MECHANICAL PROPERTIES OF TEXTILE COMPOSITES WITH VARIABILITY IN MICROMECHANICAL PROPERTIES

M. Y. Matveev*, A. C. Long, I. A. Jones

Polymer Composites Group, Faculty of Engineering, The University of Nottingham, University Park, Nottingham, NG7 2RD, UK
** emxmm@nottingham.ac.uk*

Keywords: damage modelling, variability, stochastic analysis.

Abstract
The effects of variability of fibre strength and modulus in textile composites were analysed numerically. A single fibre strength model based on a Weibull distribution was used to predict the strengths of fibre bundles. Predicted fibre bundle strengths and Youngs moduli were substituted as the input parameters for finite element models. The fibre bundle strength model was validated on a unidirectional composite and its convergence was shown. The unit cell approach was utilized for damage modelling of a plain weave textile composite with fibre strength variability. The influence of fibre strength on effective moduli and strength of textile composites was estimated. It was shown that fibre strength variability affects overall strength.

1 Introduction
A distinctive feature of textile composites is their complex structure. The composite parts consist of textile layers bound together with matrix and/or stitched with yarns. Each layer consists of yarns woven together. In turn, yarns consist of fibre bundles. A textile composite can therefore be regarded as a hierarchical or multi-scale structure where levels are distinguished by their characteristic length. This structure lends itself to a multi-scale modelling approach, where within a hierarchical structure a heterogeneous medium on one level can be replaced by a homogeneous medium with the same properties on a higher level. This procedure, called homogenisation, is usually based on the assumption of ideal periodicity of structures on all levels. Although this approach, based on a periodic unit cell representation of composites, has shown good results in many studies it cannot predict scatter of experimental results for elastic properties and failure strengths.

Following the multi-scale approach uncertainties can be divided in groups by length scale [1]. Variability on the micro-scale level includes the distribution of fibres within yarns, fibre waviness [2], voids between fibres and variability of constituent properties; on the meso-scale yarn path [3], size and shape of yarn cross-section, nesting, voids between yarns etc.

On the micro-scale level, the strength of single fibres is highly scattered [4, 5]. This scattering is often described by a two-parameter Weibull distribution or by a Weibull distribution with a length scale parameter [5]. Curtin [6] proposed a model called the Weibull of Weibulls (WoW) model, which takes fibre length and strength variability along the fibre in the longitudinal direction into account. It assumes that the scale parameter of a Weibull distribution also follows a Weibull distribution.

Prediction of fibre bundle or unidirectional (UD) composite strength at the meso-scale level is the next step in multi-scale modelling. Two types of models can be considered. The first type, a global load sharing (GLS) concept, assumes that the load from a failed fibre is distributed equally over the all surviving fibres. However, this simple model does not take the stress concentrations near the failed fibres into account. In contrast, a local load sharing (LLS) concept proposes that the load from a failed fibre is distributed over a number of neighbouring fibres according to a sharing rule [7, 8]. The number of neighbouring fibres that take the load depends on the matrix, fibre properties and chosen theory. Curtin proposed an analytical LLS model based on the WoW approach for the fibre bundle and showed its good agreement with experimental results [6]. Okabe [9] developed a micro-scale FE model which utilized a WoW approach and showed good results in predicting strength of a UD composite. However, these approaches have never been used for textile composites. Nilakantan modelled the impact response of a fabric with yarn strengths following a normal distribution [10]. This study demonstrated the importance of strength variability on the fabric's response. Nevertheless, this study assumed that the yarn's strength is constant along its length while in reality it varies.

This paper presents a numerical multi-scale modelling approach for textile composites with variability in fibre strength and Young's modulus. The fibre bundle strength model was chosen and validated using experimental data for a UD composite. The FE model of a textile composite with fibre strength variability was created. Using the fibre bundle strength model stochastic simulations were performed to find the mechanical properties of the textile composite with variability.

2 Variability models

2.1 Strength model

The Weibull distribution and particularly the WoW concept has proved to be adequate in prediction of single fibre strength distributions and strength of UD composites. In this work the WoW concept is used to describe the strength of a single fibre in a bundle on the micro-scale. Cumulative probability of fibre failure, P_f, under loading stress σ is equal to

$$P_f = 1 - \exp(-(L/L_0)(\sigma/\sigma_0^i)^{\rho}) \tag{1}$$

where L is a fibre length, L_0 is a reference gauge length, ρ is a Weibull shape parameter and the Weibull scale parameter σ_0^i has cumulative distribution, P, as follows:

$$P = 1 - \exp\left(-(\sigma_0^i/\bar{\sigma}_0)^m\right) \tag{2}$$

where m is a Weibull shape parameter and $\bar{\sigma}_0$ is a Weibull scale parameter.

