Epoxy V_f=70% [17]	51.92	21.97	8.86	6.25	0.25	0.21
AS4 carbon/Epoxy V_f=70% [19]	151.0	10.10	5.70	3.40	0.24	0.50

Table 3. Material constants for the impregnated yarns in the selected composites

3.2 MCDM results

Next, to construct a set of criteria for the decision matrix, important and practical attributes (fabric properties) were selected as follows:

- Composite stiffness under uniaxial extension along warp (E_{uni});
- Composite stiffness under equi-biaxial loading along warp and weft (E_{bi});
- Composite in-plane shear stiffness (G);
- Ratio of the maximum Von Mises stress in matrix and longitudinal stress in yarns to their ultimate strength under a uniaxial and a biaxial loading of 1000 *N/mm;* for the matrix the above ratio for the two loading cases is denoted by ($S_{m,uni}$, $S_{m,bi}$), and for the yarns by ($S_{f,uni}$, $S_{f,bi}$).

- Ratios of the maximum Von Mises stress in the matrix and longitudinal stress in the yarns to their ultimate strength under a shear load of 100 *N/mm*; in the matrix and yarns the ratio is denoted by ($S_{m,sh}$, $S_{f,sh}$), respectively.
- The density of the composite (ρ).

Having the results from the finite element simulations of the candidate materials under the above conditions, the decision matrix of Table 4 was obtained. Subsequently, normalizing this matrix via Eq. 1 resulted in Table 5.

		The higher the better criteria			***The lower the better criteria***						
		E_{uni} (GPa)	E_{bi} (GPa)	G (GPa)	$S_{m,uni}$	$S_{m,bi}$	$S_{m,sh}$	$S_{f,uni}$	$S_{f,bi}$	$S_{f,sh}$	ρ Kg/m^3
E-glass	1*1 Plain	29.8	36.3	6.37	.564	.588	.187	.257	.246	.014	1768
	2*1 Twill	29.8	35.8	6.24	.351	.437	.128	.179	.167	.010	1768
	2*2 Twill	29.6	35.5	6.19	.243	.288	.097	.132	.126	.007	1768
AS4 Carbon	1*1 Plain	53.7	69.3	4.79	.28	.26	.194	.240	.206	.01	1514
	2*1 Twill	57.2	70.1	4.75	.189	.166	.132	.149	.139	.007	1514
	2*2 Twill	58.5	70.5	4.74	.143	.127	.100	.107	.107	.008	1514

Table 4. The decision matrix based on the performed finite element simulations of candidate fabrics

	Composite	E_{uni}	E_{bi}	G		$S_{m,uni}$	$S_{m,bi}$	$S_{m,sh}$	$S_{f,uni}$	$S_{f,bi}$	$S_{f,sh}$	ρ
E-glass	1*1 Plain	.115	.114	.193	.318	.314	.223	.242	.248	.079	.180	.115
	2*1 Twill	.115	.113	.189	.198	.233	.153	.169	.168	.054	.180	.115
	2*2 Twill	.114	.112	.187	.137	.154	.115	.124	.127	.382	.180	.114
AS4 Carbon	1*1 Plain	.208	.218	.145	.159	.142	.231	.225	.208	.056	.154	.208
	2*1 Twill	.221	.221	.144	.107	.089	.158	.140	.140	.400	.154	.221
	2*2 Twill	.226	.222	.143	.081	.068	.119	.100	.108	.030	.154	.226

Table 5. The normalized decision matrix based on the values of Table 4 and Eq. (1)

Using Table 5 and applying the calculation steps in Sections 2.1 to 2.3, the criteria weighting factors were determined based on the Entropy, MDL, and the combined methodologies, as summarized in Table 6. It should be added that for the modified digital method, E_{uni}, $S_{m,uni}$ and $S_{f,uni}$ were considered as the main attributes for assessing the performance of the candidate composite materials. Thus, in their pair-wise comparisons they were assigned a value of 3 against any other attribute, and 2 among themselves (Figure 2). Finally, using the normalized data and the obtained weighting factors, a total score via Eq. 3 for each material was obtained and subsequently the ranking was made as shown in Table 7. There is a notable consistency between results of different weighting methods, even though the score values are different among the methods. A negative score, mathematically, means that the 'the lower the better' criteria have dominated "the higher the better" criteria during scoring; regardless a candidate with a higher score is always ranked higher. In total, AS4 Carbon yarns in Epoxy resin with a 2*2 twill pattern can be selected as the top candidate for this example.

Weights →	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}
Corresponding criteria/attributes →	E_{uni}	E_{bi}	G	$S_{m,uni}$	$S_{m,bi}$	$S_{m,sh}$	$S_{f,uni}$	$S_{f,bi}$	$S_{f,sh}$	ρ
Entropy	.055	.059	.010	.111	.140	.041	.053	.046	.482	.003
MDL	.133	.078	.078	.133	.078	.078	.133	.078	.078	.133
Combined	.081	.051	.009	.165	.121	.035	.079	.040	.416	.005

Table 6. The criteria weights obtained from different methods (note that for each method the weights sum to one.)

																																														Sum	w_j
E_{uni}	3	3	2	3	3	2	3	3	2																																					24	0.13
E_{bi}	1									2	1	2	2	1	2	2	1																													14	0.08
G		1								2								1	2	2	1	2	2	1																						14	0.08
$S_{m,uni}$			2								3							3							3	3	2	3	3	2																24	0.13
$S_{m,bi}$				1								2							2						1						2	1	2	2	1											14	0.08
$S_{m,sh}$					1								2							2						1					2					1	2	2	1							14	0.08
$S_{f,uni}$						2								3							3						2					3				3				3	3	2				24	0.13
$S_{f,bi}$							1								2							2						1					2				2			1			2	1		14	0.08
$S_{f,sh}$								1								2							2						1					2				2			1		2		1	14	0.08
ρ									2								3							3						2					3				3			2		3	3	24	0.13
																																														Sum	1.00

Figure 2. Chart of pairwise comparisons made in the modified digital logic method

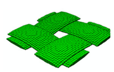
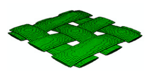
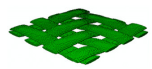
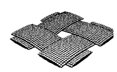
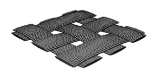
Materials		E-glass / Epoxy			AS4 Carbon / Epoxy		
Weave pattern		1*1 Plain 	2*1 Twill 	2*2 Twill 	1*1 Plain 	2*1 Twill 	2*2 Twill 
MCDM method							
Entropy	Score	-0.194	-0.125	-0.080	-0.090	-0.042	-0.022
	Rank	**6**	**5**	**3**	**4**	**2**	**1**
MDL	Score	-0.140	-0.091	-0.061	-0.075	-0.036	-0.020
	Rank	**6**	**5**	**3**	**4**	**2**	**1**
Combined	Score	-0.193	-0.123	-0.078	-0.088	-0.040	-0.020
	Rank	**6**	**5**	**3**	**4**	**2**	**1**

Table 7. The total scores and ranking of composite candidates using different weighting techniques

4 Summary and conclusion

A multiple criteria decision making (MCDM) approach based on the Entropy and the Modified Digital Logic (MDL) methods, along with a weighted sum scoring technique, was introduced for selecting the optimum composite material among a set of given candidates for a particular application. A new technique based on the combination of both the objective (Entropy) and the subjective (MDL) weights was also suggested. As an illustrative example, a design scenario for selecting a low weight and high performing 1D composite structure was studied. For this specific application, AS4 carbon fiber yarn with a 2*2 twill pattern and consolidated with epoxy was selected as the preferred option using all the three weighting methods. However, it should be mentioned that the MCDM results can be dependent on the designer's experience (through assigning different subjective weights) and the number of criteria included in the decision process. Regardless, it is believed that as long as a proper decision making matrix is created, it is straight forward to conduct the decision making process via MCDM techniques.

Finally, from a practical point of view, making a general purpose design matrix may need cumbersome and expensive experiments to fully characterize the properties of all given material candidates. The meso-level finite element modeling of fabrics is recommended as a cost effective method to facilitate the decision process in conjunction with MCDM.

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