According to the LLS model, the load from a failed fibre is distributed over a number of remaining fibres. Different theories suggest different number of fibres which take the load but usually this number is between 200 and 1000 fibres depending on properties. It was assumed that within a small domain or finite element a fibre bundle follows the GLS concept i.e. the load can be distributed equally over the remaining fibres. However, the whole bundle consisting of many elements implements some sort of LLS model. Assuming a bundle with N

fibres and fibre volume fraction V_f where each fibre has a strength S_i, i = 1...N, sorted in ascending order, the stress σ^i prior to i-th fibre failure is equal to

$$\sigma^i = \frac{V_f}{N}(N - i + 1)S_i \tag{3}$$

The global maximum of the series of stresses σ^i defines the stress prior to the catastrophic failure of the bundle, in other words the fibre bundle ultimate strength.

2.3 Young's modulus

As well as the fibre strength the Young's modulus of a fibre in the longitudinal direction also varies. The Young's modulus, E_{bundle}, of the fibre bundle consisting of N fibres and a fibre volume fraction V_f, can be found using the rule of mixtures as follows:

$$E_{bundle} = \frac{V_f}{N}\sum_{i=1}^{N} E_f^i + (1 - V_f)E_m \tag{4}$$

where E_f^i is the Young's modulus of i-th fibre and E_m is the Young's modulus of matrix.

It was assumed that a single fibre's Young's modulus follows a normal distribution for which the probability density function, f, is equal to:

$$f(E) = \frac{1}{\sqrt{2\pi s^2}} e^{-\frac{(E-E_1)^2}{2s^2}} \tag{5}$$

where E_1 is a mean value of Young's modulus and s is a standard deviation.

3 Finite element model of composite with fibre strength variability

3.1 Validation on UD test

To validate the chosen fibre strength and Young's modulus variability models FE studies of a UD composite consisting of AS4 fibres with fibre volume fraction Vf = 59% were conducted. Published experimental results [6] were used to validate the presented model. Actual sizes of composite tensile specimen are 152 x 12.5 x 1.8 mm. Eight node linear cuboid elements C3D8R were used to discretize the model. The model consists of two zones, damageable and undamagable. The latter were placed on the ends and boundary conditions were applied as shown in Figure 1.

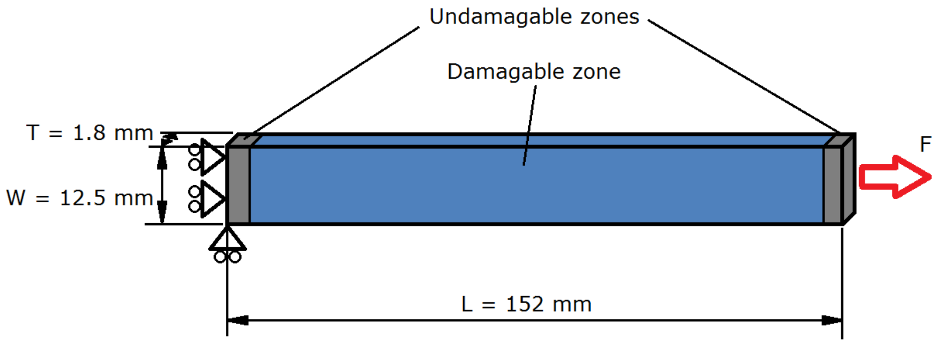

Figure 1. Model of the tensile test on UD composite

The number of fibres in each element was found and fibre strengths were assigned following equation (4) using the length of an element as input parameter L. The strength of the fibre bundle or element was found using equation (5). The other properties of UD composite are found using Chamis micromechanical formulae [11]. Published experimental results were used as the input data for the strength bundle model. AS4 carbon fibre parameters that were used in the model are following: L_0 = 12.5 mm, $\bar{\sigma}_0$ = 4275 MPa, ρ = 10.3, m = 8, E_1 = 234 GPa [6]. The standard deviation s of fibre Young's modulus from the mean value was assumed to be 11.7 GPa. The matrix Young's modulus is 2.7 GPa and strength is 69 MPa. The maximum stress criterion and abrupt degradation scheme was used to model damage in the composite [12].

Mesh convergence studies of the model with fibre strength variability showed that the overall strength converged with mesh refinement, as shown in Figure 2. Each point is an average of 20 calculations and the error bars represent one standard deviation. The optimal mesh size of 6×10^5 elements was chosen.

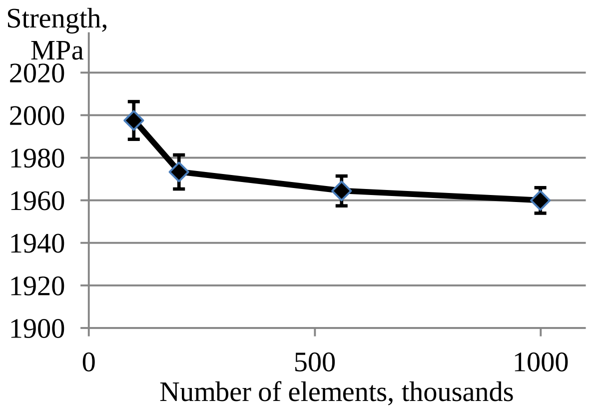

Figure 2. Convergence of FE solution with mesh refinement

Tensile strength of AS4 fibres is 4433 MPa according to the data sheet [13], which results in a nominal strength of 2615 MPa for a UD composite with fibre volume fraction 59% following the rule of mixtures. The predicted UD composite strength is 1960±7 MPa while the experimental value is 1890 MPa and Curtin's analytical prediction is 1990 MPa [6].

Next, variability of Young's modulus according to equation (5) was implemented into the FE model. Young's modulus of each element, each consisting of N fibres, was calculated with equation (4). Stochastic FE analysis showed that the Young's modulus of the UD composite does not vary significantly. Its value is 138±2 GPa. The final strength is 1958±10 MPa.
Mesh convergence of the model and good agreement of predicted and experimental results allows to use this model in further predictions for textile composite strength. The simulations showed that fibre strength variability has a significant effect on the overall strength of the UD composite while the effect of variability in fibre Young's modulus is negligible.

3.2 Strength of textile composite without variability of fibre strength

A plain weave textile composite was chosen to investigate the influence of fibre strength on overall strength. Using the TexGen geometric modelling software [14] and voxel meshing technique [15, 16] an FE model of the textile composite was generated. The unit cell of a plain weave textile composite AS4/Vinylester is shown in Figure 3. The values of its geometric parameters used in this work are as follows: L = 6.27 mm, H = 0.624 mm, h_y = 0.31 mm, w_y = 2.81 mm [17]. The fibre volume fraction is V_f = 0.42 [17]. In the original studies the lengths of the unit cell and yarn widths in the warp and weft directions are slightly different but it was assumed that the difference is not significant. Constituent properties are listed in Table 1. Yarn effective moduli and strengths were found with the Chamis micromechanical formulae [11].

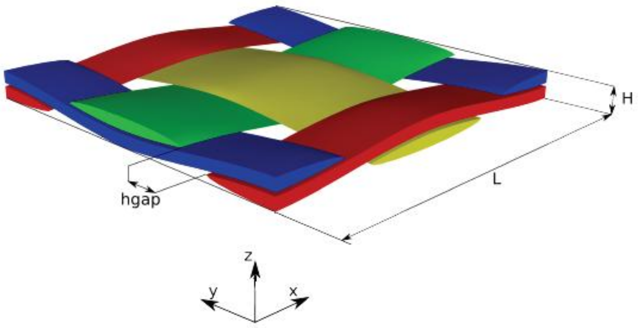

Figure 3. The unit cell of plain weave textile composite with removed matrix

Fibre properties (AS4)					Matrix properties (Vinylester)		
E_1, MPa	E_2, MPa	ν_{12}	ν_{23}	S_f, MPa	E_m, MPa	ν_m	S_m, MPa
221.0	16.60	0.26	0.30	3930.0	3.45	0.35	76.0

Table 1. Constituent properties

As the unit cell is an idealized representation of a textile composite and it has no nesting [17], periodic boundary conditions [18] were applied to the unit cell in all three directions. A continuum damage mechanics (CDM) approach has shown good agreement between experimental and predicted mechanical properties of damage of textile composites [19] therefore it was chosen to model the damage in the yarns and matrix [20]. The unit cell was discretized with voxel elements.

To validate the voxel approach a conventional (conformal) tetrahedral mesh was generated for the same unit cell. A deliberate gap between the yarns in the vertical direction was introduced to achieve good quality of the tetrahedral mesh. It reduced the Young's modulus and overall properties but allowed comparison between the two meshes. Convergence of the elastic modulus and initial failure strain are shown in Figure 4. It was shown that the voxel mesh can be used instead of a conformal mesh. Moreover, the voxel mesh approach allows the creation of a mesh of the textile composite geometry without any artificial gap between the yarns. However, it requires more elements to achieve sufficiently converged results.

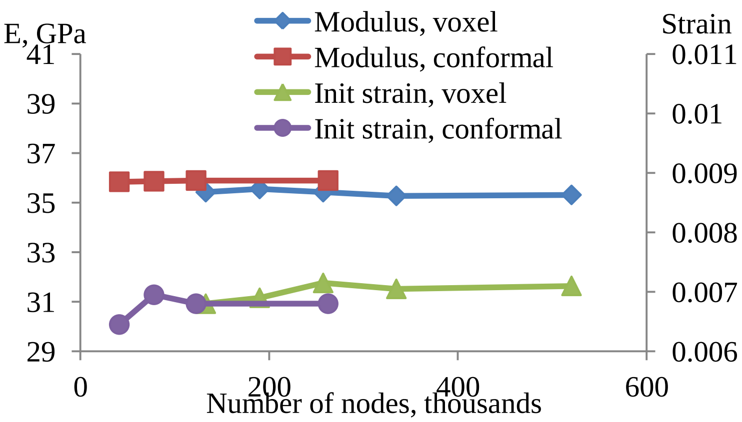

Figure 4. Convergence of voxel and tetrahedral meshes

Experimental and predicted stress-srain curves for plain weave textile under tensile loading are shown in Figure 5. It can be seen that predicted final strength 534 MPa is overpredicted compare to experimental value in 480 MPa but predicted stress-strain curve and experimental results are in good agreement.

3.2 Effect of strength variability on textile composites

The influence of fibre strength variability was studied on the unit cell model of the textile composite. The strength of each voxel was chosen according to the approach described above with the same parameters. Young's modulus variability was neglected due to its minor effect in the UD composite study. Stochastic simulations were performed on the same mesh as for the model without variability. Results of stochastic simulations are shown in Figure 6.

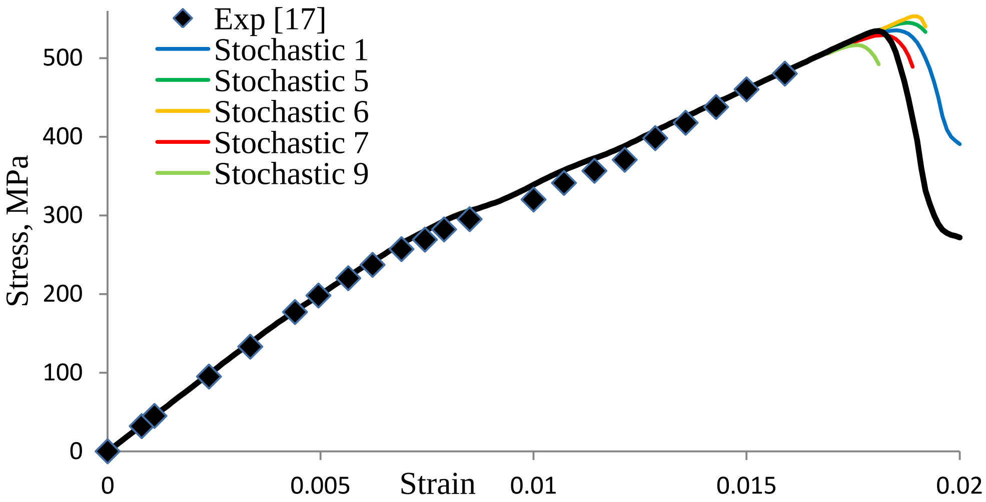

Figure 5. Experimental and predicted stress-strain curves for composite with and without variability

The final strength of the composite with fibre strength variability is lower than the composite without it due to the presence of weak fibres and hence the presences of elements with lower strength. The damage is more likely to initiate at the elements with lower strength and propagate from these elements. The longitudinal stresses in the longitudinal yarns of the textiles with and without variability at strain level 0.0174 are shown in Figure 6. It can be seen that damage has already occurred in the composite with variability while the longitudinal yarns in composite without variability are intact until the strain level reaches 0.0178.

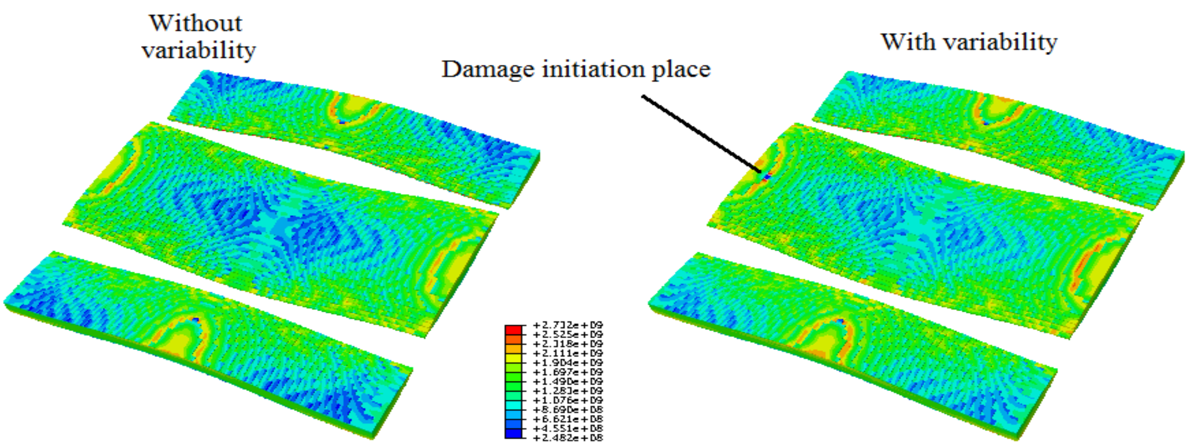

Figure 6. Longitudinal stresses in the longitudinal yarns at strain level 0.0174, Pa

The damage states of matrix of the textile composites with and without variability at strain level 0.0178 are shown in Figure 7. Both examples of textiles with variability exhibit matrix damage while the textile without variability does not exhibit such damage on the surface of the unit cell.

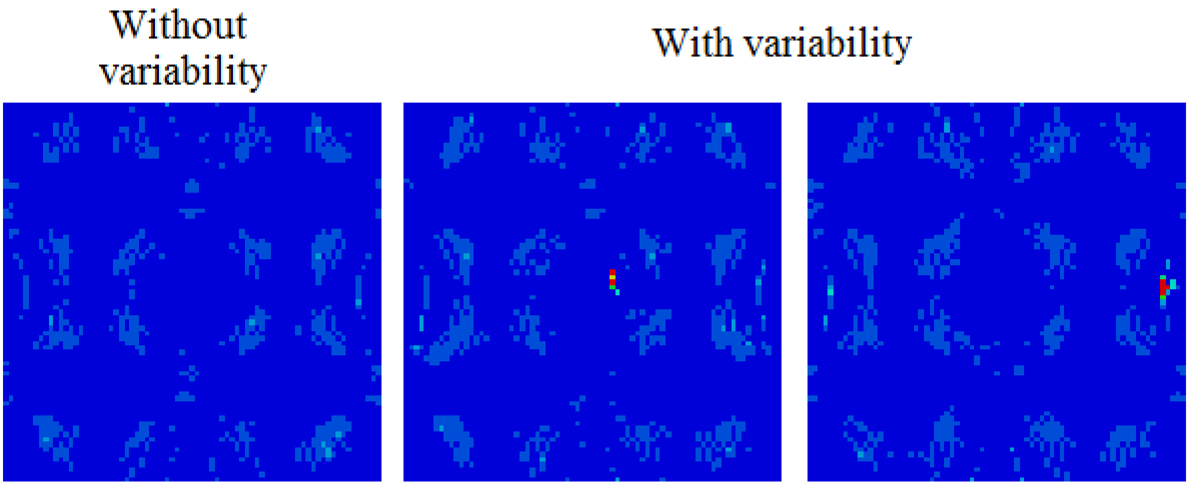

Figure 7. The damage state of the matrix at strain level 0.0178, red colour shows fully damaged material

The final strength and strain at failure of the composite without the variability is 534MPa and 0.0181, respectively. The corresponding values are 525±25 MPa and 0.0183±0.0004 for the composite with the fibre strength variability. It is still higher than the experimental values that are 480 MPa and 0.0159.

4 Conclusions

A fibre strength model was implemented and validated for a UD composite. Stochastic simulations showed that the model gives reasonable results and can be used for textile composites. The importance of fibre strength variability was identified. It was shown that the effect Young's modulus variability on the strength and modulus of UD composites is negligible.

The FE model of the unit cell of a textile composite was generated automatically using the TexGen software. The CDM approach was successfully used to predict the strength without any variability under tensile loading. The fibre strength model was implemented in the unit cell model of the plain weave textile composite. Stochastic studies of the textile composite with variability in fibres strength showed that this variability has significant influence. The mean final strength of the composite with variability is lower than the composite without it. However, the predicted final strength value is higher than the experimental value. One possible reason is the scale effect which takes place in the UD composites and should appear in textile composites as well. Further investigations on a domain larger than one unit cell are required.

References

[1] Sriramula S., Chryssanthopoulos M.K. Quantification of uncertainty modelling in stochastic analysis of FRP composites. *Composites Part A – Applied Science and Manufacturing*, **40(11)**, pp. 1673-1684 (2009)

[2] Yushanov S.P., Bogdanovich A.E. Fiber waviness in textile composites and its stochastic modeling. *Mechanics of Composite Materials*, **36(4)**, pp. 297-318 (2000)

[3] Endruweit A., Long A.C. Influence of stochastic variations in the fibre spacing on the permeability of bi-directional textile fabrics. *Composites Part A - Applied Science and Manufacturing*, **37(5)**, pp. 679-694 (2006)
[4] Pickering K.L., Murray T.L. Weak link scaling analysis of high-strength carbon fibre. *Composites Part A - Applied Science and Manufacturing*, **30(8)**, pp. 1017-1021 (1999)
[5] Andersons J., Joffe R., Hojo M., Ochiai S. Glass fibre strength distribution determined by common experimental methods. *Composite Science and Tech.*, **62(1)**, pp. 131-145 (2002).
[6] Curtin W.A. Tensile strength of fiber-reinforced composites: III. Beyond the traditional Weibull model for fiber strengths. *J Composite Materials*, **34(15)**, pp. 1301-1332, (2000)
[7] Newman W.I., Phoenix S.L. Time-dependent fiber bundles with local load sharing. *Physical Review E*, **63(2)** (2001)
[8] Smith R.L. A Probability Model for Fibrous Composites with Local Load Sharing. *Proceedings of Royal Society A*, **372(1751)**, pp. 539-553 (1980)
[9] Okabe T., Ishii K., Nishikawa M., Takeda N. Prediction of Tensile Strength of Unidirectional CFRP Composites. *Advanced Comp. Materials*, **19(3)**, pp. 229-241 (2010)
[10] Nilakantan G., Keefe M., Wetzel E.D., Bogetti T.A., Gillespie J.W. Effect of statistical yarn tensile strength on the probabilistic impact response of woven fabrics. *Composite Science and Technology*, **72(2)**, pp. 320-329 (2012)
[11] Murthy P.L.N., Chamis C.C. *ICAN Users and Programmers Manual*. NASA (1986)
[12] Blackketter D.M., Walrath D.E., Hansen A.C. Modeling Damage in a Plain Weave Fabric-Reinforced Composite-Material. *Comp. Tech. and Res.*, **15(2)**, pp. 136-142 (1993)
[13] HexTow AS4 Carbon Fibre http://www.hexcel.com/resources/datasheets/carbon-fiber-data-sheets/as4.pdf
[14] Sherburn M. *Geometric and Mechanical Modelling of Textiles*. PhD thesis. University of Nottingham, School of M3 (2007)
[15] Crookston J.J., Ruijter W., Long A.C., Warrior N.A., Jones I.A. *A comparison of mechanical property prediction techniques using conformal tetrahedral and voxel-based finite element meshes for textile composite unit cells* in "Proceeding of *22nd Boundary and Finite Element Methods Conference*", Saint-Petersburg, Russia (2007)
[16] Mishnaevsky L.L. Automatic voxel-based generation of 3D microstructural FE models and its application to the damage analysis of composites. *Material Science and Engineering A – Structures*, **407(1-2)**, pp. 11-23 (2005)
[17] Ito M., Chou T.W. An analytical and experimental study of strength and failure behavior of plain weave composites. *Journal of Composite Materials*, **32(1)**, pp. 2-30 (1998)
[18] Li S.G., Wongsto A. Unit cells for micromechanical analyses of particle-reinforced composites. *Mechanics of Materials*, **36(7)**, pp. 543-572 (2004)
[19] Zako M., Uetsuji Y., Kurashiki T. Finite element analysis of damaged woven fabric composite materials. *Composite Science and Technology*, **63(3-4)**, pp. 507-516 (2003)
[20] Ruijter W. Analysis *of mechanical properties of woven textile composite as functions of textile geometry*, PhD thesis. University of Nottingham, School of M3 (2008)